Math Day
Team Competition 1997
Colorado State University

1. At 10:00 a.m., Bob starts jogging at 3 km/hr. At noon, Kate starts jogging along the same path from Bob's starting place at 4 km/hr. How far behind will she be at 2:00 p.m.?

$$\boxed{6-2=\boxed{4\text{ km}}}$$

2. How many minutes were there in October of 1997?

$$\boxed{31\times 24\times 60=\boxed{44\,640}\text{ minutes}}$$

3. What complex numbers are equal to their squares?

$$\boxed{x=x^2,\text{ Solution is: } x=\boxed{0},\ x=\boxed{1}}$$

4. What fraction is equivalent to the decimal expansion $0.7777\ldots$?

$$\boxed{\boxed{\frac{7}{9}}=.777\,7777777777777777777777777777777777777\ldots}$$

5. What fraction is equivalent to the decimal expansion $.151515151515\ldots$?

$$\boxed{x=.1515\ldots\Longrightarrow 100x=15.15\ldots\Longrightarrow 99x=15,\text{ Solution is : } x=\boxed{\frac{5}{33}}}$$

6. How many possible ways are there to pick the top 4 finishers in a 9-person race?

$$\boxed{9\times 8\times 7\times 6=\boxed{3024}}$$

7. In how many ways can the US Senate pick a committee of 3 from among its 100 members?

$$\boxed{\binom{100}{3}=\boxed{161700}}$$

8. What is $\binom{5}{0}+\binom{5}{1}+\binom{5}{2}+\binom{5}{3}+\binom{5}{4}+\binom{5}{5}$? (Sum of binomial coefficients.)

$$\boxed{\binom{5}{0}+\binom{5}{1}+\binom{5}{2}+\binom{5}{3}+\binom{5}{4}+\binom{5}{5}=\boxed{32}=2^5}$$

9. A shirt has been marked down 10% to \$31.50. What was the original price?

$$\boxed{31.5=.9x,\text{ Solution is: } x=\boxed{\$35.00}}$$

10. What can you say about a parallelogram whose diagonals have equal length?

$$\boxed{\text{It is a rectangle.}}$$

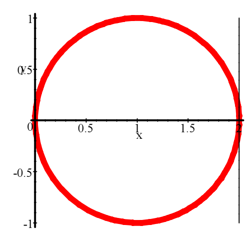
11. What is the volume of a spherical raindrop of diameter 2 millimeters?

$$\boxed{\frac{4\pi}{3}(1)^3=\boxed{\frac{4\pi}{3}}\text{mm}^2}$$

12. If 108 inches of wire are used to build a skeleton of a cube, what is the volume of the cube?

$$\boxed{\left(\frac{108}{12}\right)^3 = 9^3 = \boxed{729}\ \text{in}^3}$$

13. Given the circle $x^2 - 2x + y^2 = 0$ in the xy plane, what are the equations of the two vertical tangent lines to this circle?



$$\boxed{\boxed{x = 0} \text{ and } \boxed{x = 2}}$$

14. What is the 15th term of the arithmetic sequence that begins 17,23,29,. . .?

$$\boxed{f(n) = 11 + 6n,\ f(1) = 17,\ f(2) = 23,\ f(3) = 29,\ f(15) = \boxed{101}}$$

15. What is the smallest positive integer with exactly 5 positive integer divisors?

$$\boxed{\boxed{16} \text{ (divisors are } \{1, 2, 4, 8, 16\})}$$

16. Two wheels are connected by a belt. One has a diameter of 10 centimeters and a speed of 250 rmp. The other has a speed of 50 rpm. What is its diameter?

$$\boxed{10 \times 250 = 50x, \text{ Solution is: } x = \boxed{50} \text{cm}}$$

17. Given 5 gallons of a 20% antifreeze/water mixture, how much pure antifreeze must be added to yield a 50% antifreeze/water mixture?

$$\boxed{w = 4,\ a = 1,\ 1 + x = .5(1 + x + 4), \text{ Solution is : } x = \boxed{3} \text{ gal antifreeze}}$$

18. Find all positive roots of the equation $x^4 - 5x^2 + 4 = 0$.

$$\boxed{x^4 - 5x^2 + 4 = 0, \text{ Solution is : } x = \boxed{1}, x = \boxed{2}, x = -2, x = -1}$$

19. Which real numbers are equal to their cubes?

$$\boxed{x = x^3, \text{ Solution is : } x = \boxed{0}, x = \boxed{1}, x = \boxed{-1}}$$

20. If a third degree polynomial has leading coefficient of -2 and roots -1, 2 and 3, what is its constant term?

$$\boxed{-2(x + 1)(x - 2)(x - 3) = -2x^3 + 8x^2 - 2x \boxed{-12}}$$

21. How many strings of length 6 can be made from the letters in CONOCO?

$$\boxed{\frac{6!}{3!2!} = \boxed{60}}$$

22. Seven poker players are seated at a round table. How many rearrangements are possible, if the only

considerations are who is seated at each person's left, and at each person's right? (Because of the betting order Arlene on your left is different from Arlene on your right.)

$$\boxed{\frac{7!}{7} = \boxed{720}}$$

23. A baseball manager has selected 9 starters for a game. If the pitcher must bat last, how many different batting line-ups are possible?

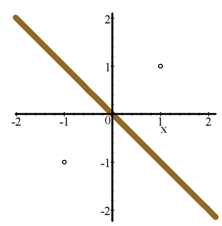
$$\boxed{8! = \boxed{40\,320}}$$

24. If eggs weigh 1.5 ounces each and a dozen eggs cost 90¢, what is the cost of a pound of eggs?
$\frac{90¢}{12 \text{ eggs}} \times \frac{1 \text{ egg}}{1.5 \text{ oz}} \times \frac{16 \text{ oz}}{1 \text{ lb}}$

$$\boxed{\frac{90¢}{12 \text{ eggs}} \times \frac{1 \text{ egg}}{1.5 \text{ oz}} \times \frac{16 \text{ oz}}{1 \text{ lb}} = \boxed{80¢} \text{ per lb}}$$

25. What is the equation of the line in the xy plane all of whose points are equidistant from the two points $(1,1)$ and $(-1,-1)$?

$$\boxed{y = -x}$$



26. A circle of diameter 4 contains a circle of diameter 1 in its interior. What is the area contained in the larger circle that is exterior to the smalll circle?

$$\boxed{\pi 2^2 - \pi \left(\frac{1}{2}\right)^2 = \boxed{\frac{15\pi}{4}}}$$

27. How far is the point (3,0) from the line $3y = 4x$?

$$\boxed{\frac{x}{3} = \frac{4}{5} \Longrightarrow x = \boxed{\frac{12}{5}} = \boxed{2.4}}$$

28. What is the area of the largest octagon that can be inscribed in a square of side 1?

$$\begin{aligned} x + 2\left(\frac{x}{\sqrt{2}}\right) &= 1, \text{ Solution is: } x = \frac{1}{\sqrt{2}+1} \\ area &= 1 - x^2 = 1 - \frac{1}{\left(\sqrt{2}+1\right)^2} \\ &= \boxed{\frac{2}{1+\sqrt{2}}} = \boxed{2\left(\sqrt{2}-1\right)} = \boxed{2\sqrt{2}-2} \end{aligned}$$

29. Neglecting the order of addition, in how many ways can 36 be written as the sum of two primes?

$$\boxed{36 = 5 + 31 = 7 + 29 = 13 + 23 = 17 + 19 \qquad \boxed{4 \text{ ways}}}$$

30. Art, Betty and Claude are now 5, 7, and 11 years old. In what year will they again all have prime

numbered ages?

In the year $\boxed{2003}$, they will have ages 11, 13, and 17.

31. Farmer Jill raises goats and geese. If she counts 100 eyes and 150 feet, how many goats and how many geese does Jill have?

$$\boxed{\begin{matrix} 2a+2b=100 \\ 4a+2b=150 \end{matrix}, \text{Solution is: } \boxed{a = \boxed{25} \text{ goats}, b = \boxed{25} \text{ geese}}}$$

32. Sam has \$9.00 in quarters and dimes. If he has twice as many quarters as dimes, how many dimes does Sam have?

$$\boxed{25 \times 2d + 10d = 900, \text{ Solution is : } d = \boxed{15}}$$

33. Factor the polynomial $2x^3 - 5x^2 + 2x - 5$.

$$\boxed{2x^3 - 5x^2 + 2x - 5 = \boxed{\left(x^2+1\right)(2x-5)}}$$

34. What is the vertex of the parabola with equation $y = x^2 - 6x + 11$?

$$\boxed{x^2 - 6x + 11 = (x-3)^2 + 2, \text{ vertex } (3,2)}$$

35. The graph of a cubic polynomial crosses the x-axis at $x = -5$, $x = 1$, and $x = 3$. What is one such polynomial?

$$\boxed{\boxed{(x+5)(x-1)(x-3)} = \boxed{x^3 + x^2 - 17x + 15} \text{ (or any nonzero multiple)}}$$

36. A basketball team has 12 players on the roster. How many different starting lineups are possible?

$$\boxed{\binom{12}{5} = \boxed{792}}$$

37. How many strings of length 8 can be made with the letters in parabola?

$$\boxed{8!/3! = \boxed{6720}}$$

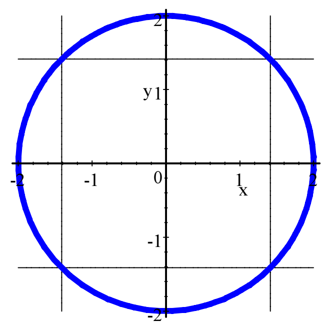
38. A pair of dice is rolled. What is the probability that the sum is either a 7 or an 11?

$$\boxed{\tfrac{6}{36} + \tfrac{2}{36} = \boxed{\tfrac{2}{9}}}$$

39. A gold watch has been reduced 10%, then 20%, and then 30% and finally sold for \$504. What was the original price?

$$\boxed{x\tfrac{9}{10} \cdot \tfrac{8}{10} \cdot \tfrac{7}{10} = \$504, \text{ Solution is : } x = \boxed{\$1000}}$$

40. What is the area of the largest square that can be inscribed in the ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{4} = 1$?



$$x = y \Longrightarrow \frac{2x^2}{4} = 1 \Longrightarrow 4x^2 = \boxed{8}$$

41. A biological brick grows 20% in length, 10% in width, and shrinks in height by 25%. Is it larger or smaller than when it started out?

$$1.2 \times 1.1 \times 0.75 = .99 \ \boxed{\text{smaller}}$$

42. A punch bowl at a Halloween party is the shape of a truncated cone of height 1 foot, base diameter 1 foot, and top diameter 18 inches. The orange plastic punch cups are also truncated cones of height 2 inches, base diameter 2 inches, and top diameter 3 inches. How many cups of punch will the punch bowl hold?

$$6^3 = \boxed{216} \text{ cups}$$

43. An orange has a diameter that is 80% fruit and 20% peel. To the nearest percent, what percentage of the volume is the peel?

$$1 - \left(\frac{8}{10}\right)^3 = .488 \approx \boxed{48.8\%}$$

44. What is the remainder if 2^{29} is divided by 29?

$$3^{29} \bmod 29 = \boxed{3} \text{ (Fermat's Little Theorem)}$$

45. What is the prime factorization of 2222?

$$2222 = \boxed{2 \times 11 \times 101}$$

46. One third of the air in a container is removed by each cycle of an air pump. What fractional part of the air remains after 5 cycles?

$$\left(\frac{2}{3}\right)^5 = \boxed{\frac{32}{243}} \approx 13\%$$

47. If a 200-Watt sound system can break a glass goblet placed 1 foot away from the speaker, how powerful a sound system would it take to break a similar glass goblet placed 5 feet from the speaker?

$$200 \times 5^2 = \boxed{5000} \quad \text{Watts}$$

48. What is the sum of the roots of the polynomial $x^2 - 7x + 3$?

$$(x-a)(x-b) = x^2 - x(a+b) + ab \text{ so } \boxed{7} \text{ is the sum of the roots}$$

49. The graph of the polynomial equation $y = x^4 - x^3 - 9x^2 + 7x + 6$ crosses the x-axis at $x = 3$. Factor the polynomial $x^4 - x^3 - 9x^2 + 7x + 6$.

$$\boxed{x^4 - x^3 - 9x^2 + 7x + 6 = (x-3)\left(x^3 + 2x^2 - 3x - 2\right)}$$

50. Find the base 2 sum of 11_2 and 101_2.

$$\boxed{11_2 + 101_2 = \boxed{1000_2}}$$

51. Find x so that the four numbers 23, 14, x, and 29 have an average of 25.

$$\boxed{\frac{23+14+x+29}{4} = 25, \text{ Solution is: } \left\{x = \boxed{34}\right\}}$$

52. Allison scored 74 on the first exam and 83 on the second exam. What must she average on the next two exams to bring her average for the four exams up to 85?

$$\boxed{\frac{74+82+2x}{4} = 85, \text{ Solution is : } \left\{x = \boxed{92}\right\}}$$

53. If two cards are drawn from a standard deck of 52 cards, what is the probability that both are kings?

$$\boxed{\frac{\binom{4}{2}}{\binom{52}{2}} = \frac{6}{1326} = \boxed{\frac{1}{221}}}$$

54. The compact disk UR2gly sells at outlet AC for \$12.95 less a discount of 15%, and at outlet DC for \$14.65 less a discount of 25%. Which outlet has the lower price?

$$\boxed{AC = 12.95 \times .85 = 11.0075, \boxed{DC} = 14.65 \times .75 = 10.9875}$$

55. If a cube has a volume of $216\,\mathrm{cm}^3$, what is its surface area?

$$\boxed{6^3 = 216, 6^2 \times 6 = \boxed{216}\,\mathrm{cm}^2}$$

56. What is the surface area of a sphere of volume $36\pi\ \mathrm{ft}^3$?

$$\boxed{\frac{4}{3}\pi r^3 = 36\pi, \text{ Solution is : } r = 3, \left[4\pi r^2\right]_{r=3} = \boxed{36\pi}\,\mathrm{ft}^2}$$

57. Name the five Platonic solids (regular polyhedra).

$\boxed{\text{Tetrahedron, cube (or hexahedron), octahedron, dodecahedron, icosahedron}}$

58. A regular tetrahedron has edges of length 6. What is the total surface area of the tetrahedron?

$$\boxed{36\sqrt{3}}$$

59. What is the sum of the first ten even integers $2 + 4 + 6 + \cdots + 20$?

$$\boxed{2 + 4 + 6 + \cdots + 20 = 2\sum_{i=1}^{10} i = 2 \times \frac{10 \times 11}{2} = \boxed{110}}$$

60. What is the smallest positive integer divisible by all of the integers 1, 2, 3, 4, 5, and 6?

$$\boxed{\mathrm{lcm}(1,2,3,4,5,6) = 2^2 \times 3 \times 5 = \boxed{60}}$$

61. The Center Ring Janitorial Supply owns a fleet of 3 vehicles: A car, which gets 30 miles per gallon, a van, which gets 20 miles per gallon, and a truck, which gets 10 miles per gallon. If in a typical week the car is driven 300 miles, the van is driven 200 miles and the truck is driven 100 miles, how many miles per

gallon is Center Ring's fleet getting?

$$\boxed{\text{Total miles} = 600, \text{ total gallons} = \frac{300}{30} + \frac{200}{20} + \frac{100}{10} = 30, \text{ mileage } \frac{600}{30} = \boxed{20} \text{ mi/gal}}$$

62. A bag of chicken feed will feed 18 chickens for 46 days. For how many days will it feed 12 chickens?

$$\boxed{46 \times \frac{18}{12} = \boxed{69} \text{ days}}$$

63. The divergent series $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$ has a musical name. What is that name?

$$\boxed{\text{Harmonic series}}$$

64. The graph of a cubic polynomial has x-intercepts 0 and 1 (only). What is a possible expression for the polynomial?

$$\boxed{\boxed{x^2(x-1)} = \boxed{x^3 - x^2} \text{ or } \boxed{x(x-1)^2} = \boxed{x^3 - 2x^2 + x} \text{ (or a nonzero multiple)}}$$

65. Given the equation $x^3 - 6x^2 - x + 30 = 0$, what is the product of the roots?

$$\boxed{x^3 - 6x^2 - x + 30 = (x+2)(x-3)(x-5), (-2) \times 3 \times 5 = \boxed{-30}}$$

$$\boxed{x^3 - 6x^2 - x + 30 = (x-a)(x-b)(x-c) = x^3 + \cdots - abc \Longrightarrow abc = \boxed{-30}}$$

66. A bag of Halloween candy contains 5 pieces of chocolate and 4 pieces of fruit bar. What is the probability that two items selected at random are both chocolate?

$$\boxed{\frac{\binom{5}{2}}{\binom{9}{2}} = \frac{10}{36} = \boxed{\frac{5}{18}}}$$

67. A committee of 3 people is to be chosen from among 3 men and 3 women. How many ways can this be done if the committee must include at least one man and at least one woman?

$$\boxed{\binom{6}{3} - \binom{3}{3} - \binom{3}{3} = \boxed{18}}$$

68. In an algebra class with 30 students, each person shakes hands with all the other students. How many handshakes are there?

$$\boxed{\frac{30 \cdot 29}{2} = \boxed{435}, \binom{30}{2} = \boxed{435}, \sum_{i=1}^{29} i = \boxed{435}}$$

69. On January 1, 1994, a savings account contained \$1000. If it earns 4% compounded annually, what will the balance be January 1, 1996?

$$\boxed{(1.04)^2 \, 1000 = \boxed{\$1081.60}}$$

70. A solid statue is made by melting $10\,\text{cm}^3$ of metal and pouring it into a mold. A larger model needs to be constructed by increasing each of its linear dimensions by a factor of 5. How much metal will the new statue require?

$$\boxed{10 \times 5^3 = \boxed{1250} \text{ cm}^3}$$

71. Express the surface area S of a cube as a function of its volume V.

$$\boxed{S = 6x^2 = 6\left(x^3\right)^{2/3} = \boxed{6V^{2/3}}}$$

72. Ten liters of paint are required to paint the outside of a cubic box of volume 125 cubic meters. How much

paint is needed to paint the outside of a cubic box of volume $8\,\mathrm{m}^3$?

$$\boxed{\frac{4}{25} \times 10 = \boxed{\frac{8}{5}} = \boxed{1.6} \text{ liters}}$$

73. A 3-dimensional cube has 8 vertices and 12 edges. How many vertices and how many edges does a 4-dimensional cube have?

$$\boxed{\text{16 vertices and 32 edges}}$$

74. What is the smallest 3-digit prime?

$$\boxed{101}$$

75. What is the greatest common divisor of 105 and 112?

$$\boxed{\gcd(105, 112) = \boxed{7}}$$

76. The age of a mathematician and her son added together is 48. In two years the mother will be three times as old as her son. How old is the mathematician?

$$\boxed{\left\{\begin{array}{c} m + s = 48 \\ m + 2 = 3(s + 2) \end{array}\right\}, \text{Solution is : } s = 11, m = \boxed{37} \text{ years}}$$

77. An automobile travels at 60 kilometers per hour for 1 minute, $30\,\mathrm{km}\,/\,\mathrm{h}$ for 2 minutes, and then $20\,\mathrm{km}\,/\,\mathrm{h}$ for 3 minutes. How far did it travel during the 7 minutes?

$$\boxed{60 \times \frac{1}{60} + 30 \times \frac{2}{60} + 20 \times \frac{3}{60} = \boxed{3} \text{ kilometers}}$$

78. What is the coefficient of x^2 in the expansion of $(2x + 3)^4$?

$$\boxed{(2x + 3)^4 = 16x^4 + 96x^3 + \boxed{216}\,x^2 + 216x + 81}$$

$$\boxed{\binom{4}{2} 3^{4-2} 2^2 = \boxed{216}}$$

79. What is the reduced form of the fraction $\frac{51}{221}$?

$$\boxed{\frac{51}{221} = \frac{3 \times 17}{13 \times 17} = \boxed{\frac{3}{13}}}$$

80. The sum of two positive integers is 7 and their product is 12. What are the integers?

$$\boxed{\left\{\begin{array}{c} a + b = 7 \\ ab = 12 \end{array}\right\}, \text{Solution is : } \{b = 4, a = 3\}}$$

81. The probability of picking a dog to finish in the top 3 at the dog track is $\frac{1}{4}$. What is the probability of picking 3 straight losers?

$$\boxed{\left(\frac{3}{4}\right)^3 = \boxed{\frac{27}{64}}}$$

82. How many rearrangements of the letters a, b, c, d, e, f have a listed before b and b listed before c?

$$\boxed{\frac{6!}{3!} = \boxed{120}}$$

83. If $\binom{n}{1} + \binom{n}{2} = 36$, what is n?

$$\boxed{\binom{n}{1} + \binom{n}{2} = 36, \text{Solution is : } \boxed{n = 8}}$$

84. Store A sells candy bars 3 for \$1.00. Store B sells candy bars individually for 40 cents, but you get 5 for the price of 4. On Monday John bought some candy bars at store A. On Tuesday Jill bought some candy bars at store B. They compared notes and found that they had gotten the same number of candy bars, and each had paid the same amount of money. What is the least amount of money that each of them could have spent?

$\boxed{\text{They each bought 6 candy bars and spend } \boxed{\$2.00}}$

85. A spherical balloon's diameter increases by 20%. By what percentage does the surface area change?

$\boxed{(1.2)^2 = 1.44 \ \boxed{44\% \text{ increase}}}$

86. What is the distance between two opposite vertices of a cube of edge $\sqrt{3}$?

$\boxed{\text{ANS: } \boxed{3}}$

87. An octahedron is a regular solid whose 8 faces are equilateral triangles. What is the distance between a pair of opposite vertices, assuming each edge of the octahedron is of length $\sqrt{2}$?

$\boxed{\text{ANS: } \boxed{2}}$

88. What is the volume of the tetrahedron whose vertices are $(0,0,0)$, $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$?

$\boxed{\text{ANS: } \boxed{1/6}}$

89. A computer sequentially computes integers by the following rule: If n is a square then multiply by 2, otherwise subtract 1. Starting at $n = 9$, what is the integer after 6 iterations?

$\boxed{9 \to 18 \to 17 \to 16 \to 32 \to 31 \to \boxed{30}}$

90. What is the smallest number n greater than 50 such that n divided by 17 has remainder 1 and n divided by 13 also has remainder 1?

$\boxed{17 \times 13 + 1 = \boxed{222}}$

91. Ann keeps flies and spiders in a box in her dorm room during the Halloween season. There are a total of 20 creatures with 140 legs. How many flies and how many spiders does she have?

$$\boxed{\left\{ \begin{array}{c} f + s = 20 \\ 6f + 8s = 140 \end{array} \right\}, \text{ Solution is : } \boxed{f = \boxed{10}, s = \boxed{10}}}$$

92. John and Jill traded positions several times while rowing a canoe through the Boundary Waters of Minnesota. With Jill at the rear, the canoe went fast enough to complete the entire trip in 10 hours, and with John at the rear the trip would have taken 14 hours. The trip actually took 12 hours. For how many hours did Jill sit in the back of the canoe?

$$1 = \frac{1}{10}t + \frac{1}{14}(12 - t), \text{ Solution is: } t = \boxed{5 \text{ hours}}$$

93. For what choice of c does the polynomial $cx^2 - 4x + 1$ have exactly one real root?

$\boxed{16 - 4c = 0 \ \boxed{c = 4}}$

94. According to Decarte's Rule of Signs, how many positive roots are there of the polynomial equation

$2x^3 + 5x^2 - x - 8 = 0$?

$\boxed{\boxed{1} \text{ (One sign change in the sequence } + + - -)}$

95. The equation $x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right)$ provides an algorithm for approximating $\sqrt{2}$. Starting with $x_1 = 1$, what is x_3 (as a rational number)?

$\boxed{x_2 = \frac{1}{2}(1+2) = \frac{3}{2},\ x_3 = \frac{1}{2}\left(\frac{3}{2} + \frac{2}{3/2}\right) = \boxed{\frac{17}{12}} = \boxed{1\frac{5}{12}}}$

96. In how many ways can 6 boys and 6 girls be teamed into pairs, if each pair must contain one girl?

$\boxed{6! = \boxed{720}}$

97. On a four-question true/false exam, correct answers are worth 3 points, wrong answers 0, and blanks count 1 point. How many different responses will result in a total score of 4?

$\boxed{\text{Form 1111 or 3100, so } 1 + \frac{4!}{2!} = \boxed{13}}$

98. How many subsets of $\{1, 2, 3, 4, 5, 6, 7\}$ contain both 2 and 4?

$\boxed{2^5 = \boxed{32}}$

99. The wholesale cost of slacks is \$40. The price is marked up 50%, then reduced 20% on sale. What is the sale price?

$\boxed{40 \cdot (1.5) \cdot (.8) = \boxed{\$48.00}}$

100. What is the diameter of a circle with area 64π cm^2?

$\boxed{64\pi = (D/2)^2\pi, \text{ Solution is:} D = \boxed{16} \text{ cm}}$

101. What is the perimeter of an isosceles triangle with base 20 and area 240?

$\boxed{h = 24,\ s^2 = h^2 + 100,\ s = 26,\ P = 20 + 2 \times 26 = \boxed{72}}$

102. What is the perimeter of a right triangle with legs 8 and 15?

$\boxed{s^2 = 8^2 + 15^2,\ s = 17,\ P = 8 + 15 + 17 = \boxed{40}}$

103. What are the dimensions of a rectangle with area 120 and perimeter 44?

$\boxed{\left\{\begin{array}{c} ab = 120 \\ 2a + 2b = 44 \end{array}\right\}, \text{ Solution is : } \boxed{\{a = 10, b = 12\}}}$

104. What is the smallest number greater than 50 such that when divided by 5 the remainder is 1 and when divided by 4 the remainder is 3?

$\boxed{\boxed{51} = 5 \times 10 + 1 = 4 \times 12 + 3}$

105. How many integers between 50 and 100 are divisible by 7?

$\boxed{8 \times 7 = 56,\ 15 \times 7 = 105,\ 15 - 8 = \boxed{7}}$

$\boxed{\{56, 63, 70, 77, 84, 91, 98\}}$

106. Marty bought a farm, a house, and a barn for $\$504,000$. If the house cost twice as much as the barn, and the farm twice as much as the house and barn together, how much did each cost?

$$\begin{aligned} 504000 &= f + b + h \\ h &= 2b \\ f &= 2(h+b) \end{aligned}$$

$$\boxed{barn = \boxed{\$56,000}, house = \boxed{\$112,000}, farm = \boxed{\$336,000}}$$

107. Jennifer explained her age by saying, "$\frac{2}{5}$ of my age less $\frac{1}{9}$ of what it will be a year from now is equal to $\frac{1}{3}$ of what my age was 5 years ago." What is her age now?

$$\boxed{\tfrac{2}{5}a - \tfrac{1}{9}(a+1) = \tfrac{1}{3}(a-5), \text{ Solution is : } a = \boxed{35}}$$

108. The equation $x_{n+1} = \frac{1}{2}\left(x_n + \frac{3}{x_n}\right)$ provides an algorithm for approximating $\sqrt{3}$. Starting with $x_1 = 1$, what is x_3 (as a rational number)?

$$\boxed{x_2 = \tfrac{1}{2}(1+3) = 2, x_3 = \tfrac{1}{2}\left(2 + \tfrac{3}{2}\right) = \boxed{\tfrac{7}{4}} = \boxed{1\tfrac{3}{4}}}$$

109. If $f(x) = 2x - 3$, what is $f(f(f(2)))$?

$$\boxed{f(2) = 1, \quad f(f(2)) = -1, \quad f(f(f(2))) = \boxed{-5}}$$

110. What is the coefficient of x^3 in the expansion of $(x+2)^5$?

$$\boxed{(x+2)^5 = x^5 + 10x^4 + \boxed{40}\,x^3 + 80x^2 + 80x + 32}$$

111. At the local Dairy Queen, the "Monster Sundae" can be ordered with any of six flavors of ice cream plus any of all of the following topings: Nuts, whip creme, cherries, hot fudge, banana slices, gum balls, and oreo cookies. If you order one such sundae every Saturday, how many weeks will it be before you must order the same sundae twice?

$$\boxed{6 \cdot 2^7 = \boxed{768} \text{ weeks}}$$

112. How many committees of 2 men and 2 women can be formed from a group of 4 men and 5 women?

$$\boxed{\tbinom{4}{2}\tbinom{5}{2} = 6 \cdot 10 = \boxed{60}}$$

113. How many 5-digit numbers can be constructed entirely out of 3's and 7's?

$$\boxed{2^5 = \boxed{32}}$$

114. Bonnie gets a salary of $32,000 with a 5% yearly raise. To the nearest $1000, what will her salary be after 3 years?

$$\boxed{32000 \cdot (1.05)^3 = \$37044.00 \approx \boxed{\$37000}}$$

115. A circular pizza is diameter 16 inches is cut into 8 congruent slices. What is the perimeter of each slice?

$$\boxed{c = 16\pi, p = \tfrac{16\pi}{8} + 2 \times 8 = \boxed{2\pi + 16} \text{ inches}}$$

116. What is the length of the arc on a circle of radius 5 subtended by a central angle of $72°$?

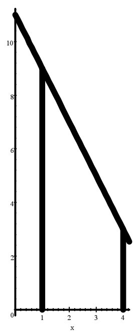
$$\boxed{\tfrac{72}{360} \times 5 \times 2\pi = \boxed{2\pi}}$$

117. What is the area of the right triangle whose hypotenuse is 17 if one of the legs has length 8?

$x^2 + 8^2 = 17^2$, Solution is : $x = 15$, $Area = \frac{1}{2} \times 8 \times 15 = \boxed{60}$

118. What is the area of the trapezoid bounded by the x -axis, the vertical lines $x = 1$, $x = 4$, and the line

$y = -2x + 11$?



$A = \frac{1}{2}(9 + 3)3 = \boxed{18}$

119. Write 1997 as a product of primes.

$\boxed{1997}$ (prime)

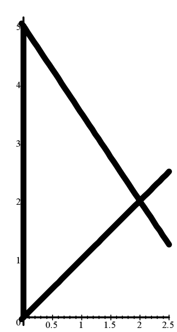
120. If 2 is the first prime, what is the seventh prime?

$2, 3, 5, 7, 11, 13, \boxed{17}, 19, 23, 29$

121. What number is halfway between $\frac{2}{3}$ and $\frac{3}{4}$?

$\frac{1}{2}\left(\frac{2}{3} + \frac{3}{4}\right) = \boxed{\frac{17}{24}}$

122. What is the area of the triangle bounded by the lines $x = 0$, $y = x$, and $2y + 3x = 10$?



Vertices $(0, 0)$, $(0, 5)$, $(2, 2)$, base 5, height 2, area $\boxed{5}$

123. Use the approximation $2^{10} \approx 10^3$ to approximate 2^{43} as a number in scientific notation $2^{43} \approx c \times 10^n$, where n is an integer and c is a number between 1 and 10.

$2^{43} = 8 \times \left(2^{10}\right)^4 \approx 8 \times \left(10^3\right)^4 = \boxed{8 \times 10^{12}} = 8000000000000$

$$\boxed{2^{43} = 8.7961 \times 10^{12} \text{ so estimate is a bit on the low side}}$$

124. The sum of 4 consecutive integers is 74. What is the smallest of the 4 integers?

$$\boxed{74 = \boxed{17} + 18 + 19 + 20}$$

125. How many different 6-letter words can be formed by rearranging the letters in SCHOOL?

$$\boxed{\frac{6!}{2!} = \boxed{360}}$$

126. Which mathematician amazed his grade school teacher by quickly summing the integers from 1 to 100?

$$\boxed{\text{Gauss}}$$

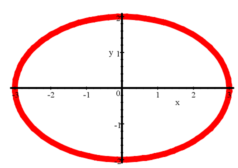
127. The average of 3 numbers is 19. What is the average of these 3 numbers together with 27?

$$\boxed{\frac{3\times 19+27}{4} = \boxed{21}}$$

128. Sally invested \$4,000, part at 5% and the rest at 4%. If the annual interest income from both investments was \$175, how much was invested at 5%?

$$\boxed{.05x + .04(4000 - x) = 175, \text{ Solution is : } x = \boxed{\$1500.00}}$$

129. What is the area enclosed by the ellipse $\dfrac{x^2}{3^2} + \dfrac{y^2}{2^2} = 1$?



$$\boxed{3 \times 2 \times \pi = \boxed{6\pi}}$$

130. The height of a rectangle is 25% less than its base. The perimeter of the rectangle is 42 inches. Find the area of the rectangle.

$$\boxed{2x + 2 \cdot \tfrac{3}{4}x = 42, \text{ Solution is : } x = 12,\ Area = 12 \cdot 9 = \boxed{108}\text{in}^2}$$

131. Express the perimeter P of a square as a function of its area A.

$$\boxed{A = \left(\tfrac{P}{4}\right)^2, \text{ Solution is : } \boxed{P = 4\sqrt{A}}}$$

132. List the following three numbers in increasing order: 2^8, 3^5, 6^3.

$$\boxed{\boxed{6^3} = 216 < \boxed{3^5} = 243 < \boxed{2^8} = 256}$$

133. What mathematician popularized the use of δ and ϵ in proofs involving limits?

$$\boxed{\text{ANS: Augustin-Louis } \boxed{\text{Cauchy}}}$$

134. What is the area of an equilateral triangle inscribed in a circle of radius 4 inches?

$$\boxed{\boxed{12\sqrt{3}}\text{ in}^2}$$

135. The number 6 is perfect because $6 = 1 + 2 + 3$ is the sum of the proper divisors. Name the next perfect

number.

$$\boxed{\boxed{28} = 1+2+4+7+14}$$

136. How many distinct complex roots does the polynomial x^6+1 have?

$$\boxed{6}$$

137. Give an equation in the form $ax+by=c$, where a, b, and c are integers, for the line through $(5,-2)$ that is perpendicular to the line $3x-2y=5$.

$$\boxed{\boxed{2x+3y=4}}$$

138. If the first term of a geometric sequence is $\frac{1}{3}$ and the second term is $\frac{1}{2}$, what is the fifth term?

$$\boxed{\frac{1}{3}\left(\frac{3}{2}\right)^4 = \boxed{\frac{27}{16}}}$$

139. If a mantel clock strikes the hours, how many times will it strike during a 24-hour period?

$$\boxed{2\sum_{i=1}^{12} i = \boxed{156}}$$

140. Which of the following best describes how many 6-symbol license plates are possible if the symbols come from the letters A-Z together with the digits 0-9? (a) 2–3 million (b) 20–30 million (c) 200–300 million (d) 2–3 billion

$$\boxed{36^6 = 21767\,82336 \approx \boxed{\text{2–3 billion}} \text{ or } \boxed{\text{(d)}}}$$

141. John got a 10% raise last year and another 10% raise this year and his current salary is $\$24,200$. What was his salary before last year's raise?

$$\boxed{\text{ANS: } x\left(11/10\right)^2 = 24200\text{, so } x = \frac{100}{121}(24200) = \boxed{\$20,000}}$$

142. A circle of radius 1 is divided into 5 pieces. One of the pieces is 1/2 as large as each of the other four. What is the area of the smallest piece?

$$\boxed{\frac{\pi}{9}}$$

143. A 6-foot man casts a 4-foot shadow. A flag pole next to him casts a 50-foot shadow. How tall is the flag pole?

$$\boxed{\boxed{75}\text{ feet}}$$

144. The diagonal of a table with a square top is 6 feet. What is the area of the table top?

$$\boxed{\boxed{18}\text{ ft}^2}$$

145. Three unit circles are mutually tangent and enclose a triangular region R. Find the area of R.

$$\boxed{\sqrt{3}-\frac{\pi}{2}}$$

146. Five straight lines are drawn in the plane. What is the largest possible number of points of intersection?

$$\boxed{\binom{5}{2} = \boxed{10}}$$

147. List the following three numbers in increasing order: $x = 1+2+3+\cdots+100$, $y = 1\times2\times3\times4\times5\times6\times7$,

and $z = 4^6$.

$$\boxed{z} = 4^6 = 4096 < \boxed{y} = 7! = 5040 < \boxed{x} = \sum_{i=1}^{100} i = 5050$$

148. Pierr de Fermat was not a professional mathematician, although such famous results as Fermat's Last Theorem and Fermat's Little Theorem were named in his honor. What was his profession?

$\boxed{\text{Lawyer}}$

149. What mathematician introduced the dy/dx notation in calculus?

$\boxed{\text{ANS: Gottfried Wilhelm } \boxed{\text{Leibniz}}}$

150. The number 12 is abundant because 12 is less than the sum $1 + 2 + 3 + 4 + 6 = 16$ of its proper divisors. Name the next two abundant numbers after 12.

$$\boxed{\boxed{18} < 1 + 2 + 3 + 6 + 9 = 21, \boxed{20} < 1 + 2 + 4 + 5 + 10 = 22}$$

151. Assuming $0 < a < b$, express $\frac{a^b b^a}{a^a b^b}$ in terms of one quotient raised to a positive exponent.

152. If $f(x) = x^{5/4}$, what is $f(16)$?.

$$\boxed{f(x) = x^{5/4},\ f(16) = \boxed{32}}$$

153. What is the base 8 representation of the number 83?

$$\boxed{83 = 64 + 16 + 3 = 1 \times 8^2 + 2 \times 8 + 3 = \boxed{123} \text{ or } \boxed{123_8}}$$

154. What is the sum of the first 100 positive integers?

$$\boxed{\sum_{i=1}^{100} i = \boxed{5050}}$$

155. In how many ways can one arrange the letters in OBOE?

$$\boxed{\tfrac{4!}{2!} = \boxed{12}}$$

156. What is the area of a regular hexagon with sides of length 2?

$$\boxed{6\sqrt{3}}$$

157. Find the area of the circle inscribed in a regular hexagon with sides of length 2.

$$\boxed{3\pi}$$

What is the size of an interior angle of an octagon?

$$\boxed{135^\circ = \tfrac{3}{4}\pi}$$

158. Two sides of a nontrivial triangle have lengths 5 and 8. What is the smallest integer length for the third side?

$$\boxed{4}$$

159. A square has area α in^2 and has perimeter α in. What is α?

$$\boxed{\alpha = x^2 = 4x \text{ so } x = 4,\ \alpha = \boxed{16}}$$

160. The plane can be completely tiled using equilateral triangles, or using squares, or using regular hexagons.

These give the three regular tessellations of the plane. A semi-regular tessellation is a tiling that uses at least two different regular polygons, and where every vertex is congruent to every other vertex. How many semi-regular tessellations are there?

$$\boxed{8}$$

161. List the following three numbers in increasing order: $\sqrt{2}$, $\sqrt[3]{3}$, $\sqrt[6]{10}$.

$$\boxed{\boxed{\sqrt{2}} = 1.4142 < \boxed{\sqrt[3]{3}} = 1.4422 < \boxed{\sqrt[6]{10}} = 1.4678}$$

$$\boxed{\left(\boxed{2^{1/2}}\right)^6 = 8 < \left(\boxed{3^{1/3}}\right)^6 = 9 < \left(\boxed{10^{1/6}}\right)^6 = 10}$$

162. A British mathematician who teaches at Princeton University has recently announced a proof of Fermat's Last Theorem. Name the mathematician.

$$\boxed{\text{Andrew Wiles}}$$

163. After what mathematician was the Cartesian coordinate system named?

$$\boxed{\text{ANS: (René) } \boxed{\text{Descartes}}}$$

164. What are the next two prime numbers greater than 50?

$$\boxed{51 = 3 \times 17, \boxed{53}, 55 = 5 \times 11, 57 = 3 \times 19, \boxed{59}}$$

165. Factor completely the integer $2^5 + 3^5$.

$$\boxed{2^5 + 3^5 = 275 = \boxed{5^2 11}}$$

166. Rewrite $\dfrac{5x - 13}{x^2 - 5x + 6}$ in the form $\dfrac{A}{Bx + C} + \dfrac{D}{Ex + F}$, where A, B, C, D, E, and F are integers.

$$\boxed{\frac{5x - 13}{x^2 - 5x + 6} = \boxed{\frac{3}{x - 2} + \frac{2}{x - 3}}}$$

167. Solve the equation $x\sqrt{.04} = 3$.

$$\boxed{\text{Solution is : } \boxed{x = 15.0}}$$

$$\boxed{\frac{a^b b^a}{a^a b^b} = \left(\tfrac{a}{b}\right)^b \left(\tfrac{b}{a}\right)^a = \boxed{\left(\tfrac{a}{b}\right)^{b-a}}}$$

168. What are the next 4 terms in the sequence that begins $1, 1, 2, 3, 5, 8$?

$$\boxed{\boxed{13, 21, 34, 55} \text{ (Fibonacci sequence)}}$$

169. A golf bag contains 2 white golf ball, 4 yellow balls, and 4 orange balls. Two golf balls are selected at random. What is the probability that both are white?

$$\boxed{\frac{\binom{2}{2}}{\binom{10}{2}} = \boxed{\frac{1}{45}}}$$

170. How many 3-member teams can be formed from a group of 5 women and 2 men?

$$\boxed{\binom{7}{3} = \boxed{35}}$$

171. Given an isosceles right triangle with a hypotenuse of length $\sqrt{50}$, what is its area?

$$\boxed{\frac{25}{2}}$$

172. What is the area of the smallest right triangle with all sides positive integers?

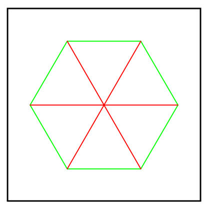
$$\boxed{3-4-5 \text{ right triangle has area } 3\cdot 2 = \boxed{6}}$$

173. The base of an isosceles triangle is 4 and the opposite vertex moves up and down. If the area of the triangle is plotted as a function of its height, what is the shape of graph?

$$\boxed{\text{ANS: } A = \frac{1}{2}(4)\,h = 2h \text{ or } \boxed{\text{straight line}}}$$

174. What is the area of a regular hexagon of edge 1?

$$\boxed{\text{ANS: 6 triangles of base 1, height } \sqrt{3}/2 = 6\times\left(\frac{1}{2}\right)\times 1\times\frac{\sqrt{3}}{2} = \boxed{\frac{3}{2}\sqrt{3}}}$$



175. A ladder leans against a vertical wall. The bottom of the ladder is 5 feet from the wall and the top of the ladder is 12 feet above the floor. How long is the ladder?

$$\boxed{\text{ANS: } \sqrt{5^2+12^2} = \boxed{13}\text{ ft}}$$

176. List the following three numbers in increasing order: 2^{10}, $6!$, 10^3.

$$\boxed{\boxed{6!} = 720 < \boxed{10^3} = 1000 < 2^{10} = \boxed{1024}}$$

177. What was the license plate number the Indian mathematician Ramanujan referred to when he said, "On the contrary, that's a very interesting number. It's the first number that can be written as the sum of 2 cubes in 2 different ways."

$$\boxed{12^3+1^3 = 10^3+9^3 = \boxed{1729}}$$

178. What mathematician first resolved the Königsberg bridge problem?

$$\boxed{\text{ANS: } \boxed{\text{Euler}}}$$

179. During the 1970's Steve Jobs and Steve Wozniak starting selling electronic equipment out of a garage. What company did they start?

$$\boxed{\text{ANS: } \boxed{\text{Apple}}\text{ Computer}}$$

180. What is the product of the next two primes numbers greater than 25?

$$\boxed{29\times 31 = \boxed{899} = 30^2 - 1^2}$$

181. What is the base 10 value of the base 5 number 222_5?

$$\boxed{2 + 2 \times 5 + 2 \times 5^2 = 2 + 10 + 50 = \boxed{62}}$$

182. What fraction is represented by the repeating decimal $0.\overline{45}$?

$$\boxed{\frac{45}{99} = \boxed{\frac{5}{11}}}$$

183. Partition 18 into two parts whose product is 77.

$$\boxed{\left\{ \begin{array}{c} x + y = 18 \\ xy = 77 \end{array} \right\}, \text{Solution is : } \boxed{x = 7, y = 11}}$$

184. In how many ways can ALLAN misspell his name, assuming he uses all the right letters (the right number of times)? (Spell out A-L-L-A-N.)

$$\boxed{\frac{5!}{2!2!} - 1 = \boxed{29}}$$

185. The average of the four numbers 11, 15, 17, and x is 23. What is x?

$$\boxed{\frac{11+15+17+x}{4} = 23, \text{Solution is : } \boxed{x = 49}}$$

186. If $m > 0$ and the points $(m, 3)$ and $(1, m)$ lie on a line with slope m, find m.

$$\boxed{\frac{3-m}{m-1} = m, \text{Solution is : } \boxed{m = \sqrt{3}}}$$

187. Let $y = mx + b$ be the image when the line $x - 3y + 11 = 0$ is reflected across the x-axis. What is $m + b$?

$$\boxed{y = -\frac{1}{3}x - \frac{11}{3}, m + b = -\frac{1}{3} - \frac{11}{3} = \boxed{-4}}$$

188. Find the fourth vertex of a rectangle, three of whose vertices are $(-2, 1)$, $(-1, -1)$, and $(1, 0)$.

$$\boxed{\text{The fourth vertex is } \boxed{(0, 2)}}$$

189. What is the midpoint of the line segment joint $(2, 8)$ and $(-4, 6)$?

$$\boxed{(\frac{2-4}{2}, \frac{6+8}{2}) = \boxed{(-1, 7)}}$$

190. Where does the circle of radius 5 centered at the origin intersect the line passing through the origin with slope $-3/4$?

$$\boxed{\left\{ \begin{array}{c} x^2 + y^2 = 25 \\ y = -\frac{3}{4}x \end{array} \right\}, \text{Solution is : } \boxed{\{y = 3, x = -4\}, \{y = -3, x = 4\}}}$$

191. What is the largest integer $\leqq \sqrt{700}$?

$$\boxed{\sqrt{700} = 26.45751311, \text{ so } \boxed{26}}$$

192. List the following solids in order increasing volume: A sphere of radius 3, a cube of edge 5, and a pyramid whose base is a square of edge 8 and height 6.

$$\boxed{\frac{4}{3}\pi 3^3 = 113.1 < 5^3 = 125 < \frac{1}{3}8^2 6 = 128}$$

$\boxed{\boxed{\text{Sphere}} < \boxed{\text{Cube}} < \boxed{\text{Pyramid}}}$

193. Paul Erdös (pronounced "Air′-dish") wrote hundreds of papers in mathematics, many of which were coauthored by various mathematicians representing nearly every country on Earth. In which country was Paul Erdös born?

$\boxed{\text{Hungary}}$

194. A mathematician named Wolfram started a company named Wolfram Research. What is its primary product?

$\boxed{\text{ANS: } \boxed{\text{Mathematica}}}$

195. What is the smallest positive integer n such that $n^2 - n + 41$ is not prime?

n	1	2	3	4	5	$\cdots$	40	$\boxed{41}$
$n^2 - n + 41$	41	43	47	53	61	$\cdots$	1601	$1681 = 41^2$

196. Find the next two terms of the sequence that begins $3, 7, 12, 18, 25$.

$\boxed{\boxed{33}, \boxed{42}}$

$\boxed{3 + 4 = 7,\ 7 + 5 = 12,\ 12 + 6 = 18,\ 18 + 7 = 25,\ 25 + 8 = \boxed{33},\ 33 + 9 = \boxed{42}}$

197. What is $144°$ equal to in radians?

$$144° = \boxed{\frac{4\pi}{5}}$$

198. What is the minimum value of the function $f(x) = x^2 - 6x + 7$?

$\boxed{x^2 - 6x + 7 = (x - 3)^2 - 2 \text{ has minimum value } \boxed{-2}}$

199. The 24 ways to write rearrangements of the letters MATH are listed in alphabetical order. Where in this list does MATH appear?

$\boxed{\boxed{\text{14th on the list}} \text{ right after MAHT and before MHAT}}$

200. In how many ways can you have \$10 worth of dimes and quarters?

$\boxed{21}$

201. A square is inscribed in a circle, which in turn is inscribed in a square. What percentage of the area of the large square is inside the small square?

$\boxed{\text{ANS: } \boxed{50\%}}$

202. A triangular section of Old Town is divided into a smaller triangle and two trapezoids by two streets parallel to one of the boundary streets. The heights of the two trapezoids are equal to the height of the small triangle, and the area of the middle trapezoid is 12 acres. How many acres are there in the larger trapezoid?

$\boxed{\text{ANS: } \boxed{20} \text{ acres}}$

203. How many edges does an octahedron have?

$\boxed{\frac{8 \cdot 3}{2} = \boxed{12}}$

204. A Spanish port was protected by large cannons, and each cannon had a pile of cannon balls nearby stacked neatly in the shape of a tetrahedron. If each bottom layer contained a total of 21 cannon balls, how many cannon balls were in each pile?

$$\boxed{21 + 15 + 10 + 6 + 3 + 1 = \boxed{56}}$$

205. How many edges does a regular tetrahedron have?

$$\boxed{\frac{3 \cdot 4}{2} = \boxed{6}}$$

206. List the following three numbers in increasing order: $\frac{3}{7}, \frac{4}{9}, \frac{7}{16}$

$$\boxed{\boxed{\frac{3}{7}} = .428\,57 < \boxed{\frac{7}{16}} = .437\,5 < \boxed{\frac{4}{9}} = .444\,44}$$

$$\boxed{\text{Note that } \frac{7}{16} = \frac{3+4}{7+9} \text{ is between } \frac{3}{7} \text{ and } \frac{4}{9}}$$

207. List the following three numbers in increasing order: $\sqrt{50}$, $\sqrt[3]{200}$, $\sqrt[4]{1300}$.

$$\boxed{\boxed{\sqrt[3]{200}} = 5.848 < \boxed{\sqrt[4]{1300}} = 6.004\,6 < \boxed{\sqrt{50}} = 7.071\,1}$$

$$\boxed{\boxed{\sqrt[3]{200}} < \sqrt[3]{216} = 6 = \sqrt[4]{1296} < \boxed{\sqrt[4]{1300}} < 7 = \sqrt{49} < \boxed{\sqrt{50}}}$$

208. In the early 1900s a self-taught Indian mathematician sent some of his unusual mathematical formulas to the English mathematician G. H. Hardy, who recognized the depth of the formulas and invited this mathematician to England. This exceptional Indian mathematician was the subject of a public television documentary. A biography, The man Who Knew Infinity, was published in 1991. Name this mathematician.

$$\boxed{\text{ANS: Srinivasa } \boxed{\text{Ramanujan}}}$$

209. According to Descartes' Rule of Signs, how many positive real roots does the polynomial equation $3x^4 + 10x^2 + 5x - 4 = 0$ have?

$$\boxed{\text{ANS: } \boxed{\text{one}} \text{ (since there is one sign change)}}$$

210. What is the least common multiple of 91 and 119?

$$\boxed{\operatorname{lcm}(91, 119) = \frac{91 \times 119}{7} = \boxed{1547}}$$

211. If the absolute value of $x + 1$ is equal to the absolute value of $x - 1$, what is x?

$$\boxed{|x + 1| = |x - 1|, \text{ Solution is : } \boxed{x = 0}}$$

212. What is the sum of all the integers greater than 10 and less than 20?

$$\boxed{\sum_{i=11}^{19} i = \boxed{135} = \frac{19 \cdot 20}{2} - \frac{10 \cdot 11}{2}}$$

$$\boxed{11 + 12 + 13 + 14 + 15 + 16 + 17 + 18 + 19 = \boxed{135}}$$

213. The sum of the squares of two positive integers is 225, and the difference of their squares is 63. What are the numbers?

$$\boxed{\left\{ \begin{array}{c} x^2 + y^2 = 225 \\ x^2 - y^2 = 63 \end{array} \right\}, \text{ Solution is : } \boxed{x = 9, y = 12}}$$

214. Two dice are rolled. What is the probability that the sum is a 3, a 6, or an 8?

$$\boxed{\frac{2}{36} + \frac{5}{36} + \frac{5}{36} = \boxed{\frac{1}{3}}}$$

215. How many 3-digit numbers can be made (with no repetitions) using only the digits 1, 3, 5, 7, 9?

$$\boxed{5 \cdot 4 \cdot 3 = \boxed{60}}$$

216. The 12 faces of a regular dodecahedron are pentagons. How many edges does a regular dodecahedron have?

$$\boxed{\text{ANS: } \frac{12 \times 5}{2} = \boxed{30}}$$

217. A wooden cube of edge 4 inches is painted red. The cube is then cut into 64 one-inch cubes by making 9 saw cuts. How many of the one-inch cubes have exactly 3 red faces?

$$\boxed{\text{ANS: 8 corners} \times \text{1 cube per corner} = \boxed{8} \text{ cubes}}$$

218. The distance between the points $(2, 4)$ and $(7, c)$ is 13. Find all the possible values for c.

$$\boxed{(2-7)^2 + (4-c)^2 = 13^2, \text{ Solution is : } \boxed{\{c = 16\}, \{c = -8\}}}$$

219. Give the points of intersection of the two curves $y = 20 - 6x$ and $y = 8 - 6x + 3x^2$.

$$\boxed{\left\{ \begin{array}{c} y = 8 - 6x + 3x^2 \\ y = 20 - 6x \end{array} \right\}, \text{ Solution is : } \boxed{(2, 8), (-2, 32)}}$$

220. Find a point equidistant from the points $(-1, -1)$, $(1, 1)$, and $(1, -1)$.

$$\boxed{(0, 0)}$$

221. List the following three numbers in increasing order: e^π, π^e, 3^3.

$$\boxed{\boxed{\pi^e} = 22.459 < \boxed{e^\pi} = 23.141 < \boxed{3^3} = 27}$$

222. List the following three numbers in increasing order: π, $22/7$, and 3.14?

$$\boxed{\boxed{3.14} < \boxed{\pi} \approx 3.141592654 < \boxed{22/7} \approx 3.1429}$$

223. In 1962 a mathematician named Edward O. Thorp wrote a book entitled Beat the Dealer that claimed to give a winning strategy, based upon large-scale computer simulations, for a certain card game. What is the name of that card game?

$$\boxed{\boxed{\text{Blackjack}} \text{ or } \boxed{21}}$$

224. What famous mathematical object was named after the mathematician David Hilbert?

$$\boxed{\text{ANS: } \boxed{\text{Hilbert Space}}}$$

225. Completely factor $10! = 3628\,800$ as a product of powers of primes.

$$\boxed{10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 = \boxed{2^8 3^4 5^2 7}}$$

226. A shelf will hold 6 calculus textbooks and 10 algebra textbooks, or 3 calculus textbooks and 15 algebra

textbooks. How many calculus books alone will the shelf hold?

$$\left\{ \begin{array}{c} 6c + 10a = 1 \\ 3c + 15a = 1 \end{array} \right\}, \text{Solution is : } c = \frac{1}{12}, a = \frac{1}{20} \boxed{\text{Shelf will hold 12 calculus books}}$$

227. Partition 25 into two parts such that the difference of their square roots is 1.

$$\text{Solution is : } \boxed{9, 16}$$

228. What is x if $128^x = 32^7$?

$$\left(2^7\right)^x = \left(2^5\right)^7 \Rightarrow \boxed{x = 5}$$

$$128^{\boxed{5}} = 34\,359\,738\,368,\ 32^7 = 34\,359\,738\,368$$

229. Barbara got a 73 on the first exam and 81 on the second. What must she average on the next two exams to have an overall average of 85 on the four exams?

$$\frac{73+81+2x}{4} = 85, \text{Solution is : } x = \boxed{93}$$

230. If the probability that the Nuggets beat the Suns is 0.3, what is the probability that the Nuggets beat the Suns 4 times in a row?

$$(0.3)^4 = \boxed{.0081}$$

231. How far apart are the two points with the curves $y = x + 6$ and $y = x^2$ intersect?

$$\left\{ \begin{array}{c} y = x + 6 \\ y = x^2 \end{array} \right\}, \text{Solution is : } \{(-2, 4), (3, 9)\}, \text{distance } \boxed{5\sqrt{2}}$$

232. In which quadrant do the two lines $y = 4 - 2x$ and $y = 18 + 5$ intersect?

$$\text{Point } (-2, 8) \text{ is in the } \boxed{\text{second quadrant}}$$

233. How far apart are the two points at which the curves $x = 5$ and $x^2 + y^2 = 169$ intersect?

$$\left\{ \begin{array}{c} x = 5 \\ x^2 + y^2 = 169 \end{array} \right\}, \text{Solution is : } (5, 12), (5, -12), \text{distance } = \boxed{24}$$

234. If the line $y = mx$ touches the curve $y = x^2 + 1$ in exactly 1 point, what are the possibilities for m?

$$m = \boxed{\pm 2}$$

235. Give the slope-intercept equation for a line through the origin that contains no other points with integer coordinates.

$$\boxed{y = \alpha x, \alpha \text{ any irrational number}}$$

236. What is the greatest integer in the sum $\frac{11}{3} + \frac{3}{11}$?

$$\frac{11}{3} + \frac{3}{11} = \frac{130}{33} = 3.939393939, \text{ so greatest integer is } \boxed{3}$$

237. What is the smallest perfect cube larger than 100?

$$4^3 = 64, 5^3 = \boxed{125}$$

238. This 20th century American mathematician introduced game theory as a mathematical discipline,

conceived the idea of a self-stored computer program, and worked on the Manhattan project that developed the atomic bomb. Name this person.

ANS: John [von Neuman]

239. George Boole developed a mathematical system called Boolean algebra. What is the value of $1 + 1$ under this system?

ANS: $1 + 1 = \boxed{1}$

240. Find an integer between 100 and 1000 that is both a perfect square and a perfect cube.

ANS: $\boxed{729} = 9^3 = 27^2$