

Math Day 1996

Team Competition

Colorado State University

1. A rectangle's length is increased by 30% and its width is decreased by 30%. How does its area change?

$$\text{ANS: } 1.3L \cdot .7W = .91LW \text{ or decreases by } 9\%$$

2. A pyramid is build out of cubical blocks by placing 36 blocks on the floor, 25 blocks on top of the bottom layer, and so forth. How many cubes are required to build the pyramid?

$$\text{ANS: } \sum_{i=1}^6 i^2 = 91, \frac{6 \cdot 7 \cdot 13}{6} = 91$$

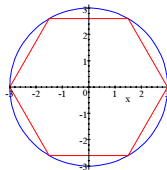
3. If a pair of fair dice is rolled, what is the probability that the sum is 5?

$$\text{ANS: } \frac{4}{36} = \frac{1}{9}$$

4. What fact about the number 1729 did the Indian mathematician Ramanujan consider interesting?

$$\text{ANS: Smallest positive integer = sum of cubes in 2 ways: } 12^3 + 1^3 = 10^3 + 9^3$$

5. A regular hexagon is inscribed in a circle of radius 3 cm. What is the perimeter of the hexagon?



$$\text{ANS: } 6 \cdot 3 = 18 \text{ cm}$$

6. What positive number is five times as big as its reciprocal?

$$\text{ANS: } x = \frac{5}{x} \text{ implies } x = \sqrt{5}$$

7. If the absolute value of $x^2 + 4$ is equal to the absolute value of $x^2 - 12$, what is x ?

$$\text{ANS: } |x^2 + 4| = |x^2 - 12|, \text{ Solution is : } x = 2, x = -2$$

8. Two wheels are connected by a drive belt. One has a diameter of 15 centimeters and a speed of 135 rpm. The other has a speed of 72 rpm. What is its diameter?

$$\text{ANS: } 15 \cdot 135 = 75x, \text{ Solution is : } x = 27 \text{ cm}$$

9. Six people are in the same room. If everyone shakes hands with everyone else in the room, how many handshakes take place?

$$\text{ANS: } 5 + 4 + 3 + 2 + 1 = 15$$

10. Find the Greatest Common Divisor of 121 and 143.

$$\text{ANS: gcd } 121, 143 = \boxed{11}$$

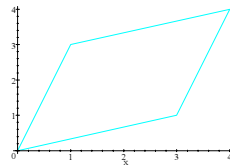
11. Out of a group of ten members, in how many different ways can a president, vice-president, and treasurer be elected?

$$\text{ANS: } 10 \cdot 9 \cdot 8 = \boxed{720}$$

12. Find all the integer solutions of the equation $x^4 - 14x^2 - 32 = 0$.

$$\text{ANS: } x^4 - 14x^2 - 32 = (x - 4)(x + 4)(x^2 + 2), \text{ so } \boxed{x = 4, -4}$$

13. The vertices of a quadrilateral are at the points $(0, 0)$, $(3, 1)$, $(4, 4)$, and $(1, 3)$. What is the area of the quadrilateral?



$$\text{Area} = \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} = \boxed{8}$$

14. If $f(x) = 3x - 4$, what is $f(f(1.5))$?

$$\text{ANS: } f(f(1.5)) = \boxed{-2.5}$$

15. What famous mathematical object was named after the mathematician David Hilbert?

$$\text{ANS: } \boxed{\text{Hilbert Space}}$$

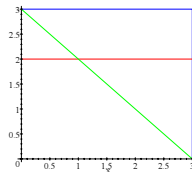
16. A shirt has been marked down 25% twice. The current price is \$18. What was the original price?

$$\text{ANS: } (.75)(.75)x = 18, \text{ Solution is } : x = \boxed{\$32.00}$$

17. Two evenly-matched baseball teams, the Braves and the Yankees, start a 7-game series. In how many different ways can the Yankees win the series four games to two?

$$\text{ANS: The Yankees win game 6 plus 3 of the first 5, so } \frac{5!}{3!2!} = \binom{5}{3} = \boxed{10}$$

18. An ant and a drop of honey are at diagonally opposite corners of a rectangular box of dimensions 1 by 2 by 3. What is the shortest distance the ant can travel to reach the (stationary) drop of honey by crawling along walls?



$$\text{ANS: } \sqrt{3^2 + 3^2} = \sqrt{18} = \boxed{3\sqrt{2}}$$

19. Name a mathematician who made early contributions to differential calculus.

ANS: Pierre Fermat or John Wallis or Isaac Newton

or Gottfried Leibniz or Isaac Barrow

20. Two positive integers have a sum of 9. What is the smallest possible value for the sum of their cubes?

ANS: $4^3 + 5^3 = 189$

21. How many minutes were there in October of 1996?

ANS: $31 \cdot 24 \cdot 60 = 44640$

22. What is the largest prime less than 500?

ANS: `nextprime(491)` = 499

23. How many possible ways are there to pick the first place, second place, and third place in a 10-person race?

ANS: $10 \cdot 9 \cdot 8 = 720$

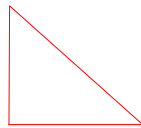
24. If $f(x)$ is a linear function with $f(2) = -3$ and $f(5) = 6$, then what is $f(1)$?

ANS: $f(1) = -6$

25. What are the dimensions of a rectangle with area 110 and perimeter 42?

ANS: $ab = 110$
 $2a + 2b = 42$, Solution is : $a = 11$ by $b = 10$

26. Visually, an 8-9-12 triangle appears to be a right triangle. Is it actually acute, or is it obtuse?



$\approx 89.602109^\circ$ or $145 = 8^2 + 9^2 > 12^2 = 144$

27. One fifth of the air in a container is removed by each cycle of a vacuum pump. What fractional part of the air remains after 3 cycles?

ANS: $\left(\frac{4}{5}\right)^3 = \frac{64}{125}$

28. Find C so that the equation $x^2 - 6x + C = 0$ has exactly one real root.

ANS: $x = 3 \pm \sqrt{9 - C}$, same when $C = 9$

29. What is the 12th term of the arithmetic sequence that begins 59, 55, 51, ...?

ANS: $f(n) = 63 - 4n, f(12) = 15$

30. What Greek philosopher raised paradoxes that argued that motion is impossible?

ANS: Zeno of Elea

31. Jane's initial salary of \$25,000 was cut 10% last year, but this year she got a 10% raise. What is her current salary?

ANS: $\$25,000 (1 - .1) (1 + .1) = \$24,750$

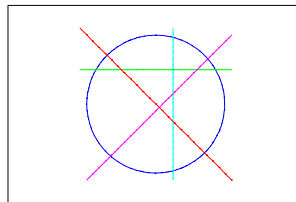
32. Two baseball teams, the Braves and the Yankees, begin a 7-game series. The odds makers give the green Braves 5 to 3 odds of winning the first game. Assuming the odds makers are good at their job, what is the probability that the Braves will win the first game?

ANS: $\frac{5}{8}$

33. In how many ways can 3 different math books, 2 different stat books, and 1 physics book be arranged on a shelf, assuming the math books must be together and the stat books must be together?

ANS: $3!(3! 2! 1!) = 72$

34. What is the maximum number of pieces into which a circular pizza can be cut using 4 chops of a knife (with no intermediate rearrangements of the pieces)?



ANS: 11

35. How bits are there in a nibble?

ANS: 4 (a nibble is half a byte, which is 8 bits)

36. If the interior angle of a regular polygon is 165° , how many sides does it have?

ANS: Central angle is 15° and $\frac{360}{15} = 24$ sides

37. How many primes are there between 100 and 110?

ANS: Four (101, 103, 107, 109)

38. What mathematician popularized the use of δ and ϵ in proofs involving limits?

ANS: Augustin-Louis Cauchy

39. If $f(x) = x^2 - 1$, what is $f(f(2))$?

$$\text{ANS: } f(f(f(2))) = \boxed{63}$$

40. If $f(x) = x^{4/3}$, what is $f(125)$?

$$\text{ANS: } f(125) = \boxed{625}$$

41. What is the coefficient of x^3 in the expansion of $(3x + 2)^4$?

$$\text{ANS: } (3x + 2)^4 = 81x^4 + \boxed{216}x^3 + 216x^2 + 96x + 16$$

$$\text{ANS: } 3^3 \cdot 2 \cdot \binom{4}{3} = \boxed{216}$$

42. If a single 60-Watt bulb provides sufficient light to read a newspaper 5 feet from the bulb, how many 60-Watt bulbs are required in a light fixture 15 feet from the newspaper in order to provide the same apparent level of brightness?

$$\text{ANS: } \left(\frac{15}{5}\right)^2 = \boxed{9} \text{ bulbs}$$

43. A 14-inch pizza is cut into 8 congruent slices. What is the perimeter of each slice?

$$\text{ANS: } C = 14\pi \text{ so } P = \frac{14\pi}{8} + 2 \cdot 7 = \boxed{\frac{7}{4}\pi + 14} \text{ in}$$

44. Reduce the fraction 63/91 to simplest terms.

$$\text{ANS: } \frac{63}{91} = \frac{9 \cdot 7}{13 \cdot 7} = \boxed{\frac{9}{13}}$$

45. What mathematician introduced the dy/dx notation in calculus?

$$\text{ANS: } \text{Gottfried Wilhelm } \boxed{\text{Leibniz}}$$

46. John got a 20% raise last year and another 20% raise this year and his current salary is \$23,040. What was his salary before last year's raise?

$$\text{ANS: } x(1.2/10)^2 = 23040, \text{ so } x = \frac{100}{144}(23040) = \boxed{\$16,000}$$

47. What is the area of a regular octagon of edge 1?

$$\text{ANS: } \text{Large square} - \text{four corners} = \left(1 + 2\left(\frac{1}{\sqrt{2}}\right)\right)^2 - 4\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)^2 = \boxed{2 + 2\sqrt{2}}$$

48. Sarah averages 50% on multiple choice exams. What is the probability that she gets at least 3 correct on a 5-question exam?

$$\text{ANS: } \left(\frac{1}{2}\right)^5 + 5\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right) + 10\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 = \boxed{\frac{1}{2}}$$

49. What is the Least Common Multiple of 63 and 49?

$$\text{ANS: } \text{lcm}(63, 49) = 7^2 \cdot 9 = \boxed{441}$$

50. How many different 6-place licence plates are possible if the first two places are letters and the last 4 places are digits?

$$\text{ANS: } 26^2 \cdot 10^4 = \boxed{6,760,000}$$

51. How many of the 1024 subsets of the set $1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ contain the integer 2?

ANS: Half or 512

52. What mathematician first resolved the Königsberg bridge problem?

ANS: Leonhard Euler

53. Name the smallest integer whose fourth power is less than 1000.

ANS: $(-5)^4 = 625$, $(-6)^4 = 1296$, etc. -5 is the smallest

54. To the nearest one-hundredth of an inch, what is the smallest diameter log needed for cutting a square post of dimensions 4 in by 4 in?

ANS: $4\sqrt{2} = 5.65685424949238 \approx 5.66$ in

55. Bo scored 87 on the first exam and 83 on the second. What is the lowest possible score on the third exam so that Bo's average for the three exams will be 90?

ANS: $\frac{x+87+83}{3} = 90$, Solution is $x = 100$

56. What number is halfway between $1/4$ and $1/3$?

ANS: $(\frac{1}{4} + \frac{1}{3})/2 = \frac{7}{24}$

57. Find a point equidistant from the three points $(-2, -2)$, $(2, 2)$, and $(2, -2)$.

ANS: $(0, 0)$

58. A bag of rabbit feed will feed 12 rabbits for 24 days. For how many days will it feed 16 rabbits?

ANS: $(24)\frac{12}{16} = 18$ days

59. Expand $(x^2 + 2y^3)^3$.

ANS: $(x^2 + 2y^3)^3 = x^6 + 6x^4y^3 + 12x^2y^6 + 8y^9$

60. What did the Norwegian mathematician Niels Henrik Abel prove about general fifth-degree polynomials?

ANS: Cannot be solved in terms of radicals involving the coefficients

61. A biological brick grows 20% in length, 10% in width, and shrinks in height by 25%. Is it larger or smaller than when it started out, and by what percentage has it changed?

ANS: $(1.2)(1.1)(.75) = .99$ 1% smaller

62. A wooden cube of edge 5 inches is painted red. The cube is then cut into 125 one-inch cubes by making 12 saw cuts. How many of the one-inch cubes have exactly 2 red faces?

ANS: 12 edges 3 cubes per edge = 36 cubes

63. Kyle uses pure guessing on a TRUE/FALSE exam. Which of the following options give Kyle the

best chance to score (at least) 50%? (A) A seven-question exam (guess correctly on 4, 5, 6, or 7 questions). (B) A five-question exam (guess correctly on 3, 4, or 5 questions). (C) The chances are equal.

ANS: (A) $P(\text{at least } 4/7) = \frac{1}{2}$ (B) $P(\text{at least } 3/5) = \frac{1}{2}$ (C) Chances are equal

64. How many different strings of length 7 can be formed using the letters in ELLIPSE?

ANS: $\frac{7!}{2!2!} = 1260$

65. Which regular polyhedron has the same number of faces as vertices?

ANS: Tetrahedron 4 faces and 4 vertices

66. Write 1996 as a product of primes.

ANS: $1996 = 2^2 \cdot 499$ or $2 \cdot 2 \cdot 499$

67. In how many ways can the U.S. Senate select a committee of 4 from among its 100 members?

ANS: $\binom{100}{4} = 3,921,225$

68. Over 350 years ago Pierre Fermat stated in a margin note that he had a proof that $x^n + y^n = z^n$ has no positive integer solutions for $n > 2$. No proof was ever found. Who recently has claimed to have solved this problem?

ANS: Andrew Wiles

69. Factor the polynomial $2x^3 - 5x^2 - 14x + 8$.

ANS: $2x^3 - 5x^2 - 14x + 8 = (2x - 1)(x + 2)(x - 4)$

70. The product of three consecutive integers is 210 and their sum is 18. What are the three numbers?

ANS: Average is $\frac{18}{3} = 6$, Solution is 5, 6, 7

71. Shortcut Lawn Care Services owns a fleet of 4 vehicles: One car, which gets 35 miles per gallon, a van, which gets 20 miles per gallon, and two pickups, each of which gets 15 miles per gallon. If in a typical week the car is driven 210 miles, the van 100 miles, and each truck is driven 240 miles, how many miles per gallon is the fleet getting, rounded to the nearest mile per gallon?

ANS: $\text{mi} = 780$, $\text{gal} = 6 + 5 + 32 = 43$, $\text{mpg} = \frac{780}{43} = 18.1395348837209$

72. What is the sum of the roots of the polynomial $x^2 - 53x + 5$?

ANS: $(x - a)(x - b) = x^2 - x(a + b) + ab$, so $a + b = 53$

73. What is the area of the ellipse $\frac{x^2}{36} + \frac{y^2}{49} = 1$?

ANS: $6 \cdot 7\pi = 42\pi$

74. What is the 21st term of an arithmetic sequence whose first three terms are -24 , -21 , and -18 ?

$$\text{ANS: } f(n) = -27 + 3n, f(1) = -5 = -24, f(21) = \boxed{36}$$

75. According to Descartes' Rule of Signs, how many positive real roots does the polynomial equation $3x^4 + 10x^2 + 5x + 4 = 0$ have?

ANS: (since there are no sign changes)

76. A gold watch has been reduced by 20%, then by 30%, and finally sold for \$648.00. What was the original price? $800 \cdot .8 \cdot .7 = 448.0$

$$\text{ANS: } (.8)(.7)x = 448, \text{ Solution is : } x = \boxed{\$800.00}$$

77. Sarah has a license plate with three letters and three digits, e.g. . What is the probability that at least one of the three digits is a 4?

$$\text{ANS: } 1 - \left(\frac{9}{10}\right)^3 = \frac{271}{1000} = \boxed{.271}$$

78. A quadrilateral kite is made with a right angle at the top, angles of 2α left and right, and an angle of α at the bottom. What is α ?

$$\text{ANS: } 5\alpha = 360^\circ - 90^\circ \quad \alpha = \boxed{54^\circ} \text{ or } \boxed{\frac{3\pi}{10}}$$

79. Bo and Jo raise goats and geese. If Bo counts 90 feet and Jo counts 56 eyes, how many goats and how many geese do Bo and Jo have?

$$\text{ANS: } \left\{ \begin{array}{l} 4a + 2b = 90 \\ 2a + 2b = 56 \end{array} \right\}, \text{ Solution is : } a = \boxed{17 \text{ goats}}, b = \boxed{11 \text{ geese}}$$

80. Find the Greatest Common Divisor of 63 and 91.

$$\text{ANS: } \text{gcd}(63, 91) = \boxed{7}$$

81. A diagonal of a rectangle is $\sqrt{13}$ and the area of the rectangle is 6. What are the dimensions of the rectangle?

$$\text{ANS: } \text{Solve } x^2 + y^2 = 13, xy = 6, \text{ Solution is : } y = 2, x = 3 \boxed{2 \quad 3}$$

82. Six poker players are seated at a round table. How many rearrangements are possible, if the only considerations are who is seated at each person's left, and at each person's right?

$$\text{ANS: } \frac{6!}{6} = \boxed{120}$$

83. What is the statement of Fermat's Last Theorem?

ANS:

84. The average of your first 5 exams is 83. What is your new average if you score 77 on your 6th exam?

$$\text{ANS: } \frac{5 \cdot 83 + 77}{6} = \boxed{82}$$

85. Solve the equation $x\sqrt{0.25} = 5$.

$$\text{ANS: } x\sqrt{0.25} = 5, \text{ Solution is : } x = \boxed{10.0}$$

86. A bicycle travels 10 miles per hour for 1 mile and 15 miles per hour for 1 mile. What is the average speed of the bike for the 2 miles?

$$\text{ANS: } d = rt \quad t = \frac{1}{10} + \frac{1}{15} \quad r = \frac{2}{\frac{1}{10} + \frac{1}{15}} = \boxed{12} \text{ mph}$$

87. If $3^p = 7$ then what is 9^p ?

$$\text{ANS: } 9^p = (3^2)^p = (3^p)^2 = 7^2 = \boxed{49}$$

88. An orange has a diameter that is 80% fruit and 20% peel. What percentage of the volume is the peel?

$$\text{ANS: } 1 - \left(\frac{4}{5}\right)^3 = \frac{61}{125} = .488 = \boxed{48.8\%}$$

89. Find x so that the average of the three numbers 12, x , and 50 is 30.

$$\text{ANS: } \frac{12+x+50}{3} = 30, \text{ Solution is : } x = \boxed{28}$$

90. The first famous woman mathematician was Hypatia, who lived from 370 to 415 A.D. Although none of her writings survive, it is believed that her work was included integer solutions to equations. What branch of mathematics deals with such problems?

$$\text{ANS: } \boxed{\text{Number Theory}} \text{ or } \boxed{\text{Diophantine Analysis}} \text{ (Simmons, } \textit{Calculus Gems})$$

91. The compact disk UR2ugly sells at outlet AC for \$15.49 less a discount of 20%, and at outlet DC for \$16.95 less a discount of 30%. Which outlet has the lower price?

$$\text{ANS: } \text{AC, } 15.49 \cdot .8 = \$12.39 \quad \boxed{\text{DC}}, 16.95 \cdot .7 = \$11.86$$

92. An equilateral triangle has vertices at (0,0) and (8,0). Give one set of possible coordinates for the third vertex.

$$\text{ANS: } (4,y) \text{ where } 4^2 + y^2 = 8^2, \text{ so vertex at } \boxed{(4, 4\sqrt{3})} \text{ or } \boxed{(4, -4\sqrt{3})}$$

93. Sarah is an 70% free throw shooter. What is the probability that she misses two in a row?

$$\text{ANS: } \left(\frac{3}{10}\right)^2 = \frac{9}{100} = \boxed{.09}$$

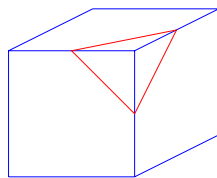
94. A Spanish port was protected by large cannons, and each cannon had a pile of cannon balls nearby stacked neatly in the shape of a tetrahedron. If each bottom layer contained a total of 28 cannon balls, how many cannon balls were there altogether in each pile?

$$\text{ANS: } 28 + 21 + 15 + 10 + 6 + 3 + 1 = \boxed{84}$$

95. If 2 is the first prime, what is the fifteenth prime?

$$\text{ANS: } 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, \boxed{47}$$

96. A cube has pyramids cut and discarded from each corner by passing planes through the midpoints of the edges adjacent to each of its vertices. How many edges does the new solid have?



$$\text{ANS: } 8 \text{ vertices } \cdot 3 \text{ edges/vertex} = \boxed{24} \text{ edges}$$

97. A committee of 3 people is to be chosen from among 4 men and 5 women. How many ways can this be done if the committee must include at least one man and at least one woman?

$$\text{ANS: } \binom{9}{3} - \binom{4}{3} - \binom{5}{3} = \boxed{70} = \binom{4}{2} \binom{5}{1} + \binom{4}{1} \binom{5}{2}$$

98. It is well known that the Hindus introduced zero into our number system, but that concept was also developed independently by other civilizations. What Central American civilization used a symbol for zero that resembled a half-open eye?

ANS: The Maya's (Source: Boyer and Merzbach, *A History of Mathematics*)

99. The average of your first 5 exams is 83. What is your new average if your lowest exam score of 71 is dropped?

$$\text{ANS: } \frac{5 \cdot 83 - 71}{4} = \boxed{86}$$

100. What is the largest perfect cube smaller than 2000?

$$\text{ANS: } 11^3 = 1331, 12^3 = \boxed{1728}, 13^3 = 2197$$

101. Find the vertex of the parabola $y = 3x^2 - 12x + 7$.

$$\text{ANS: } 3(x^2 - 4x + 4) + 7 - 12 = 3(x - 2)^2 - 5 \text{ vertex } \boxed{(2, -5)}$$

102. Bo has \$3.85 in quarters and dimes. If Bo has three times as many dimes as quarters, how many dimes does Bo have?

$$\text{ANS: } \left\{ \begin{array}{l} 25q + 10d = 385 \\ 3q = d \end{array} \right\}, \text{ Solution is : } d = \boxed{21}, q = 7$$

103. What is the sum of the roots of the polynomial $x^3 - 13x + 7$?

$$\text{ANS: } (x - a)(x - b)(x - c) = x^3 - (a + b + c)x^2 + (ab + ac + bc)x - abc \quad \boxed{\text{zero}}$$

104. What is the least common denominator of the two fractions $\frac{5}{91}$ and $\frac{3}{84}$.

$$\text{ANS: } \text{lcm}(84, 91) = \boxed{1092} = 2^2 \cdot 3 \cdot 7 \cdot 13$$

105. Name the Scottish mathematician who invented logarithms.

ANS: Napier

106. On January 1, 1996, an investment bond was purchased for \$1000. If it earns 10% compounded

annually, what will the balance be on January 1, 2000?

$$\text{ANS: } 1.1^4 \cdot 1000 = \boxed{\$1464.10}$$

107. The 12 faces of a regular dodecahedron are pentagons. How many edges does a regular dodecahedron have?

$$\text{ANS: } \frac{12 \cdot 5}{2} = \boxed{30}$$

108. Sarah is an 70% free throw shooter. What is her expected score if she shoots 2 free throws?

$$\text{ANS: } 2\left(\frac{7}{10}\right)^2 + 1\left(\frac{7}{10}\right)\left(\frac{3}{10}\right) + 1\left(\frac{3}{10}\right)\left(\frac{7}{10}\right) = \frac{7}{5} = \boxed{1.4}$$

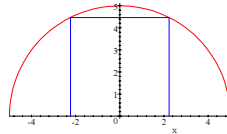
109. A pyramid is build out of cubical blocks by placing 64 blocks on the floor, 49 blocks on top of the bottom layer, and so forth. How many cubes are required to build the pyramid?

$$\text{ANS: } \sum_{i=1}^8 i^2 = \boxed{204} = \frac{8 \cdot 9 \cdot 17}{6}$$

110. The number 12 is called *abundant* because the sum of the proper divisors $1 + 2 + 3 + 4 + 6 = 16$ is greater than 12. What is the next abundant number?

$$\text{ANS: } \boxed{18} < 1 + 2 + 3 + 6 + 9 = 21$$

111. A square is inscribed in a semicircle of radius 5 feet. Find the area of the square.



$$\text{ANS: } \sin \theta = 2 \cos \theta \text{ Solution is: } \theta = \arctan 2, (5 \sin \theta)^2 = \boxed{20} \text{ ft}^2$$

112. What is the smallest positive integer n such that $n^2 - n + 11$ is *not* prime.

$$\text{ANS: } \boxed{11} \text{ since } 11, 13, 17, 23, 31, 41, 53, 67, 83, 101 \text{ are prime but } 11^2 - 11 + 11 = 11^2$$

113. How far apart are the two points at which the curves $x^2 + y^2 = 169$ and $x = 5$ intersect?

$$\text{ANS: } \left\{ \begin{array}{l} x = 5 \\ x^2 + y^2 = 169 \end{array} \right\}, \text{ Points } (5, 12) \text{ and } (5, -12), \text{ distance } \boxed{24}$$

114. A computer sequentially computes integers by the following rule: If n is odd then add 5 and divide the result by 2, otherwise add 3. Starting at $n = 13$, what is the integer after 6 iterations?

$$\text{ANS: } 13 \rightarrow \frac{13+5}{2} = 9 \rightarrow \frac{9+5}{2} = 7 \rightarrow \frac{7+5}{2} = 6 \rightarrow 9 \rightarrow 7 \rightarrow 6$$

115. When this Hungarian mathematician died recently, the New York Times acknowledged his contributions to mathematics that included some 1500 published papers. Name this person.

$$\text{ANS: Paul } \boxed{\text{Erdős}}$$

116. What is the greatest integer less than the sum $\frac{17}{7} + \frac{7}{17}$?

$$\text{ANS: } \frac{17}{7} + \frac{7}{17} = 2.84035 = \boxed{2} + .84035$$

117. An automobile is driven 20,000 miles with 5 tires rotated often for even wear. How many miles are there on each tire?

$$\text{ANS: } \frac{20000 \cdot 4}{5} = \boxed{16,000} \text{ miles}$$

118. What is the smallest value of the expression $x + \frac{1}{x}$ if x is a positive real number?

$$\text{ANS: } \left[x + \frac{1}{x} \right]_{x=1} = \boxed{2}$$

119. The polynomial $5x^7 + 21x^5 + 35x^3 + 35x + 18$ has one real root. How many imaginary roots does it have?

$$\text{ANS: } \boxed{\text{six}}$$

120. If a right triangle has hypotenuse c and legs a and b , then $c^2 = a^2 + b^2$. After whom is this result named?

$$\text{ANS: } \boxed{\text{Pathagoras}}$$

121. Six people are in the same room. If everyone shakes hands with everyone else in the room, how many handshakes take place?

$$\text{ANS: } 5 + 4 + 3 + 2 + 1 = \boxed{15}$$

122. Sarah is an 70% free throw shooter. What is her expected score if she shoots a one and one (if she makes the first she gets a second chance)?

$$\text{ANS: } 2\left(\frac{7}{10}\right)^2 + 1\left(\frac{7}{10}\right)\left(\frac{3}{10}\right) = \frac{119}{100} = \boxed{1.19}$$

123. How many positive divisors (including itself) does the number 81 have?

$$\text{ANS: } \boxed{5} \text{ divisors are } \{1,3,9,27,81\}$$

124. A regular icosahedron has 20 faces, each of which is an equilateral triangle. If the midpoint of each face is connected with an edge to the midpoint of each adjacent face, what solid do these new edges determine?

$$\text{ANS: } \boxed{\text{Dodecahedron}} \text{ (12 faces, each face is a pentagon.)}$$

125. Neglecting order of addition, in how many ways can 25 be written as a sum of 3 distinct primes?

$$\text{ANS: } 25 = 3 + 5 + 17 = 5 + 7 + 13, \boxed{2 \text{ ways}}$$

126. This cipher scrambled messages by assigning numbers to letters and creating ciphertext by adding 3 and reducing the result modulo 26. Name this cipher.

$$\text{ANS: } \boxed{\text{Caesar}} \text{ cipher (used by Julius Caesar)}$$

127. One of the most influential mathematicians of all time was the ninth century Arab named Mohammed ibn-Musa al-Khwarizmi. His last name survives in mathematics today as the term

“algorithm”. His most important work was *Al-jabr wa'l mugabālah*. What mathematical term was derived from this title?

ANS: Algebra (Source: Boyer and Merzbach, A History of Mathematics)

128. Bo is going to the store to buy candy that will cost somewhere between 40 cents and 54 cents. What is the fewest number of coins Bo can carry in order to be certain to have exact change to buy the candy?

ANS: penny, penny, penny, penny, nickel, dime, dime, quarter 8 coins

129. A 6-foot man casts a 2-foot shadow. A flag pole next to him casts a 30-foot shadow. How tall is the flag pole?

ANS: $\frac{6}{2} = \frac{x}{30}$, Solution is : $x = 90$ ft

130. Express the volume V of a cube as a function of the area A of one of its faces.

ANS: $V = A^{3/2}$ or $V = A\sqrt{A}$

131. The formula $e^{i\pi} + 1 = 0$ relates five of the most popular numbers in mathematics. What is π rounded to 10 significant digits?

ANS: $\pi = 3.14159265358979 \approx 3.141592654$

132. If a cube has a volume of 343, what is its surface area?

ANS: $7^3 = 343$, $7^2 \cdot 6 = 294$

133. Four positive integers have a sum of 10. What is the maximum possible value for the sum of their squares?

ANS: $1^2 + 1^2 + 1^2 + 7^2 = 52$

134. Solve the system of equations

$$y + z = 1$$

$$x + z = 1$$

$$x + y = 1$$

ANS: By symmetry, $x = y = z = \frac{1}{2}$

135. During the 1970's Steve Jobs and Steve Wozniak starting selling electronic equipment out of a garage. What company did they start?

ANS: Apple Computer

136. Out of a group of ten members, in how many different ways can a president, vice-president, and treasurer be elected?

ANS: $10 \cdot 9 \cdot 8 = 720$

137. 5000 raffle tickets are to be sold for \$1 each. The winner receives \$2500. If you purchase 1

ticket, how much are your expected earnings?

$$\text{ANS: } \frac{2500}{5000} - 1 = \boxed{-\$0.50}$$

138. Express the perimeter P of a square as a function of its area A .

$$\text{ANS: } A = \left(\frac{P}{4}\right)^2 \quad P = 4\sqrt{A}$$

139. 100 cadets throw their hats high into the air and randomly pick up hats afterwards. How many cadets are expected to pick up their own hats?

$$\text{ANS: } 100\left(\frac{1}{100}\right) = \boxed{1} \text{ cadet}$$

140. Alpher, Beta, and Gamow are now 7, 11, and 13 years old, respectively. What in the next year in which they again all have prime-numbered ages?

ANS: In $\boxed{2002}$ they will have ages 13, 17, and 19.

141. An icosahedron has 20 faces and 12 vertices. A diagonal of such a solid is a line segment joining two vertices not lying in the same face. How many diagonals are there?

$$\text{ANS: } \frac{6 \cdot 12}{2} = \boxed{36} \text{ Each vertex connected to 5 other vertices,}$$

so each vertex lies on 6 diagonals.

142. What is the greatest integer in the sum $\frac{19}{5} + \frac{5}{19}$?

$$\text{ANS: } \frac{19}{5} + \frac{5}{19} = 4.06315789473684 = \boxed{4} + .06315789473684$$

143. A mathematician named Wolfram started a company named Wolfram Research. What is its primary product?

ANS: $\boxed{\text{Mathematica}}$

144. George Boole developed a mathematical system called Boolean algebra. What is the value of $1 + 1$ under this system?

$$\text{ANS: } 1 + 1 = \boxed{1}$$

145. According to Descartes' Rule of Signs, how many negative real roots does the polynomial equation $x^4 - 8x^2 + 6x - 7 = 0$ have?

ANS: $\boxed{\text{one}}$ (since there is one sign change if x is replaced by $-x$)

146. At 7:00 a.m., Chad starts jogging at 5 km/hr. At 8:00 a.m. Marzelle starts jogging from Chads's starting place at 7 km/hr. How far behind is Marzelle at 10:00 a.m.?

$$\text{ANS: } 5 - 2 \cdot 2 = \boxed{1} \text{ km}$$

147. A computer sequentially computes integers by the following rule: If n is odd then replace n by $3n + 1$, otherwise replace n by $n/2$. If n starts at 5, what is n after 5 iterations?

$$\text{ANS: } 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow \boxed{1}$$

148. What is the surface area of a sphere of volume 972π ?

$$\text{ANS: } \frac{4}{3}\pi r^3 = 972\pi, \text{ Solution is : } r = 9, 4\pi r^2 = \boxed{324\pi}$$

149. Given the equation $x^3 - 7x^2 + 14x - 71$, what is the product of its real and complex roots?

$$\text{ANS: } (x - a)(x - b)(x - c) = x^3 - (a + b + c)x^2 + (ab + ac + bc)x - abc, \text{ so } abc = \boxed{71}$$

150. Every mathematician has an Erdős number. How is this number defined?

ANS: Paul Erdős has an Erdős number of 0.

Those who published a joint paper with Erdős have an Erdős number of 1.

Those who have a joint paper with someone with Erdős number n

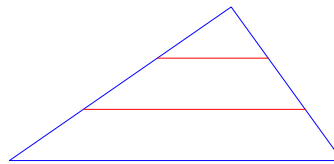
has an Erdős number $\leq n + 1$. The Erdős number is the length of

the shortest such path.

151. How many different strings of length 7 can be formed using the letters in ELLIPSE?

$$\text{ANS: } \frac{7!}{2!2!} = \boxed{1260}$$

152. A triangular section of Old Town is divided into a smaller triangle and two trapezoids by two streets parallel to one of the boundary streets. The heights of the two trapezoids are equal to the height of the small triangle, and the area of the middle trapezoid is 12 acres. How many acres are there in the larger trapezoid?



$$\text{ANS: } \boxed{20} \text{ acres}$$

153. A deck of 52 cards is thoroughly shuffled and the cards are turned over two at a time. How many pairs (two cards of the form $\boxed{A} \boxed{A}$ or $\boxed{5} \boxed{5}$) do you expect to see?

$$\text{ANS: } 26 \left(\frac{3}{51} \right) = \boxed{\frac{26}{17}}$$

154. How many diagonals are there in a regular 17-sided polygon?

$$\text{ANS: } \frac{14 \cdot 17}{2} = \boxed{119} \text{ Each of 17 vertices shares a diagonal with 14 other vertices.}$$

155. A pair of dice is rolled 10 times. What is the expected number of times the total is seven?

$$\text{ANS: } 30 \left(\frac{1}{6} \right) = \boxed{5}$$

156. This British mathematician led a team of mathematicians and cryptologists during World War II that broke ciphertext generated by the German Enigma machine. Name this person.

ANS: Alan Turing

157. What is the smallest positive integer divisible by all of the integers 1, 2, 3, 4, 5, 6, 7, 8, and 9?

$$\text{ANS: } \text{lcm}(1, 2, 3, 4, 5, 6, 7, 8, 9) = 2520 = 2^3 \cdot 3^2 \cdot 5 \cdot 7$$

158. When is the next prime year?

$$\text{ANS: } \text{nextprime}(1996) = 1997$$

159. CAD describes a type of computer software. What do the letters stand for?

ANS: Computer-Aided Design

160. If $\binom{n}{1} + \binom{n}{2} = 45$, what is n ?

$$\text{ANS: } \binom{n}{1} + \binom{n}{2} = 45, \text{ Solution is: } n = 9$$

161. If it takes 8 minutes to inflate a large spherical balloon to a radius of 2 meters, how long will it take to inflate a large spherical balloon to a radius of 3 meters?

$$\text{ANS: } 8 \cdot \frac{3^3}{2^3} = 27 \text{ minutes}$$

162. If g is a function such that $g(1) = 3$, $g(2) = -1$, and

$$g(n+1) = g(n) + 2g(n-1)$$

for $n \geq 2$, what is $g(4)$?

$$\text{ANS: } g(3) = g(2) + 2g(1) = -1 + 6 = 5, g(4) = g(3) + 2g(2) = 5 - 2 = 3$$

163. What is the area of the triangle bounded by the x -axis, the y -axis, and the line $y = 2x + 4$?

$$\text{ANS: } \frac{1}{2} \cdot 2 \cdot 4 = 4$$

164. Write the expression $\frac{7x+11}{x+5(x-3)}$ in the form $\frac{a}{x+5} + \frac{b}{x-3}$.

$$\text{ANS: } \frac{7x+11}{(x+5)(x-3)} = \frac{3}{x+5} + \frac{4}{x-3}$$

165. A set of points in the complex plane is determined by iteration of the function $z \rightarrow z^2 - \lambda$, where z and λ are complex numbers. What is the name of this set?

ANS: Mandelbrot set named after Benoit B. Mandelbrot

166. 200 cadets throw their hats high into the air and randomly pick up hats afterwards. How many cadets are expected to pick up their own hats?

$$\text{ANS: } 200 \left(\frac{1}{200} \right) = 1 \text{ cadet}$$

167. You currently earn \$6.00 per hour delivering pizza. You are due for a raise, and you figure the probability of a \$.50 raise is 70% and the probability of a \$1.00 raise is 30%. What is your expected new salary?

$$\text{ANS: } 6.00 + .7(.50) + .3(1.00) = \$6.65$$

168. If 2 cards are drawn from a standard deck of 52 cards, what is the probability that one is a heart and the other is a diamond?

$$\text{ANS: } \frac{\binom{13}{1}\binom{13}{1}}{\binom{52}{2}} = \frac{13}{102}$$

169. What is the radius of the sphere whose volume is eight times its surface area?

$$\text{ANS: } \frac{4}{3}\pi r^3 = 8 \cdot 4\pi r^2, \text{ Solution is: } r = 24$$

170. What is the smallest integer greater than 50 that when divided by 15 has a remainder of 1 and when divided by 16 also has a remainder of 1?

$$\text{ANS: } 15 \cdot 16 + 1 = 241$$

171. The sum of four consecutive integers is 170. What is the smallest of the four integers?

$$\text{ANS: Average is } \frac{170}{4} = \frac{85}{2} = 42\frac{1}{2}, \text{ subtract } \frac{3}{2} \text{ to get } 41$$

172. The interior of a triangle is painted black. The midpoints of the three edges are connected, and the interior of this triangle is painted white. The midpoints of each of the remaining black triangles are connected and the interiors are painted white. Continuing this process leaves a bit of black paint. What is the name of the resulting set of black points?

$$\text{ANS: Sierpinski triangle}$$

173. Maple is a widely-used computer algebra system. In what country was it developed?

$$\text{ANS: Canada}$$

174. Factor the polynomial $x^3 + 4x^2 + 6x + 4$ as a product of a linear and a quadratic, using the fact that -2 is a root.

$$\text{ANS: } x^3 + 4x^2 + 6x + 4 = (x + 2)(x^2 + 2x + 2)$$

175. Given 5 gallons of a 10% antifreeze/water mixture, how much pure antifreeze must be added to yield a 50% antifreeze/water mixture?

$$\text{ANS: } \frac{\frac{1}{2} + x}{5 + x} = \frac{1}{2}, \text{ Solution is : } x = 4 \text{ gal}$$

176. Name the five Platonic solids (regular polyhedra).

$$\text{Tetrahedron, cube (hexahedron), octahedron, dodecahedron, icosahedron}$$

177. Cubic polynomials have three zeros, which in general are complex numbers. Knowing that 5 is a zero of the polynomial $x^3 - 4x^2 + 5x - 50$, what is the product of the two remaining real or complex zeros?

$$\text{ANS: } (x - 5)(x^2 + x + 10), \text{ roots: } \frac{5}{- \frac{1}{2} + \frac{1}{2}i\sqrt{39}}, \left(-\frac{1}{2} + \frac{1}{2}i\sqrt{39}\right) \left(-\frac{1}{2} - \frac{1}{2}i\sqrt{39}\right) = 10$$

$$\text{ANS: } (x - a)(x - b)(x - 5) = x^3 - 5x^2 - x^2b + 5xb - ax^2 + 5ax + abx - 5ab \quad ab = 10$$

178. How many integers between 1000 and 1100 are divisible by 23?

$$\text{ANS: } 44 \quad 23 = 1012, 47 \quad 23 = 1081, 48 \quad 23 = 1104 \quad \boxed{4}$$

179. What is the area of an equilateral triangle inscribed in a circle of radius 4 inches?

$$\text{ANS: } \left(4 - 4 \cos \frac{2\pi}{3}\right) 4 \sin \frac{2\pi}{3} = \boxed{12\sqrt{3}} \text{ in}^2$$

180. For whom was the computer language Ada named?

ANS: Ada Lovelace

181. In how many ways can the U.S. Senate select a committee of 4 from among its 100 members?

$$\text{ANS: } \binom{100}{4} = \boxed{3,921,225}$$

182. A pair of dice is rolled. What is the probability that the sum is either a 3 or a 4?

$$\text{ANS: } \frac{2}{36} + \frac{3}{36} = \boxed{\frac{5}{36}}$$

183. What is the diameter of a circle with area $25\pi \text{ cm}^2$?

$$\text{ANS: } 25\pi = (D/2)^2\pi, D = \boxed{10}$$

184. A triangle has sides of length 8, 5, and 5. Find the area of the triangle.

$$\text{Altitude is 3, so } Area = \frac{1}{2}(8)(3) = \boxed{12}$$

185. A bag of Halloween candy contains 4 pieces of chocolate and 4 pieces of fruit bar. What is the probability that two items selected at random are both chocolate?

$$\text{ANS: } \binom{4}{2} \binom{8}{2} = \boxed{\frac{3}{14}}$$

186. Who designed, but never build, an early “analytic engine” that led eventually to the Automatic Sequence Controlled Calculator developed jointly by IBM and Harvard University in 1944?

ANS: Charles Babbage

187. What is the perimeter of an isosceles triangle with base 8 and area 12?

$$\text{ANS: } h = 3, 4^2 + 3^2 = s^2 \quad s = 5, 8 + 2 \times 5 = \boxed{18}$$

188. How many integers between 100 and 1000 are divisible by 23?

$$\text{ANS: } 23 \times 5 = 115, 23 \times 43 = 989, 43 - 5 + 1 = \boxed{39}$$

189. Two 3-digit integers consist of the same digits, but in the reverse order. What is the largest possible difference between the two numbers?

$$\text{ANS: } 991 - 199 = 981 - 189 = \dots = 901 - 109 = \boxed{792}$$

190. This research organization has employed applied mathematicians such as Ronald Graham, Richard Hamming, and Claude Shannon, each of whom have made major contributions to

applied discrete mathematics. Name this organization.

ANS: Bell Labs or AT&T Bell Laboratories

191. A computer sequentially computes integers by the following rule: If n is a square then multiply by 2, otherwise subtract 3. Starting at $n = 13$, what is the integer after 6 iterations?

ANS: $12 \rightarrow 9 \rightarrow 18 \rightarrow 15 \rightarrow 12 \rightarrow 9 \rightarrow 18$

192. Two cars start 3 miles apart and drive toward each other. One car goes 55 mph, the other 65 mph. After how many seconds do the two cars meet?

ANS: $t = \frac{d}{r} = \frac{3}{55+65} = \frac{1}{40} \text{ hr} = 1.5 \text{ min} = 90 \text{ sec}$

193. According to the Rational Root Theorem, what are all the possible rational roots of the polynomial $2x^3 - 5x^2 - 11x - 4$?

ANS: $\pm \frac{1}{2}, \pm 1, \pm 2, \pm 4$

194. Find an integer between 100 and 1000 that is both a perfect square and a perfect cube.

ANS: $729 = 9^3 = 27^2$

195. What former University of Colorado student went on to help found Apple Computer?

ANS: Steve Wozniak

196. In how many ways can ALLAN misspell his name, assuming he uses all the right letters (the right number of times)? (Spell out A-L-L-A-N.)

ANS: $\frac{5!}{2!2!} - 1 = 29$

197. If 84 centimeters of wire are used to build the skeleton of a cube, what is the volume of the cube?

ANS: $\left(\frac{84}{12}\right)^3 = 7^3 = 343 \text{ cm}^3$

198. The probability of picking a dog to finish in the top 3 at the dog track is $\frac{1}{5}$. What is the probability of picking 2 straight losers?

ANS: $\left(\frac{4}{5}\right)^2 = \frac{16}{25}$

199. What is the volume of the largest cube that can fit inside a sphere of radius $\sqrt{3}$?

ANS: $Volume = 2^3 = 8$

200. A golf bag contains 5 white balls, 2 yellow balls, and 2 orange balls. If two golf balls are selected at random, what is the probability that both are white?

ANS: $\frac{\binom{5}{2}}{\binom{9}{2}} = \frac{5}{18}$

201. What former CSU Mathematics Professor was known as an "Euler Spoiler" for finding a counterexample to a famous conjecture of Euler and who helped develop the Bose-Chaudhuri-Hocquenghem error-correcting codes?

ANS: R. C. Bose

202. How many integers between 101 and 999 are divisible by 3 or by 5?

ANS: $\frac{999-102}{3} + 1 = 300$, $\frac{995-105}{5} + 1 = 179$, $\frac{990-105}{15} + 1 = 60$, $300 + 179 - 60 = 419$

203. What is the least positive residue of $5^{22} \pmod{23}$?

ANS: $5^{22} \pmod{23} = 1$ (Fermat's Little Theorem)

204. CAS describes a type of computer software. What do the letters stand for?

ANS: Computer Algebra System

205. What is x if $512^x = 16^5$?

ANS: $(2^9)^x = (2^4)^5 \Rightarrow 9x = 20 \Rightarrow x = \frac{20}{9}$

206. To the nearest minute, at what time between 9:30 a.m. and 10:00 a.m. are the minute hand and the hour hand at right angles?

ANS: $45 + \frac{m}{12} = m + 15$, Solution is : $m = \frac{360}{11} : 32.7272727272727$ 9:33 a.m.

207. Brigitte plants a pumpkin seed. The area that is covered by the vine doubles every month. After 5 months the entire garden is covered. When was exactly half of the garden covered with the vine?

ANS: One month earlier or after 4 months

208. What is the greatest integer in $\frac{7}{27} + \frac{27}{7}$?

ANS: $\frac{7}{27} + \frac{27}{7} = \frac{778}{189} = 4.11640211640212$ 4

209. A popular novel has 423 pages and 128592 words. What is the average number of words per page?

ANS: $\frac{128592}{423} = 304$

210. A Stanford University mathematician/computer scientist who developed the TeX typesetting system and who wrote a three-volume series of books called *The Art of Computer Programming* stated, "Every bit of mathematics I have ever learned, I have found a use for someplace." What is this person's name?

ANS: Donald Knuth

211. How many colors are possible on a computer that advertises 8-bit color?

ANS: $2^8 = 256$

212. A golf bag contains 9 golf balls, some yellow and the rest orange. Four balls are drawn at random from the bag and two are yellow, the other two orange. What is the probability that exactly half of the balls in the bag are yellow?

ANS: 0

213. What is the perimeter of a right triangle with legs 8 and 15?

ANS: $s^2 = 8^2 + 15^2 = 289$ $s = 17$ $P = 8 + 15 + 17 = 40$

214. If the surface area of a sphere is equal to 1, what is its volume?

ANS: $r = \frac{1}{2\sqrt{\pi}}$ so $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi \left(\frac{1}{2\sqrt{\pi}}\right)^3 = \frac{1}{6\sqrt{\pi}}$ or $\frac{\sqrt{\pi}}{6\pi}$

215. A multiple-choice exam has 20 questions with 5 choices per question. If you answer the questions randomly, what is the expected number of correct responses?

ANS: $20 \times \frac{1}{5} = 4$

216. This 20th century American mathematician introduced game theory as a mathematical discipline, conceived the idea of a self-stored computer program, and worked on the Manhattan project that developed the atomic bomb. Name this person.

ANS: John von Neuman

217. What is the least positive integer solution to the system

$$x \equiv 3 \pmod{5}$$

$$x \equiv 2 \pmod{7}$$

of congruences?

ANS: 23 $23 \pmod{5} = 3$, $23 \pmod{7} = 2$

ANS: 23 $\{3, 8, 13, 18, 23, 28, 33\} \cap \{9, 16, 23, 30\} = \{23\}$

218. What is the least positive residue of $5^{-1} \pmod{7}$?

ANS: 3 $5 \times 3 \pmod{7} = 15 \pmod{7} = 1$ so $5^{-1} \pmod{7} = 3$

219. Derive is a widely-used computer algebra system. What was the original name of the version written for Apple IIe computers?

ANS: mu-Math or μ -Math

220. The average of the four numbers 14, 16, 19, and x is 23. What is x ?

ANS: $\frac{14+16+19+x}{4} = 23$, Solution is : $x = 43$

221. A spherical balloon's radius increases by 10%. By what percentage does the surface area change?

ANS: $1.1^2 = 1.21$ 21% increase

222. How many feet per second is 75 miles per hour?

$$\text{ANS: } 75 \cdot \frac{5280}{3600} = \boxed{110} \text{ ft/sec}$$

223. What is the base 5 representation of the number 65?

$$\text{ANS: } 65 = 2 \times 5^2 + 3 \times 5 + 0 \quad \boxed{230} \text{ or } \boxed{230_5}$$

224. The 2×2 matrix A is given by $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$. What is A^{-1} ?

$$\text{ANS: } \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$$

225. Most of the current Public-Key Cryptography systems are based on the use of large primes, modular arithmetic, and the difficulty of factoring large numbers. What is the name of the most popular such algorithm?

ANS: $\boxed{\text{RSA}}$ algorithm or $\boxed{\text{Rivest-Shamir-Adleman}}$