Homework 2

Due: ~~Monday, September 22~~ Friday, September 26

1. Use induction to prove that, for each positive integer n,

$$\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}.$$

2. As in HW1#1, for each natural positive integer n, draw n generic lines, and let $R(n)$ be the resulting number of regions in the plane.

   Let $P(n)$ be the assertion that

$$R(n) = \frac{(n^2+n+2)}{2}.$$

   We will show that $P(n)$ holds for all n; we have already checked it for $1 \le n \le 4$.

   (a) Suppose you've already drawn n lines, and then draw one more.
   The n lines divide the last line up into pieces. How many pieces?

   (b) How many new regions were created by drawing the last line?

   (c) Suppose that $P(n)$ is true. Use parts (a) and (b) to prove that $P(n+1)$ is true.

   (d) Conclude that $P(n)$ is true for all $n \ge 1$, by induction.

3. Recall the Fibonacci numbers F_n from HW1#3. Use induction to show that, for each $n \ge 1$,

$$\sum_{j=1}^{N} F_j^2 = F_N F_{N+1}.$$

4. Let α be a positive real number. Use induction to show that, for each $n \ge 1$,

$$(1+\alpha)^n \ge 1 + n\alpha.$$