Modeling Spatio-Temporal Systems with Skew Radial Basis Functions: Theory, Algorithms and Applications

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The discovery of knowledge in large data sets can often be formulated as a problem in nonlinear function approximation. The inherent challenge in such an approach is that the data is often high dimensional, scattered and sparse. Given a limited number of exemplars one would like to construct models that can generalize to new regions or events. Additionally, underlying physical processes may not be stationary and the nature of the nonlinear relationships may evolve. Ideally, a good model will also be able to adapt and remain valid over extended regions in space and time.

In this work we propose a new Radial Basis Function (RBF) algorithm for constructing nonlinear models from high-dimensional scattered data. The algorithm progresses iteratively adding a new function at each step to refine the model. The placement of the functions is driven by one or more statistical hypotheses tests that reveals geometric structure in the data when it fails. At each step the added function is fit to data contained in a spatio-temporally defined local region to determine the parameters, in particular, the scale of the local model. Unlike prior techniques for nonlinear function fitting over scattered data, the proposed method requires no ad hoc parameters and it behaves effectively like a black box. Thus, the number of basis functions required for an accurate fit is determined automatically by the algorithm. An extension of the algorithms to multivariate case, i.e., the dimension of the range of the mapping is greater or equal to two, is also carried out. This approach produces more parsimonious models by exploiting the correlation among the various range dimensions. The convergence properties of the algorithms are shown from different perspectives.

To further enhance the order and conditioning of the models we introduce several new compactly supported RBFs for approximating functions in $L^p(\mathbb{R}^d)$ via over-determined least squares. We also propose a skew-radial basis function expansion for the empirical model fitting problem to achieve more accuracy and lower model orders. This is accomplished by modulating or skewing, each RBF by an asymmetric shape function which increases the number of degrees of freedom available to fit the data. We show that if the original RBF interpolation problem is positive definite, then so is the skew-radial basis function when it is viewed as a bounded perturbation of the RBF.

We illustrate the utility of the theoretic and algorithmic innovations via several applications including modeling data on manifolds, prediction of financial and chaotic time-series and prediction of the maximum wind intensity of a hurricane. In addition, the skew-radial basis functions are shown to provide good approximations to data with jumps. While the algorithms presented here are in the context of RBFs, in principle they can be employed with other methods for function approximation such as multi-layer perceptrons.