MATHDAY 2012
TEAM COMPETITION
EXCERPTS

Department of Mathematics
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Round 1

1. An urn contains 5 red balls and 3 green balls. What is the probability that Jack picks a red ball and a green ball, in that order, assuming he doesn't put the red ball back in after choosing it?
2. How many two-digit numbers are odd?
3. How many prime numbers are there?
4. How many yards are there in three miles?
5. In how many ways can four people be seated around a circular table?
6. Eric's favorite number is 63; Matt's favorite is 35. What is the least common multiple of 63 and 35?
7. Given four ounces of an 25% olive oil, 75% vinegar mixture, how many ounces of oil must be added to get a 50/50 mixture?
8. 200 elk are tagged and released. Two weeks later 150 elk are observed and 5 of them are tagged. Estimate how many elk are there.
9. What is the prime factorization of 36?
10. The arithmetic mean of a set of 50 numbers is 38. If two numbers of the set, namely 45 and 55, are removed, the arithmetic mean of the remaining set of numbers is?
11. A statement in mathematics is always true. Correct?
12. Find the larger of the roots of $x^2 - 12x + 27$.
13. If Tim gets \$1 of allowance on day one and, from then on, gets an allowance each day equal to double the previous allowance, how much money will he have after seven days?

Round 2

1. Which American mathematician is a pioneer of the use of mathematics for digital communication?
2. Given a tetrahedron, what do you get if you start with the number of faces, subtract the number of edges, and then add the number of vertices?
3. Simplify $\sqrt{72}$
4. What is the term that mathematicians use for the size of a set?
5. Find x and y such that $(2, 5)$ is the midpoint of the line segment between the points (x, y) and $(-5, 6)$.
6. A rectangular prism (a fancy word for a rectangular box) with side lengths 3, 6, and 12 is cut into 2 by 3 by 3 blocks. How many such blocks are there?
7. If 2 is the first prime number, what is the sixth?
8. Euler's number e starts with $2.\ldots$. What are the next two digits?
9. How many numbers between 0 and 60 are prime?
10. What is the largest number of obtuse angles that a quadrilateral can have?
11. If the radius of a circle is increased by 100%, the area is increased by how many percent?
12. What is the largest 3-digit prime?
13. I am thinking of a number n (actually, an integer). The number n is prime. The number n is greater than 5. More importantly, Carl Gauss once proved that the regular n-gon can be constructed by ruler and compass. What is that number n?

Round 3

1. If A is 20% of B, B is 50% of D, and C is 20% of D, then what is $\frac{C}{A}$?
2. What is the last digit of 2^{51}?
3. Find the side length of a cube whose volume is numerically the same as its surface area.
4. Jesse has a season's subscription to the local drama theater. His seat is in the 5th row from the front and the 12th row from the back of the theater. His seat is 8 seats from the right aisle and 12 seats from the left aisle. If each row has the same number of seats, how many seats are in the entire theater?
5. A man spent 1/3 of his money and then lost 2/3 of the remainder. He was left with \$12. How much did he start with?
6. A shirt has been marked down 5% to \$42.75. What was the original price?
7. A drawer has 14 unmatched pairs of socks. How many socks must I grab on the way to work to make sure I grab a pair?
8. 64 spheres of a liquid come together to form one large sphere. What is the radius of the new sphere if the small spheres each had volume $\frac{4}{3}\pi$.
9. Bob has \$3.75 in dimes and quarters. How many coins does he have if he has one more quarter than dime?
10. A square is inscribed in a circle, which in turn is inscribed in a square. What percentage of the area of the large square is inside the small square?

11. Patrick and Sofya are having a party. Their goal is to have at least two people at the party with the same birth month. What is the minimum number of guests they must have to guarantee that this will happen

12. Give all solutions to the equation $\cos(x) = 0$.

13. Three positive integers sum to 10. What is the minimum possible value for the sum of their squares?

ANSWERS:

Round 1

1. $\frac{15}{56}$
2. 45
3. Infinitely many.
4. 5280
5. 6
6. 315
7. 2
8. 6000
9. $2 \cdot 2 \cdot 3 \cdot 3$
10. 37.5
11. no, it is either always true or always false but not in-between
12. 9
13. \$127

Round 2

1. Claude Shannon
2. 2
3. $\sqrt{72}$ $6\sqrt{2}$
4. cardinality
5. $x = 9, y = 4$
6. 12
7. 13
8. 71 OR 72, if rounding is considered (both answers are OK)
9. 17
10. 3
11. 300% (NOT: 400%)
12. 997
13. $n = 17$

Round 3

1. 2
2. 8
3. 6
4. $304 = 16 \times 19$
5. 54 since $x \times \frac{2}{3} \times \frac{1}{3} = 12$
6. \$45
7. 15 (pigeonhole)
8. $\frac{4}{3}\pi$. 4
9. 21
10. 50%
11. 13
12. $x = \pi/2 + k\pi$, k any integer
13. 34 $(= 3^2 + 3^2 + 4^2)$