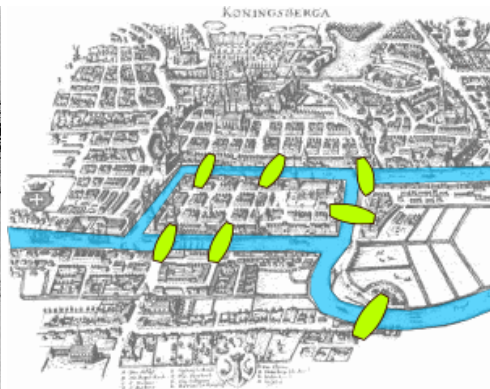

Friday, August 29

1. This problem is about the Pythagorean theorem.
 - (a) Consider an *isosceles* right triangle \triangle with side lengths a and a and hypotenuse length c .
 - i. Let \square be a square of side length c . Show how to divide \square into 4 copies of \triangle .
 - ii. Show that, in this case,
$$a^2 + a^2 = c^2.$$
 - (b) Consider a right triangle \triangle with side lengths a and b and hypotenuse length c ; assume that $b > a$.
 - i. Let \square be a square of side length c . Show how to divide \square into 4 copies of \triangle and a square of side length $b - a$.
 - ii. Show that, in this case,
$$a^2 + b^2 = c^2.$$
2. Inscribe a circle of radius 1 in a regular hexagon.
 - (a) What is the area of an equilateral triangle whose perpendicular bisector has length 1?
 - (b) Use this to find an upper bound for π . Explain your reasoning.
3. Inscribe a regular hexagon inside a circle of radius 1.
 - (a) What is the area of an equilateral triangle whose sides have length 1?
 - (b) Use this to find a lower bound for π . Explain your reasoning.
4. The Pregel river through Königsberg, Prussia* divided the city into two main regions and two islands. These four regions were connected by seven bridges.



Is it possible to take a walk which crosses each bridge exactly once? Explain.

*Now Kaliningrad, Russia