Monday, August 25

1. For a natural number $n \geq 2$, draw n distinct points on a circle; draw a line segment connecting each pair of points; and let $R(n)$ be the resulting number of regions in the disk.
   (a) Compute $R(n)$ for $n \in \{2, 3, 4, 5\}$.
   (b) Conjecture a formula for $R(n)$ in general.
   (c) Check your conjecture for $n = 6$.

2. For each natural number n, let
$$M(n) = 2^n - 1.$$
   (a) Compute $M(n)$ for $n \in \{2, 3, 4, 5, 6, 7, 8\}$.
   (b) Conjecture a relationship between the primality of n and the primality of $M(n)$:

   If n is prime, then $M(n)$ is ______________.

   while

   If n is not prime, then $M(n)$ is ______________.

   (c) In the part (b), is it possible for just one of these statements to be true? Explain.
   (d) Prove one of the conjectures from (b).

3. The Fibonacci numbers F_n are defined by
$$\begin{aligned} F_0 &= 0 \\ F_1 &= 1 \\ F_n &= F_{n-2} + F_{n-1} \text{ if } n \geq 2. \end{aligned}$$
   (a) Compute F_n for $n \in \{1, 2, \cdots, 10\}$.
   (b) For each natural number n, let
$$\begin{aligned} S_n &= \sum_{j=1}^{n} F_j \\ &= F_1 + F_2 + \cdots + F_n \end{aligned}$$
   Compute S_n for $n \in \{1, 2, \cdots, 10\}$.
   (c) Give a conjectural formula for S_n in terms of the Fibonacci numbers.