

# MPE Review

## Section I: Algebra

At Colorado State University, the College Algebra sequence extensively uses the graphing features of the Texas Instruments TI-83 or TI-84 graphing calculator. Whenever possible, the questions on the Math Placement Exam and in this review have been designed to accommodate the student who has no knowledge of these graphing features. However, students may find it helpful to learn about these features. Texas Instruments maintains a list of TI-83/84 tutorial websites at <http://education.ti.com/educationportal/sites/US/sectionHome/tutorials.html>.

### POLYNOMIAL MANIPULATION

Polynomial expressions contain letters that symbolize numbers and obey the usual laws of arithmetic. In adding, subtracting or multiplying polynomials, we apply these laws and combine like terms.

**Example:** Simplify the polynomial expression  $(x - 3)(2x + 5) - 6x(x - 1)$ .

*Solution:*  $(x - 3)(2x + 5) - 6x(x - 1) = x(2x + 5) - 3(2x + 5) - 6x(x - 1)$

$$= 2x^2 + 5x - 6x - 15 - 6x^2 + 6x$$

$$= -4x^2 + 5x - 15$$

#### Problems:

1. Simplify:  $(3x - 7)(2x - 5) - x(4x - 3)$
2. Simplify:  $(7x + 2)^2 - 5x[-3(x + 2)]$
3. Simplify:  $(2x^2 - 5x + 3)(3x^2 + x - 2)$
4. Simplify:  $2(x + 3)^3 - (x - 1)(x + 1)$

### SOLVING ONE LINEAR EQUATION IN ONE UNKNOWN

An equation (a statement equating two expressions) can be manipulated by adding (or subtracting) the same quantity from both sides, or by multiplying (or dividing) both sides by the same non-zero quantity. To solve a simple linear equation we manipulate the equation into the form

$$Ax = B,$$

then divide by  $A$  to conclude that

$$x = \frac{B}{A}$$

**Example:** Solve for  $x$  in the following equation:  $3(x - 2) - x = -5x - 4(2 - x)$ .

*Solution:* First perform all indicated multiplications:  $3x - 6 - x = -5x - 8 + 4x$ .

Next simplify each side separately:  $2x - 6 = -x - 8$ .

Now isolate all  $x$  terms on one side of the equation and all constant terms on the other by adding  $x$  to both sides and adding 6 to both sides

$$3x = -2.$$

Divide by 3 to obtain the solution

$$x = -\frac{2}{3}.$$

**Problems:**

5. Solve for  $x$ :  $2(x + 4) = 5x - 12$   
 6. Solve for  $u$ :  $-6[u - 4(u + 2)] = 5u - 4$   
 7. Solve for  $R$ :  $2R - 2(R + 3) = 4[2R - 3(1 + R)]$

**MATHEMATICAL MODELS**

A mathematical model is a formula or algebraic equation formed to model real-world situations.

To model a real-world situation:

- Read the problem carefully. What information is given to you? What is the problem looking for? Let  $x$  (or any other variable) represent one of the quantities of the problem.
- Write an expression with the information given.
- Evaluate the expression.
- Reread the problem. Does your answer make sense with the context of the problem?

**Example:** The length of the rectangular tennis court is 6 feet longer than twice the width. If the court's perimeter is 228 feet, what are the courts dimensions?

*Solution:* Let  $x$  = the width of the court. The length must be  $= 6 + 2x$ . We are given that the perimeter = 228. The formula for perimeter is:  $2W + 2L = P$ .

$$\begin{array}{rcl} 2(x) + 2(6 + 2x) = 228 & \text{Since } x = 36, & \\ 2x + 12 + 4x = 228 & \text{length} = 6 + 2(36) & \\ 6x = 216 & \text{length} = 78 & \\ x = 36 & & \end{array}$$

The dimensions of the tennis court are 36 ft. by 78 ft.

**Problems:**

8. For large parties, many restaurants automatically add an 18% gratuity to the bill. If the total bill, including gratuity is \$163.43, what was the bill before adding the gratuity?  
 9. A job pays an annual salary of \$44,550, which includes a year-end bonus of \$750. If paychecks are issued twice a month, what is the gross amount of each paycheck?

**INEQUALITIES/ABSOLUTE VALUES**

Manipulate inequalities in the same way as equalities with one notable exception. The direction of the inequality changes when multiplying or dividing the inequality by a negative number. Such is the case in the following:

$$8 > 3 \text{ so } (-2)(8) < (-2)(3), \text{ since } -16 < -6.$$

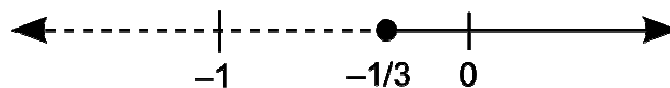
**Example:** Simplify the following:  $2x + 5 \geq -x + 4$ .

*Solution:* Simplify the following:  $2x + 5 \geq -x + 4$ .

Add  $x$  to both sides:  $3x + 5 \geq 4$ .

Subtract 5 from both sides,  $3x \geq -1$ , so that  $x \geq -\frac{1}{3}$ .

Note that we divided by a positive number, therefore, no change in the inequality sign occurs. The solution of an inequality is a set of numbers and can be graphed on the number line. In the above example, the solutions are graphed as the (closed) half-line to the right of  $-\frac{1}{3}$  (inclusive).



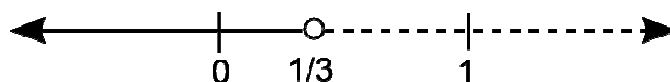
**Example:** Simplify the following:  $-2x + 5 > x + 4$ .

**Solution:** Subtract  $x$  from both sides:  $-3x + 5 > 4$ .

Subtract 5 from both sides:  $-3x > -1$ .

Then  $x < \frac{1}{3}$ .

Note that we divided by a negative number; therefore, a sign change occurs in our inequality. In this example, all  $x < \frac{1}{3}$  is graphed as the (open) half-line to the left of  $\frac{1}{3}$  excluding  $\frac{1}{3}$ .



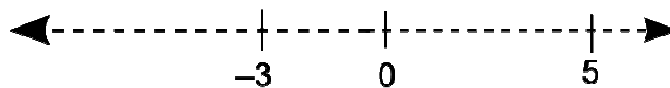
**Problems:**

- 10. Find the solution set of  $5(x - 3) + 1 > -4(x - 1)$ .
- 11. Find the solution set of  $-3x + 2 \leq 5(2x + 3)$ .
- 12. Find the solution set of  $\frac{5}{7}(x - 3) \geq 4x + \frac{22}{7}$ .
- 13. Find the solution set of  $(x + 2)(2x - 3) < 2(x + 1)(x + 5)$ .

The absolute value of a number  $x$  is defined as

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Hence,  $|5| = 5$  and  $|-3| = 3$ . The absolute value of  $x$  can be visualized on a number line as the distance the point  $x$  lies from the origin. Graphically we see that  $-3$  is three units from the origin and  $5$  is five units from the origin.

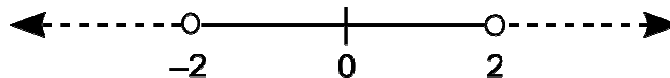


**Example:** Find the set of  $x$ -values satisfying  $|x| < 2$ .

**Solution:** When  $x \geq 0$ ,  $|x| = x$  and the inequality becomes  $x < 2$ . Thus, the non-negative  $x$ -values that satisfy  $|x| < 2$  are given by  $0 \leq x < 2$ .

When  $x < 0$ ,  $|x| = -x$  and the inequality becomes  $-x < 2$ , or  $-2 < x$ . Thus, the negative  $x$ -values that satisfy  $|x| < 2$  are given by  $-2 < x < 0$ .

It follows that the set of all  $x$ -values satisfying  $|x| < 2$  consists of all numbers  $x$  strictly between  $-2$  and  $2$  (excluding  $-2$  and  $2$ ). The set of  $x$ -values satisfying  $|x| < 2$ , is the collection of numbers within two units of the origin on the number line.



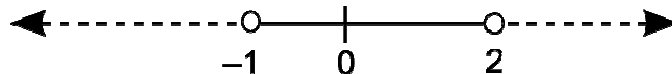
**Example:** Find the solution set of  $|2x - 1| < 3$ .

**Solution:** Any solution of  $|2x - 1| < 3$  must satisfy both of the inequalities  $2x - 1 < 3$  and

$-3 < 2x - 1$ . Write these two inequalities together as

$$-3 < 2x - 1 < 3$$

and then solved at the same time. Add 1 to all members of the inequality to get  $-2 < 2x < 4$ . Divide by 2 to find that the solution set is the interval  $-1 < x < 2$  of all  $x$ -values strictly between  $-1$  and  $2$ .



**Problems:**

14. Find the solution set of  $|6 - 2x| \leq 3$ .

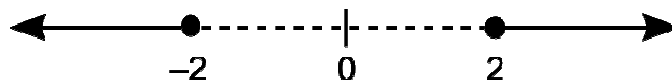
15. Find the solution set of  $\left| \frac{1}{2}x - 3 \right| \leq 4$ .

**Example:** Find the set of  $x$ -values satisfying  $|x| \geq 2$ .

*Solution:* When  $x \geq 0$ ,  $|x| = x$  and the inequality becomes  $x \geq 2$ . Thus, the non-negative  $x$ -values that satisfy  $|x| \geq 2$  are given by  $x \geq 2$ .

When  $x < 0$ ,  $|x| = -x$  and the inequality becomes  $-x \geq 2$ , or  $x \leq -2$ . Thus, the negative  $x$ -values that satisfy  $|x| \geq 2$  are given by  $x \leq -2$ .

Thus, the set of all  $x$ -values satisfying  $|x| \geq 2$  is comprised of two disjoint sets. One set is the real numbers  $x$  greater than or equal to 2. The other set is the real numbers  $x$  less than or equal to  $-2$ . The set of  $x$ -values satisfying  $|x| \geq 2$  consists of the numbers two or more units from the origin on the number line.



**Example:** Find the solution set of  $|2x - 1| \geq 3$ .

*Solution:* Any solution of  $|2x - 1| \geq 3$  must satisfy either the inequality  $2x - 1 \geq 3$  or the inequality  $2x - 1 \leq -3$ . These two inequalities cannot be written together and should not be solved at the same time.

$$\begin{aligned} \text{When } 2x - 1 &\geq 3, \\ 2x &\geq 4, \\ \text{so } x &\geq 2 \end{aligned}$$

$$\begin{aligned} \text{When } 2x - 1 &\leq -3, \\ 2x &\leq -2, \\ \text{so } x &\leq -1. \end{aligned}$$

The solution set of  $|2x - 1| \geq 3$  is comprised of two pieces. One piece is the real numbers  $x$  greater than or equal to 2. The other is the real numbers  $x$  less than or equal to  $-1$ .

Notice that the solution set for  $|2x - 1| < 3$  is an interval and the solution set for  $|2x - 1| \geq 3$  is the complement of the same interval.

**Problems:**

16. Find the solution set of  $|5 - 2x| \geq 3$ .

17. Find the solution set of  $\left| 7 - \frac{2}{3}x \right| > 2$ .

## GRAPHING ON THE PLANE

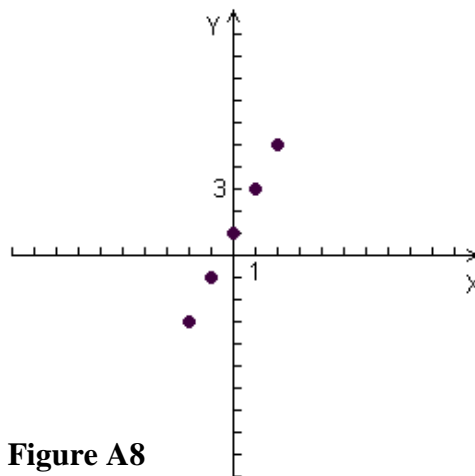
The graph of any equation  $y = mx + b$ , where  $m$  and  $b$  are specific numbers, is a straight line. Equations of this form are called linear equations.

**Example:** Find some points on the graph of  $y = 2x + 1$ .

**Solution:** First compute a table of values which satisfy the given equation.

$x$	-2	-1	0	1	2
$y$	-3	-1	1	3	5

Plotting these pairs gives the graph shown in Figure A8.



**Figure A8**

The graph of  $y = 2x + 1$  is the straight line through these points.

### Problems:

18. Sketch the graph of  $y = x^2 + 1$ .

19. Sketch the graph of  $y = \frac{1}{1+x}$ . (Consider  $x = -1$ .)

20. Sketch the graph of  $y = \frac{x^2 - 1}{1+x}$ . (Consider  $x = -1$ .)

21. Sketch the graph of  $y = x^2 - 2x + 1$ .

22. Sketch the graph of  $x = -y^2 + 2y - 1$ .

## LINES IN THE PLANE

In plane geometry, you learn that two distinct points lie on exactly one straight line. Its slope characterizes the direction of a line. Given any two points on a line, define the slope of the line is by

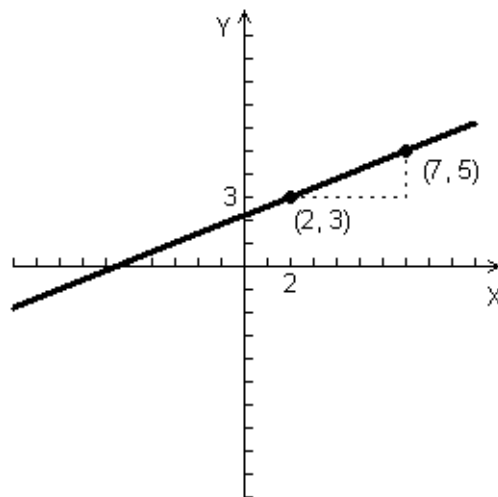
$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$$

According to this definition, the slope of a horizontal line is zero (0) and the slope of a vertical line is undefined.

**Example:** Consider the line joining (2, 3) and (7, 5). The slope of this line is

$$\frac{5-3}{7-2} = \frac{2}{5}$$

Graphically we have Figure A9.



**Figure A9**

### Problems:

23. What is the slope of the line joining (3, 5) and (7, 3)?

24. What is the slope of the line joining (2, 3) and  $(2+h, 3+4h+h^2)$ ?

Through a given point  $P = (x, y)$ , there is exactly one line with slope  $m$ . If the coordinates of  $P$  and the slope  $m$  are known, we can easily find an equation relating the  $x$ - and  $y$ -coordinates of any point  $(x, y)$

on the line. In general, the line through  $P = (x_0, y_0)$  with slope  $m$  consists of all points whose coordinates  $(x, y)$  satisfy

$$m = \frac{y - y_0}{x - x_0}$$

which can be rewritten as  $y - y_0 = m(x - x_0)$ .

This is the **point-slope** form of an equation of a line. It displays the coordinates of one point on the line and the slope  $m$  of the line.

**Example:** An equation of a line is  $y - 3 = 2(x + 4)$ . The line then passes through the point  $(x_0, y_0) = (-4, 3)$  and has slope of  $m = 2$ .

**Problems:**

25. Find an equation of the line through  $(-3, -4)$  with slope of  $\frac{5}{3}$ .

26. Find the slope and the coordinates of a point on the line whose equation is  $y - 2 = \frac{1}{3}(x + 5)$ .

27. Find an equation of the line through  $(6, 2)$  with slope 0.

28. Find an equation of the line through  $(5, 3)$  and  $(5, -11)$ .

Now we consider another useful form of an equation of a line, the **slope-intercept** form.

**Example:** If the point-slope form of an equation of a line is  $y - 3 = 2(x - 4)$ , then we can rewrite the equation as

$$y = 2x - 8 + 3 \text{ or } y = 2x - 5.$$

In this form one can easily tell that the point  $(0, -5)$  is on the line and that the slope, given by the coefficient of  $x$ , is 2. More generally the slope-intercept form is

$$y = mx + b,$$

where  $m$  is the slope and  $(0, b)$  is the point where the line intercepts the  $y$ -axis.

**Problems:**

29. Write in slope-intercept form an equation for the line through  $(5, 1)$  with slope  $-2$ .

30. Write  $3x - 7y = 4$  in the slope-intercept form.

31. What is the slope of the line  $2x + 5y = 8$ ? What is the  $y$ -intercept?

**Example:** The following three points lie on the same line:  $(-2, -3.8)$ ,  $(3, y_1)$ , and  $(5, -1)$ . What is the value of  $y_1$ ?

*Solution:* First, find the slope of the line using the two points  $(-2, -3.8)$  and  $(5, -1)$ .

$$m = \frac{-1 - (-3.8)}{5 - (-2)} = \frac{2.8}{7} = 0.4$$

Find the equation of the line using the slope and either one of the given points.

$$y - (-1) = 0.4(x - 5)$$

$$y + 1 = 0.4x - 2$$

$$y = 0.4x - 3$$

Now we can use this equation to find the value of  $y_1$ .

$$y_1 = 0.4(3) - 3 = -1.8$$

**Problems:**

32. The following three points lie on the same line:  $(-4, 4)$ ,  $(2, y_1)$ , and  $(6, 1)$ . What is the value of  $y_1$ ?
33. The following three points lie on the same line:  $(-5, 1.5)$ ,  $(x_1, -2.7)$ , and  $(1, -11.1)$ . What is the value of  $x_1$ ?

Another way to think of the slope of a line is the **average rate of change**. The average rate of change over an interval is the change in  $y$  across the interval divided by the change in  $x$  across the same interval.

**Example:** The table below could be a mathematical model for some situation. What is the average rate of change over the interval from 6 to 12?

<b>x</b>	0	3	6	9	12	15	18
<b>y</b>	19	17.75	17	15.5	13.5	10	0

*Solution:* 
$$\frac{\text{change in } y}{\text{change in } x} = \frac{13.5 - 17}{12 - 6} = \frac{-3.5}{6} = -\frac{7}{12}$$

**Problems:** Use the table above to answer the following problems.

34. What is the average rate of change over the interval from 0 to 3?
35. What is the average rate of change over the interval from 6 to 9?
36. What is the average rate of change over the interval from 3 to 15?

**SYSTEMS OF LINEAR EQUATIONS**

Review your reference text for a variety of techniques for solving a system of two linear equations in two unknowns and for solving three linear equations in three unknowns.

**Example:** Consider the two equations

$$x - y = 2$$

and

$$3x - y = 4.$$

The associated lines have different slopes (1 and 3) and therefore intersect at a single point. The coordinates of this point give the solution to this pair of equations.

**Example:** Now consider the two equations

$$x - 2y = 4$$

and

$$3x - 6y = 10.$$

The associated lines have the same slope,  $\frac{1}{2}$ , but different  $y$ -intercepts,  $-2$  and  $-\frac{5}{3}$ , respectively. Hence, these lines are parallel and do not intersect. Since the lines have no point in common, this pair of equations has no solution.

**Example:** Finally consider the equations

$$x - 2y = 4$$

and

$$2x - 4y = 8.$$

These two equations not only have the same slope, but they represent the same line. In this case, there are infinitely many solutions, one corresponding to each point  $(x, y)$  on the line.

**Problems:**

37. Solve for  $x$  and  $y$ :  $3x - 2y = 1$   
 $5x - 3y = 3.$

38. Solve for  $x$ ,  $y$ , and  $z$ :

$$\begin{aligned} x + 4y + 3z &= 0 \\ 5x + 23y + 13z &= -3 \\ 2x - 7y - 3z &= 15. \end{aligned}$$

39. In one week, Fearless Frank drove 100 miles on the freeway and 150 miles in town. His car consumed 18.2 gallons of gasoline. The next week his journeys took him over 180 miles of freeway and 30 miles of local driving, and he used only 10.9 gallons of gas. What were his highway m.p.g. and his local m.p.g.?

## DISTANCE FORMULA

To compute the distance,  $d$ , between an arbitrary pair of points, use the distance formula. Consider a point  $P$  with coordinates  $(x_1, y_1)$  and a point  $Q$  with coordinates  $(x_2, y_2)$ .

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

**Example:** Find the distance between  $(-1, 3)$  and  $(4, -9)$ . Let  $P = (-1, 3)$  and  $Q = (4, -9)$ .

*Solution:*

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[4 - (-1)]^2 + (-9 - 3)^2} \\ &= \sqrt{(5)^2 + (-12)^2} \\ &= \sqrt{25 + 144} = \sqrt{169} = 13 \end{aligned}$$

### Problems:

40. Determine the distance between  $(4, 2)$  and  $(-\frac{3}{2}, 2)$ .  
 41. Determine the distance between  $(4, -4)$  and  $(-4, 4)$ .  
 42. Find the length of the diagonal of a square of area 16.

## EQUATION OF A CIRCLE

The standard form of the equation of a circle with center  $(h, k)$  and radius  $r$  is

$$(x - h)^2 + (y - k)^2 = r^2.$$

**Example:** What is the equation for the circle in Figure A10?

*Solution:* The center of the circle in Figure A10 is the point  $(4, -4)$ , the radius is 5. The equation of this circle is:

$$(x - 4)^2 + (y + 4)^2 = 25.$$

### Problems:

43. What is the equation for the circle in Figure A11?  
 44. Graph:  $(x - 1)^2 + (y - 4)^2 = 9$   
 45. Graph:  $(x + 3)^2 + (y - 5)^2 = 2.25$

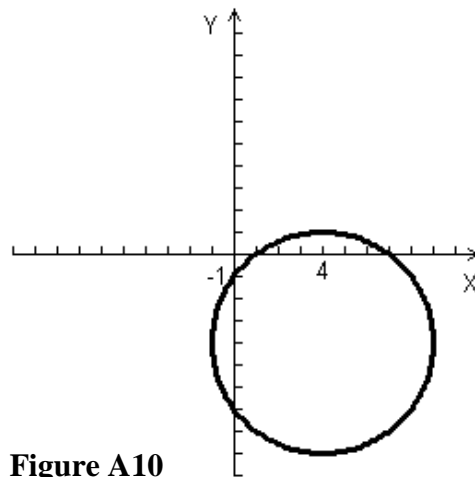


Figure A10

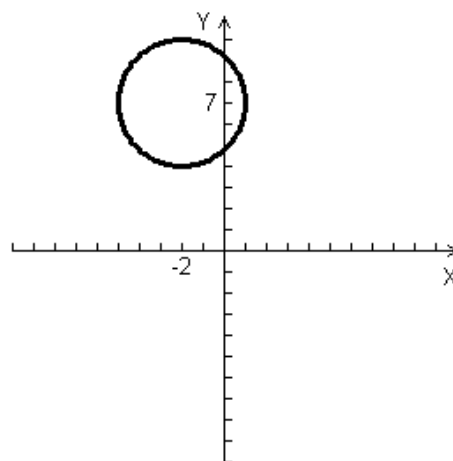


Figure A11



## PIECEWISE FUNCTIONS

A piecewise function is a function that is defined by two (or more) equations over a specific domain. The absolute value of a number  $x$  is defined as a piecewise function:

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

**Example:** Emma's cellular phone plan costs \$45 each month and includes 400 free minutes of call time. Emma has to pay an extra 25 cents for each minute she uses over 400. The following piecewise function represents how much her phone bill,  $y$ , is in dollars depending on how many minutes,  $x$ , of call time she used.

$$y = \begin{cases} 45 & x \leq 400 \\ 45 + .25(x - 400) & x > 400 \end{cases}$$

**Example:** The following table represents Emma's phone usage for the past 3 months. What are the missing entries?

Number of minutes used, $x$	379	538	400
Phone bill, $y$ , in dollars	$y_1$	$y_2$	$y_3$

*Solution:* Since  $379 < 400$ , the first equation is used to find  $y_1$ :  $y_1 = 45$ . The phone bill is \$45. Since  $538 > 400$ , use the second equation to find  $y_2$ :  $y_2 = 45 + .25(538 - 400) = 79.5$ . The phone bill is \$79.50. In addition, since  $400 = 400$ , the first equation is used to find  $y_3$ :  $y_3 = 45$ . The phone bill is \$45.

### Problems:

46. Consider the piecewise defined function:  $s = \begin{cases} t^2 - 3t - 8 & t < -1 \\ 5 - 2t & t \geq -1 \end{cases}$ . What are the missing entries

in this table?

$t$	-5	-3	-1	2
$s$	32	$s_1$	$s_2$	$s_3$

## SOLVING POLYNOMIALS BY FACTORING

Factoring is the process of writing a polynomial as the product of two or more polynomials. See your reference text for a review of the techniques for factoring polynomials. If the polynomial equation has zero on one side and a factored expression on the other side, it can be solved using the zero-product property (if  $ab = 0$ , then either  $a = 0$ ,  $b = 0$ , or both).

A quadratic equation may be expressed in the form  $ax^2 + bx + c = 0$  and then may possibly be solved by factoring.

**Example:** The equation  $x^2 - 3x + 2 = 0$  can be written in factored form as  $(x - 2)(x - 1) = 0$ . Since the product  $x - 2$  with  $x - 1$  is required to be 0, one or the other factor must equal 0. Thus,  $x = 2$  or  $x = 1$ .

**Example:** The equation  $x^3 + 4x^2 - 3x - 12 = 0$  can be factored by grouping,  $(x^3 + 4x^2) - (3x + 12) = x^2(x + 4) - 3(x + 4) = (x + 4)(x^2 - 3) = 0$ . Thus,  $x = -4$  or  $x = \pm\sqrt{3}$ .

**Problems:**

- 47. Solve:  $x^2 - 4x - 12 = 0$
- 48. Solve:  $x^2 + 18 = 9x$
- 49. Solve:  $(3x + 1)(3x - 1) = (x + 9)(x + 4) - 42$
- 50. Solve:  $x^3 + 3x^2 - 25x - 75 = 0$
- 51. Solve:  $4x^3 + 12x^2 - x - 3 = 0$

## COMPLEX NUMBERS

Complex numbers involve the imaginary unit  $i = \sqrt{-1}$ . Note that  $i^2 = -1$ ,  $i^3 = -i$ ,  $i^4 = 1$ . Thus, every positive integral power of  $i$  is equal to one of the four numbers  $i$ ,  $-1$ ,  $-i$ , or  $1$ . A complex number is any number of the form  $a + bi$ , where  $a$  and  $b$  are real numbers.

**Example:**  $i^{583} = [i^4]^{145} i^3 = i^3 = -i$

**Example:**  $(5 + 2i)(3 - 7i) = 15 - 35i + 6i - 14i^2$   
 $= 15 + 14 - 29i$   
 $= 29 - 29i$

**Example:**  $\frac{3+i}{2-i} = \frac{(3+i)(2+i)}{(2-i)(2+i)} = \frac{6+5i+i^2}{4-i^2} = \frac{5+5i}{5} = 1+i$ .

**Problems:**

- 52. Simplify:  $(6 + 4i)(5 + 7i) + 2i + 10$
- 53. Write  $\frac{3-4i}{1+2i}$  in the form  $a + bi$ .
- 54. Simplify:  $i^{648}$

## QUADRATIC FORMULA

Complex numbers are sometimes solutions to quadratic equations. When we cannot factor a given quadratic conveniently, (such as  $x^2 + x + 1$ ) we refer to the **quadratic formula** for finding the roots of a quadratic equation. The formula applies to quadratic equations written in the form

$$ax^2 + bx + c = 0$$

The quadratic formula says that the solutions to this equation are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In the earlier example  $x^2 - 3x + 2 = 0$ , the coefficients are  $a = 1$ ,  $b = -3$ , and  $c = 2$ . So, by the quadratic formula, solutions are

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(2)}}{2} = \frac{3 \pm \sqrt{1}}{2} \text{ which gives us}$$

$$x_1 = 2$$

and

$$x_2 = 1.$$

When the quadratic formula is applied to some quadratic equations, negative numbers occur under the square root sign. Since negative numbers do not have square roots that are real numbers, the solutions to these equations involve the imaginary number  $i$ .

**Example:** Solve  $x^2 + 2x + 4 = 0$ .

*Solution:* Using the quadratic formula, we find the solutions are

$$x_1, x_2 = \frac{-2 \pm \sqrt{4 - 16}}{2} = \frac{-2 \pm \sqrt{-12}}{2} = -1 \pm i\sqrt{3}.$$

The quantity  $b^2 - 4ac$  that occurs under the square root in the quadratic formula is called the **discriminant** of the quadratic. It determines the number and nature of the solutions of the quadratic equation

$$ax^2 + bx + c = 0.$$

- i. When  $b^2 - 4ac > 0$ , the equation has two distinct real solutions.
- ii. When  $b^2 - 4ac = 0$ , the equation has only one real solution.
- iii. When  $b^2 - 4ac < 0$ , the equation has two distinct, complex solutions.

**Problems:**

55. Compute the discriminant and decide what type of solutions  $2x^2 - 4x + 3 = 0$  has.

56. Compute the discriminant and decide what type of solutions  $3x^2 + x - 1 = 0$  has.

57. Compute the discriminant and decide what type of solutions  $x^2 + 4 = 4x$  has.

58. Find the solutions in Problem 55.

59. Find the solutions in Problem 56.

60. A box is 2 feet high and 5 feet longer than it is wide. If its volume is 100 cubic feet, find its width.

61. A boy rides a bike 8 miles. The trip would have taken 20 minutes less if he had ridden 2 miles per hour faster. Find his actual speed.

## COMPLETING THE SQUARE

Every quadratic function can be expressed by an equation in vertex form. The expression

$$a(x - h)^2 + k = 0$$

is called the vertex (or completed square) form of the quadratic function  $ax^2 + bx + c = 0$ . The graph of a quadratic function is a parabola whose vertex is located at some point with coordinates  $(h, k)$ . The vertex of a parabola is also the maximum (if  $a < 0$ ) or minimum (if  $a > 0$ ) value of the function. An equation written in vertex form can be solved for  $x$ . See your reference text for a review of the techniques for completing the square.

**Example:** Rewrite  $3x^2 - 5x - 10 = 0$  in vertex form, and solve for  $x$ .

*Solution:* Factor by the coefficient of  $x^2$ :  $3\left[x^2 - \frac{5}{3}x - \frac{10}{3}\right] = 0$

Add and subtract the square of half the coefficient of  $x$ :

$$3\left[x^2 - \frac{5}{3}x + \left(\frac{5}{6}\right)^2 - \left(\frac{5}{6}\right)^2 - \frac{10}{3}\right] = 0$$

Factor and combine like terms:

$$3 \left[ \left( x^2 - \frac{5}{3}x + \left( \frac{5}{6} \right)^2 \right) - \left( \frac{25}{36} + \frac{120}{36} \right) \right] = 0$$

$$3 \left[ \left( x - \frac{5}{6} \right)^2 - \frac{145}{36} \right] = 0$$

$$3 \left( x - \frac{5}{6} \right)^2 - \frac{145}{12} = 0$$

This quadratic is in vertex form. This tells us that the vertex of the parabola is found at  $\left( \frac{5}{6}, -\frac{145}{12} \right)$  and this value for y is the minimum value of the function. We can continue the process and solve for x:

$$3 \left( x - \frac{5}{6} \right)^2 = \frac{145}{12}$$

$$\left( x - \frac{5}{6} \right)^2 = \frac{145}{36}$$

$$x - \frac{5}{6} = \pm \sqrt{\frac{145}{36}}$$

$$x = \frac{5 \pm \sqrt{145}}{6}$$

**Example:** Rewrite  $ax^2 + bx + c = 0$  in vertex form, and solve for x.

*Solution:*  $a \left[ x^2 + \frac{b}{a}x + \frac{c}{a} \right] = 0$

$$a \left[ x^2 + \frac{b}{a}x + \left( \frac{b}{2a} \right)^2 - \left( \frac{b}{2a} \right)^2 + \frac{c}{a} \right] = 0$$

$$a \left[ \left( x + \frac{b}{2a} \right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} \cdot \frac{4a}{4a} \right] = a \left[ \left( x + \frac{b}{2a} \right)^2 + \frac{-b^2 + 4ac}{4a^2} \right] = 0$$

$$a \left( x + \frac{b}{2a} \right)^2 + \frac{-b^2 + 4ac}{4a} = 0 \quad \text{Note: The vertex is at } \left( \frac{-b}{2a}, \frac{-b^2 + 4ac}{4a} \right).$$

$$a \left( x + \frac{b}{2a} \right)^2 = -\frac{-b^2 + 4ac}{4a} = \frac{b^2 - 4ac}{4a}$$

$$\left( x + \frac{b}{2a} \right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (\text{Does this look familiar?})$$

**Example:** The equation  $Y = -0.0171x^2 + 1.0765x + 3.8939$  models the expected crop yield on an experimental plot, where  $Y$  is the amount of crop the plot is expected to produce (in bushels) and  $x$  is the amount of fertilizer applied in pounds. How much fertilizer should be applied to produce the maximum yield? What is the maximum yield?

**Solution:** The coefficients are:  $a = -0.0171$ ,  $b = 1.0765$ , and  $c = 3.8939$ . Since  $a < 0$ , the vertex of a parabola is also the maximum value of this equation. Use the vertex formula

$$\left( \frac{-b}{2a}, \frac{-b^2 + 4ac}{4a} \right) \text{ to find the vertex.}$$

Substitute values for  $a$  and  $b$ .  $x = \frac{-b}{2a} = \frac{-1.0765}{2(-0.0171)} = 31.4766 \Rightarrow 31.4766$  pounds of

fertilizer should be applied to produce the maximum yield.

Substitute values for  $a$ ,  $b$  and  $c$ ,

$$y = \frac{-b^2 + 4ac}{4a} = \frac{-1.0765^2 + (4)(-0.0171)(3.8939)}{4(-0.0171)} = 20.8362.$$

The maximum crop yield is 20.8362 bushels.

Another way to find the maximum is to substitute  $x = 31.4766$  into the original equation:

$$-0.0171(31.4766)^2 + 1.0765(31.4766) + 3.8939 = 20.8362.$$

**Problems:**

- 62. Rewrite  $x^2 - 5x + 6 = 0$  in vertex form.
- 63. Find the vertex and solve for  $x$ :  $-6x^2 - 2x + 7 = 0$  by completing the square. Is the vertex a maximum or minimum value?
- 64. Find the vertex and solve for  $x$ :  $8x^2 + 4x - 1 = 0$  by completing the square. Is the vertex a maximum or minimum value?
- 65. The equation  $y = 2x^2 + 22x + 320$  models the number,  $y$ , of inmates (in thousands) incarcerated in Federal and State Correctional Facilities in the United States over time. In this model,  $x$  is the number of years since January 1, 1980. According to this model, how many years after 1980 were there 505,000 inmates incarcerated in Federal and State Correctional Facilities in the United States?
- 66. A theater has 550 seats. The theater will sell tickets for all seats if the tickets cost \$30 each. However, for each \$4 increase in ticket price, 20 fewer tickets are sold. What ticket price will generate the most revenue and what is the maximum revenue?

**CONSTRUCTING POLYNOMIALS**

You can easily write a polynomial with specified zeros but there are many solutions since you could include other zeros or a constant multiplier. To reconstruct a specific polynomial in standard form from the given zeros you need to know the degree of the polynomial and a point on the graph of that polynomial. See your reference text for a review of the techniques for constructing (finding) polynomials.

**Example:** Construct a polynomial of degree 3 that has the numbers  $-3$ ,  $-1$ , and  $1$  as its zeros and has  $y$ -intercept  $-6$ .

**Solution:** Since the polynomial is of degree 3, we know the given three zeros are the only zeros of the polynomial. First, multiply the factors together to get a polynomial in standard form.

$$(x+3)(x+1)(x-1) = (x^2 + 4x + 3)(x-1) = x^3 + 3x^2 - x - 3$$

Substitute in the point  $(0, -6)$  to find out which constant number,  $a$ , is a factor of the polynomial.

$$f(x) = a(x^3 + 3x^2 - x - 3) \Rightarrow f(0) = a(0^3 + 3(0)^2 - 0 - 3) = -6 \Rightarrow a = 2$$

Multiply by the constant,  $a = 2$  to find the polynomial.

$$f(x) = 2(x^3 + 3x^2 - x - 3) = 2x^3 + 6x^2 - 2x - 6$$

### Problems:

- 67.** Construct a polynomial of degree 2 that has the numbers  $-1$ , and  $3$  as its zeros and has a y-intercept of  $-2$ .
- 68.** Construct a polynomial of degree 3 that has the numbers  $-3$ ,  $-1$ , and  $2$  as its zeros and  $f(-2) = -12$ .
- 69.** Construct a polynomial of degree 3 that has the numbers  $-5$ ,  $2$ , and  $4$  as its zeros and has a y-intercept of  $4$ .

## ALGEBRAIC FRACTIONS

Algebraic fractions are manipulated according to the same rules as numerical fractions. They may be simplified by canceling factors common to the numerator and denominator.

**Example:** 
$$\frac{12a^2x + 18a^2y}{16ax + 24ay} = \frac{2a(6ax + 9ay)}{2a(8x + 12y)} = \frac{3a(2x + 3y)}{4(2x + 3y)} = \frac{3}{4}a$$

### Problems:

**70.** Simplify: 
$$\frac{2x^2 + x - 15}{(x^2 - 9)}$$

**71.** Simplify: 
$$\frac{24x^3 + 8x^2 - 10x}{-24x^2 + 2x + 5}$$

**72.** Simplify: 
$$\frac{(x+5) + (x-2)}{(x^2 + 2x - 15) + (x-2)(x-3)}$$

**73.** Simplify: 
$$\frac{[(x+h)^2 - 5(x+h)] - (x^2 - 5x)}{h}$$

Algebraic fractions are multiplied and divided in the same way as numerical fractions.

**Example:** 
$$\frac{A}{B} \cdot \frac{C}{D} = \frac{AC}{BD}$$

**Example:** 
$$\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \frac{D}{C} = \frac{AD}{BC}$$

After operating with the fractions, always try to simplify the resulting fraction.

**Example:** 
$$\frac{x^2 + 2x - 8}{x^2 + 7x + 12} \cdot \frac{x^2 + 2x - 3}{x^2 + x - 2} = \frac{(x+4)(x-2)(x+3)(x-1)}{(x+4)(x+3)(x+2)(x-1)} = \frac{x-2}{x+2}$$

### Problems:

**74.** Multiply and simplify: 
$$\frac{20x^3 - 56x^2 + 15x}{2x^2 + 2x - 12} \cdot \frac{8x^2 + 4x - 40}{8x^4 - 50x^2}$$

**75.** Multiply and simplify: 
$$\frac{5x^2 - 17x - 12}{5x^2 + 4x - 12} \cdot \frac{2x^2 + 3x - 2}{2x^2 - 7x - 4}$$

76. Divide and simplify:  $\frac{7x^2 - 9x - 10}{6x^2 - 31x + 35} \div \frac{3x^2 - x - 10}{6x^2 + 11x - 35}$

77. Divide and simplify:  $\frac{6x^2 + 19x - 20}{6x^2 - 11x - 10} \div \frac{2x^2 + 11x + 12}{4x^2 - 25}$

To add or subtract algebraic fractions, first rewrite them as fractions having the same denominator.

**Example:**  $\frac{2}{R} + \frac{3}{S} = \frac{2S}{RS} + \frac{3R}{SR} = \frac{2S + 3R}{RS}$

**Example:**

$$\begin{aligned} \frac{x-1}{x^2+x-6} - \frac{x+2}{x^2+4x+3} &= \frac{x-1}{(x+3)(x-2)} - \frac{x+2}{(x+3)(x+1)} \\ &= \frac{(x-1)(x+1)}{(x+3)(x-2)(x+1)} - \frac{(x+2)(x-2)}{(x+3)(x+1)(x-2)} \\ &= \frac{(x^2-1) - (x^2-4)}{(x+3)(x-2)(x+1)} \\ &= \frac{3}{(x+3)(x-2)(x+1)} \end{aligned}$$

**Problems:**

78. Simplify:  $\frac{5x+2}{2x^2+17x+30} + \frac{15x}{5-13x-6x^2}$

79. Simplify:  $\frac{x+5}{x^2+6x+8} - \frac{x+9}{x^2-2x-24}$

All our previous skills with fractions are required to simplify algebraic fractions whose numerators and/or denominators involve fractions.

**Example:**  $\frac{x+y}{\frac{1}{x} + \frac{1}{y}} = \frac{x+y}{\frac{(x+y)}{xy}} = \frac{x+y}{1} \cdot \frac{xy}{x+y} = xy$

**Example:**  $\frac{x-3 + \frac{2x-5}{x-3}}{x-3 - \frac{2x-3}{x-3}} = \frac{\frac{(x-3)^2 + 2x-5}{x-3}}{\frac{(x-3)^2 - 2x+3}{x-3}} = \frac{x^2-4x+4}{x-3} \cdot \frac{x-3}{x^2-8x+12} = \frac{(x-2)^2}{(x-6)(x-2)} = \frac{x-2}{x-6}$

**Problems:**

80. Simplify:  $\frac{3x}{1 + \frac{3x}{1 + \frac{3x}{x}}}$

81. Simplify:  $\frac{(2x-1) + \frac{3x+4}{2x+5}}{(2x+5) - \frac{3x+4}{2x-1}}$

82. Simplify:  $1 + \frac{x-1}{1 + \frac{x-3}{1 + \frac{1}{x}}}$

When solving equations involving fractions, multiply the equation by the least common multiple of the denominators to obtain an equation involving no fractions, and then solve the resulting equation. The final equation may have solutions, which do not satisfy the original fractional equation. Thus, all potential solutions found in this way must be checked in the original equation.

**Example:** Solve  $\frac{x+1}{x-1} = \frac{1}{x} + \frac{2}{x(x-1)}$ .

*Solution:* Multiply by  $x(x-1)$  to eliminate fractions.

$$\begin{aligned} x(x+1) &= (x-1) + 2 \\ x^2 + x &= x + 1 \\ x^2 &= 1 \\ x &= \pm 1. \end{aligned}$$

Check by substituting into the original equation. We find that  $x = 1$  is not a solution and  $x = -1$  is a solution.

**Problems:**

83. Solve:  $\frac{x+2}{x^2-3x+2} - \frac{x-6}{x^2-5x+6} = \frac{1}{x-1}$

84. Solve:  $\frac{x-6}{x^2-9x+20} = \frac{2}{x-4} - \frac{x}{x-5}$

85. Solve:  $\frac{x}{x-3} = \frac{2}{x+1} + \frac{x+9}{x^2-2x-3}$

86. Solve:  $\frac{2x-5}{x-6} = \frac{x+4}{x+5} + \frac{3x+7}{x^2-x-30}$

**INTEGER EXPONENTS**

The basic laws for working with exponents are

i)  $A^N A^M = A^{N+M}$ ,

iv)  $A^0 = 1 \quad (A \neq 0)$ ,

ii)  $(A^N)^M = A^{NM}$ ,

v)  $A^{-N} = \frac{1}{A^N}$  and

iii)  $(AB)^N = A^N B^N$ ,

vi)  $\left(\frac{A}{B}\right)^N = \frac{A^N}{B^N}$

**Example:** Simplify  $(x^2)^3 - x^2(x^4 + x^3)$

*Solution:*  $(x^2)^3 - x^2(x^4 + x^3) = x^6 - x^6 - x^5 = -x^5$

**Example:** Eliminate negative exponents and simplify  $(3x^8y^{-6})(-5x^{-4}y^6)$ .

*Solution:*  $(3x^8y^{-6})(-5x^{-4}y^6) = -15x^{8-4}y^{-6+6} = -15x^4y^0 = -15x^4$

**Example:** Eliminate negative exponents and simplify  $\frac{3B^2(A^{-4}B^2)^{-3}}{(2A)^{-2}(A^6B^5)}$ .



$$\text{Solution: } \frac{3B^2(A^{-4}B^2)^{-3}}{(2A)^{-2}(A^6B^5)} = \frac{3B^2A^{12}B^{-6}}{2^{-2}A^{-2}A^6B^5} = \frac{3A^{12}B^{-4}}{2^{-2}A^4B^5} = \frac{12A^8}{B^9}$$

**Example:** Eliminate negative exponents and simplify  $(2x^4y)^{-1}\left(\frac{x^2}{4y^4}\right)^{-2}$ .

$$\text{Solution: } (2x^4y)^{-1}\left(\frac{x^2}{4y^4}\right)^{-2} = \frac{1}{2x^4y} \cdot \frac{x^{-4}}{4^{-2}y^{-8}} = \frac{1}{2x^4y} \cdot \frac{16y^8}{x^4} = \frac{8y^7}{x^8}$$

**Problems:** Eliminate negative exponents and simplify.

$$87. (R^{-2}(S^{-3})^{-4})(R^{-1}S^{-6})$$

$$88. \left(\frac{R^8S^5}{R^{-3}S^4}\right)\left(\frac{R^{-1}S^2}{R^6S^3}\right)^{-3}$$

$$89. \frac{u^{-5}}{(u^{-1}v^{-3})^3} - \frac{u^{-6}v^2}{2u^{-4}}$$

$$90. \left(\frac{(x^4y^{-1})^{-2}}{y^{-3}x^{-4}}\right)\left(\frac{x^3}{y^{-2}}\right)^3$$

$$91. \left(\frac{x^{-4}(2y^2)^3}{x^3y^5}\right)\left(\frac{x^0}{4y}\right)^3$$

$$92. (x+y)^{-1}(x^{-1}+y^{-1})^{-1}$$

$$93. \frac{2x^{-1}+3y^{-1}}{9x^2-4y^2}$$

$$94. \frac{x^{-1}-x^{-3}}{x^2(x-1)}$$

## RATIONAL EXPONENTS AND RADICALS

Recall that for  $a > 0$  and  $n$  a positive integer,  $\sqrt[n]{a} = a^{1/n}$  means a positive number  $b$  such that  $b^n = a$ . It follows that  $\sqrt[n]{a^m} = a^{m/n}$ . Thus, radical expressions can be converted to expressions involving fractional exponents. These fractional exponents obey the same laws as integer exponents previously reviewed. For instance,

$$a^{-3/4} = \frac{1}{a^{3/4}} = \frac{1}{\sqrt[4]{a^3}}.$$

A word of warning!  $\sqrt{25} = 5$ , not  $\sqrt{25} = \pm 5$ .

**Example:** Simplify by expressing radicals as non-negative rational powers and combining powers whenever possible:  $\sqrt[3]{8a^3b^{-6}} \cdot \sqrt{9a^5b^3}$

$$\text{Solution: } \sqrt[3]{8a^3b^{-6}} \cdot \sqrt{9a^5b^3} = 8^{1/3} a^{3/3} b^{-6/3} \cdot 9^{1/2} a^{5/2} b^{3/2} = 2 \cdot 3 \cdot a^{3/3+5/2} \cdot b^{-6/3+3/2} = 6a^{7/2} b^{-1/2} = \frac{6a^{7/2}}{b^{1/2}}$$

**Problems:** Simplify by expressing radicals as non-negative rational powers and combining powers whenever possible.

95.  $\sqrt{36x^6y^3}$

96.  $\sqrt[3]{\frac{x^4y^8}{27z^9}}$

97.  $\frac{x^{-2}\sqrt[4]{16x^7y^{-12}}}{6y}$

For typographical convenience and historical reasons, it is customary to write expressions involving both fractions and radicals so that no fraction appears under a radical sign and no radical appears in a denominator. This "rationalizing the denominator" is accomplished in several ways, as illustrated in the following examples.

**Example:**  $\sqrt{\frac{18}{125}} = \frac{3\sqrt{2}}{5\sqrt{5}} = \frac{3\sqrt{2}}{5\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{10}}{25}$

**Example:**  $\sqrt[3]{\frac{1}{x}} = \frac{1}{\sqrt[3]{x}} \cdot \frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} = \frac{\sqrt[3]{x^2}}{x}$

**Example:**  $\frac{5}{3+\sqrt{2}} = \frac{5}{3+\sqrt{2}} \cdot \frac{3-\sqrt{2}}{3-\sqrt{2}} = \frac{15-5\sqrt{2}}{9-2} = \frac{15-5\sqrt{2}}{7}$

**Example:**  $\frac{x+\sqrt{y}}{x-\sqrt{y}} = \frac{x+\sqrt{y}}{x-\sqrt{y}} \cdot \frac{x+\sqrt{y}}{x+\sqrt{y}} = \frac{x^2+2x\sqrt{y}+y}{x^2-y}$

**Problems:**

98. Rationalize the denominator in  $\frac{B\sqrt{A^3}}{\sqrt{2BC^0}}$ .

99. Rationalize the denominator in  $\sqrt{24R^{-2}S^{-1}T}$ .

100. Rationalize the denominator in  $\frac{2-\sqrt{3}}{3+\sqrt{2}}$ .

101. Rationalize the denominator in  $\frac{3-\sqrt{6}}{A-(3+\sqrt{6})}$ .

## SOLVING RADICAL EQUATIONS

Equations involving radicals are often solved by rewriting the equation, raising both sides to some power to remove the radical, and then solving the resulting equation. As with fractional equations, this process may introduce extraneous roots. Potential solutions must be checked in the *original* equation.

**Example:** Solve for  $x$ :  $\sqrt{3x+4} - 8 = 0$ .

*Solution:*  $\sqrt{3x+4} = 8$

Squaring both sides we get  $3x+4 = 64$ ,

so  $3x = 60$ .

Thus,  $x = 20$  is a possible solution.

Checking in the original equation we find  $\sqrt{3 \cdot 20 + 4} = 8$ . Thus  $x = 20$  is the solution.

**Example:** Solve for  $x$ :  $\sqrt{5x-1} + 3 = 0$ .

*Solution:*  $\sqrt{5x-1} = -3$

Squaring both sides we get  $5x - 1 = 9$ ,

so  $5x = 10$ .

Thus,  $x = 2$  is a possible solution.

Check in the original equation and find  $\sqrt{10-1} + 3 = 6 \neq 0$ . (Recall that  $\sqrt{\quad}$  means non-negative square root!) Thus,  $x = 2$  is not a solution. This equation has no solution.

**Example:** Solve for  $y$ :  $\sqrt{5y+6} - \sqrt{y+3} = 3$ .

*Solution:*  $\sqrt{5y+6} = \sqrt{y+3} + 3$

Square both sides and find  $5y + 6 = (y + 3) + 6\sqrt{y+3} + 9$ .

Isolate the square root term to get  $4y - 6 = 6\sqrt{y+3}$ ,

and square again to get  $16y^2 - 48y + 36 = 36y + 108$ .

We find  $16y^2 - 84y - 72 = 0$ ,

or  $4y^2 - 21y - 18 = 0$ ,

so  $y = -3/4$  or  $6$ .

Checking by substituting into the original equation, we find that  $6$  is a solution and  $-3/4$  is not. The only solution is  $y = 6$ .

### Problems:

**102.** Solve for  $x$ :  $\sqrt{x+2} = 2x + 1$

**103.** Solve for  $x$ :  $\sqrt{x+6} - x = 4$

**104.** Solve for  $x$ :  $\sqrt{2x+5} + \sqrt{6x-11} + 4 = 0$

**105.** Solve for  $x$ :  $\sqrt{3x+7} + \sqrt{x+5} - 4 = 0$

## POWER FUNCTIONS

A function,  $f(x) = x^a$  where  $a$  is a real number is called a power function. You have already worked

with many power functions including functions like  $y = \sqrt{x} = x^{1/2}$ ,  $y = x$ ,  $y = x^2$ ,

$y = x^{2/3} = \sqrt[3]{x^2}$ .

**Example:**  $\left(\frac{1}{3}\right)^{-4} = 3^4 = 81$

**Example:**  $4^{5/4} = 4 \cdot 4^{1/4} = 4\sqrt{2}$

**Example:** Solve for  $x$ :  $x^{3/2} = 27$

*Solution:*  $x^{3/2} = 27$

$x^3 = 27^2$

$x = \sqrt[3]{729} = 9$

**Example:** Solve for  $x$ :  $x^{4/5} - 5x^{2/5} + 6 = 0$

$$\text{Solution: } x^{4/5} - 5x^{2/5} + 6 = 0$$

$$\left(x^{2/5} - 3\right)\left(x^{2/5} - 2\right) = 0$$

$$\text{So, } x^{2/5} - 3 = 0$$

$$x^{2/5} = 3$$

$$x^2 = 243$$

$$x = \pm\sqrt{243} = \pm 9\sqrt{3} \quad \text{or}$$

$$x^{2/5} - 2 = 0$$

$$x^{2/5} = 2$$

$$x^2 = 32$$

$$x = \pm\sqrt{32} = \pm 4\sqrt{2}$$

### Problems:

106. Solve for  $x$ :  $x^{2/3} = 4$

107. Solve for  $x$ :  $x^{5/4} = 3125$

108. Solve for  $x$ :  $x^{1/2} - 7x^{1/4} + 10 = 0$

109. Solve for  $x$ :  $x^{4/3} - 13x^{2/3} + 36 = 0$

### MORE RATIONAL EXPONENTS

Here are some more problems involving exponents. Simplify and rationalize the denominators.

110.  $(A^{-4}B^2C^{-6})^{1/2}$

111.  $\frac{(u^{1/2}v^{-2/3})^5}{u^{-1/2}v^{-1/3}}$

112.  $\left(\frac{18x^5}{y^3}\right)^{-1/2}$

## I. ALGEBRA - ANSWER SECTION

### POLYNOMIAL MANIPULATION

1.  $2x^2 - 26x + 35$

2.  $64x^2 + 58x + 4$

3.  $6x^4 - 13x^3 + 13x - 6$

4.  $2x^3 + 17x^2 + 54x + 55$

### SOLVING ONE LINEAR EQUATION IN ONE UNKNOWN

5.  $x = \frac{20}{3}$

6.  $u = -4$

7.  $R = \frac{-3}{2}$

### MATHEMATICAL MODELS

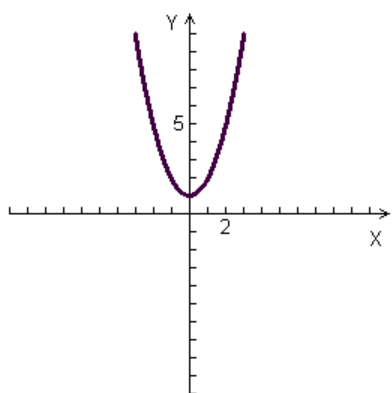
8. \$138.50

9. \$1,825

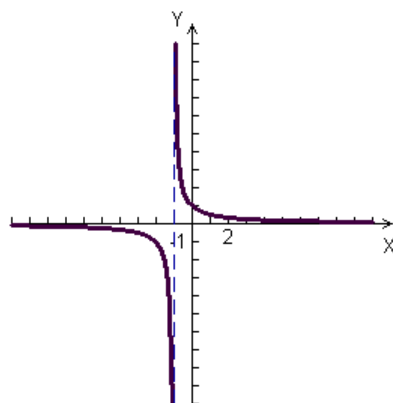
## INEQUALITIES/ABSOLUTE VALUES

- 10.**  $x > 2$       **11.**  $x \geq -1$       **12.**  $x \leq -\frac{37}{23}$       **13.**  $x > -\frac{16}{11}$   
**14.**  $\frac{3}{2} \leq x \leq \frac{9}{2}$       **15.**  $-2 \leq x \leq 14$       **16.**  $x \leq 1$  or  $x \geq 4$       **17.**  $x < \frac{15}{2}$  or  $x > \frac{27}{2}$

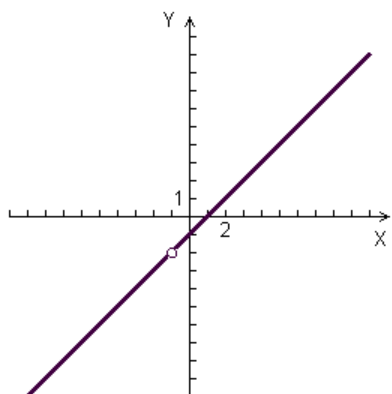
## GRAPHING IN THE PLANE



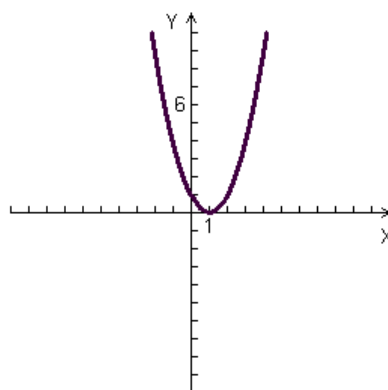
18.



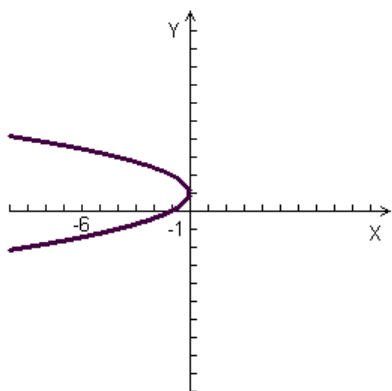
19.



20.



21.



22.

## LINES IN THE PLANE

23.  $-\frac{1}{2}$

24.  $4 + h$

25.  $y + 4 = \frac{5}{3}(x + 3)$

26.  $m = \frac{1}{3}, (x_0, y_0) = (-5, 2)$

27.  $y - 2 = 0$  or  $y = 2$

28.  $x = 5$  slope undefined

29.  $y = -2x + 11$

30.  $y = \frac{3}{7}x - \frac{4}{7}$

31.  $m = -\frac{2}{5}, b = \frac{8}{5}$

32. 2.2

33. -3

34.  $\frac{-1.25}{3} = -\frac{5}{12}$

35.  $-\frac{1}{2}$

36.  $\frac{-7.75}{12} = \frac{-31}{48}$

## SYSTEMS OF LINEAR EQUATIONS

37.  $x = 3, y = 4$

38.  $x = 4, y = -1, z = 0$

39. approximately 22 and 11

## DISTANCE FORMULA

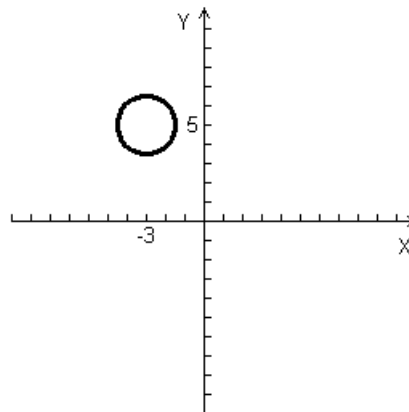
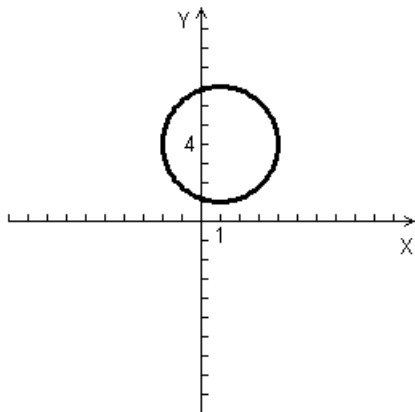
40.  $\frac{11}{2}$

41.  $8\sqrt{2}$

42.  $4\sqrt{2}$

## EQUATION OF A CIRCLE

43.  $(x + 2)^2 + (y - 7)^2 = 9$



44.

45.

## PIECEWISE FUNCTIONS

46.

$t$	-5	-3	-1	2
$s$	32	10	7	1

## SOLVING POLYNOMIALS BY FACTORING

47.  $x = 6, x = -2$

48.  $x = 6, x = 3$

49.  $x = \frac{5}{8}, x = 1$

50.  $x = -5, x = -3, x = 5$

51.  $x = -3, x = -\frac{1}{2}, x = \frac{1}{2}$

## COMPLEX NUMBERS

52.  $12 + 64i$

52.  $-1 - 2i$

54. 1

## QUADRATIC FORMULA

55. distinct complex

56. real, distinct

57. exactly one real

58.  $\frac{2 \pm i\sqrt{2}}{2}$

59.  $\frac{-1 \pm \sqrt{13}}{6}$

60. 5 feet

61. 6 miles per hour

## COMPLETING THE SQUARE

62.  $\left(x - \frac{5}{2}\right)^2 - \frac{1}{4} = 0$

63. vertex:  $\left(-\frac{1}{6}, \frac{43}{6}\right), x = \frac{-1 \pm \sqrt{43}}{6}, \text{ maximum}$

64. vertex:  $\left(-\frac{1}{4}, -\frac{3}{2}\right), x = \frac{-1 \pm \sqrt{3}}{4}, \text{ minimum}$

65. 5.58 years

66. Ticket prices of \$ 70 generate a maximum revenue of \$24,500.

## CONSTRUCTION POLYNOMIALS

67.  $f(x) = \frac{2}{3}x^2 - \frac{4}{3}x - 2$

68.  $f(x) = -3x^3 - 6x^2 + 15x + 18$

69.  $f(x) = 0.1x^3 - 0.1x^2 - 2.2x + 4$

## ALGEBRAIC FRACTIONS

70.  $\frac{2x-5}{x-3}$

71.  $-\frac{2x(6x+5)}{12x+5}$

72.  $\frac{1}{x-3}$

73.  $2x - 5 + h$

74.  $\frac{10x-3}{x(x+3)}$

75.  $\frac{(5x+3)(2x-1)}{(5x-6)(2x+1)}$

76.  $\frac{(7x+5)(2x+7)}{(2x-7)(3x+5)}$

77.  $\frac{(6x-5)(2x+5)}{(3x+2)(2x+3)}$

78.  $\frac{-89x-2}{(2x+5)(x+6)(3x-1)}$

79.  $\frac{-12}{(x+2)(x-6)}$

80.  $\frac{3x^2+9x}{3x^2+x+3}$

81.  $\frac{(2x-1)(4x^2+11x-1)}{(2x+5)(4x+9)(x-1)}$

82.  $\frac{2x}{x-1}$

83. 9

84. 1

85. no solution

86.  $-2 \pm 2\sqrt{3}$

## INTEGER EXPONENTS

87.  $\frac{S^6}{R^3}$       88.  $R^{32}S^4$       89.  $\frac{v^2(2v^7-1)}{2u^2}$       90.  $x^5y^{11}$   
91.  $\frac{1}{8x^7y^2}$       92.  $\frac{xy}{(x+y)^2}$       93.  $\frac{1}{xy(3x-2y)}$       94.  $\frac{x+1}{x^5}$

## RATIONAL EXPONENTS AND RADICALS

95.  $6x^3y^{3/2}$       96.  $\frac{x^{4/3}y^{8/3}}{3z^3}$       97.  $\frac{1}{3x^{1/4}y^4}$       98.  $\frac{A\sqrt{2AB}}{2}$   
99.  $\frac{2\sqrt{6TS}}{RS}$       100.  $\frac{6-3\sqrt{3}-2\sqrt{2}+\sqrt{6}}{7}$       101.  $\frac{3A-15+6\sqrt{6}-A\sqrt{6}}{A^2-6A+3}$

## SOLVING RADICAL EQUATIONS

102.  $\frac{1}{4}$  only      103. -2 only  
104. by inspection, there is no solution      105. -1 only

## POWER FUNCTIONS

106.  $x = \pm 8$       107. 625      108.  $x = 16, x = 625$       109.  $x = \pm 8, x = \pm 27$

## MORE RATIONAL EXPONENTS

110.  $\frac{B}{A^2C^3}$       111.  $\frac{u^3}{v^3}$       112.  $\frac{y\sqrt{2xy}}{6x^3}$