Math Day 1995

Team Competition

Colorado State University

1. What is the area of the largest rectangle that can fit inside a circle of radius 1?

   ANS: $Area = \left(\sqrt{2}\right)^2 = \boxed{2}$

2. A rectangle's length is increased by 20% and its width is decreased by 20%. How does its area change?

   ANS: $1.2L \times .8W = \boxed{.96LW}$ or $\boxed{\text{decreases by 4\%}}$

3. Six people are in the same room. If everyone shakes hands with everyone else in the room, how many handshakes take place?

   ANS: $5 + 4 + 3 + 2 + 1 = \boxed{15}$

4. Two evenly-matched baseball teams, the Green Socks and the Yellow Elbows, start a 7-game series. What is the probability that the Green Socks sweep the series in four games?

   ANS: $\left(\frac{1}{2}\right)^4 = \boxed{\frac{1}{16}}$

5. Sarah averages 50% on multiple choice exams. What is the probability that she gets at least 2 correct on a 3-question exam?

   ANS: $\left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) = \boxed{\frac{1}{2}}$

6. What fact about the number 1729 did the Indian mathematician Ramanujan consider interesting?

   ANS: Smallest positive integer = sum of cubes in 2 ways: $12^3 + 1^3 = 10^3 + 9^3$

7. A spider and a fly are at diagonally opposite corners of a closed cubical box of side length one. What is the shortest distance the spider can travel to reach the (stationary) fly by crawling along walls?

   ANS: $\boxed{\sqrt{5}}$

8. If the surface area of a cube is equal to 1, what is its volume?

   ANS: $\left(\frac{1}{\sqrt{6}}\right)^3 = \boxed{\frac{1}{36}\sqrt{6}}$

9. Out of a group of ten members, in how many different ways can a president, vice-president, and treasurer be elected?

   ANS: $10 \cdot 9 \cdot 8 = \boxed{720}$

10. This 20th century American mathematician introduced game theory as a mathematical discipline, conceived the idea of a self-stored computer program, and worked on the Manhattan project that developed the atomic bomb. Name this person.

    ANS: John $\boxed{\text{von Neuman}}$

11. What positive number is three times as big as its reciprocal?

ANS: $x = \frac{3}{x}$ implies $x = \boxed{\sqrt{3}}$

12. If the absolute value of $x^2 + 4$ is equal to the absolute value of $x^2 - 6$, what is x?

ANS: $\left|x^2 + 4\right| = \left|x^2 - 6\right|$, Solution is : $\boxed{x = 1, x = -1}$

13. Two wheels are connected by a drive belt. One has a diameter of 24 centimeters and a speed of 144 rpm. The other has a speed of 72 rpm. What is its diameter?

ANS: $24 \times 144 = 72x$, Solution is : $x = \boxed{48}$ cm

14. The vertices of a quadrilateral are at the points $(0, 0)$, $(3, 1)$, $(4, 0)$, and $(2, -4)$. What is the area of the quadrilateral?

ANS: $Area = \frac{1}{2} \cdot 4 \cdot 1 + \frac{1}{2} \cdot 4 \cdot 4 = \boxed{10}$

15. Find all the integer solutions of the equation $x^4 - x^2 - 12 = 0$.

ANS: $x^4 - x^2 - 12 = (x - 2)(x + 2)\left(x^2 + 3\right)$, so $\boxed{x = 2, -2}$

16. If a pair of fair dice is rolled, what is the probability that the sum is 7?

ANS: $\frac{6}{36} = \boxed{\frac{1}{6}}$

17. The base of an isosceles triangle is 4 and the opposite vertex moves up and down. If the area of the triangle is plotted as a function of its height, what is the shape of graph?

ANS: $A = \frac{1}{2}(4)\, h = 2h$ or $\boxed{\text{straight line}}$

18. What is the distance between two opposite vertices of a cube of edge 1?

ANS: $\boxed{\sqrt{3}}$

19. If $f(x) = 6x - 2$, what is $f(f(1.5))$?

ANS: $f(f(1.5)) = \boxed{40.0}$

20. How many minutes were there in October of 1995?

ANS: $31 \times 24 \times 60 = \boxed{44640}$

21. What famous mathematical object was named after the mathematician David Hilbert?

ANS: $\boxed{\text{Hilbert Space}}$

22. An octahedron is a regular solid whose 8 faces are equilateral triangles. What is the distance between a pair of opposite vertices, assuming each edge of the octahedron is of length 1?

ANS: $\boxed{\sqrt{2}}$

23. A shirt has been marked down 20% to \$19.20. What was the original price?

$$\boxed{\text{ANS: } x \times .8 = 19.2 \Longrightarrow x = 19.2/.8 = \boxed{\$24.00}}$$

24. The 12 faces of a regular dodecahedron are pentagons. How many edges does a regular dodecahedron have?

$$\boxed{\text{ANS: } \tfrac{12\times 5}{2} = \boxed{30}}$$

25. One fourth of the air in a container is removed by each cycle of a vacuum pump. What fractional part of the air remains after 5 cycles?

$$\boxed{\text{ANS: } \left(\tfrac{3}{4}\right)^5 = \boxed{\tfrac{243}{1024}}}$$

26. Three positive integers have a sum of 10. What is the minimum possible value for the sum of their squares?

$$\boxed{\text{ANS: } 3^2 + 3^2 + 4^2 = \boxed{34}}$$

27. Find x so that the average of the four numbers 12, x, 30, and 42 is 26.

$$\boxed{\text{ANS: } \tfrac{12+x+30+42}{4} = 26, \text{ Solution is : } x = \boxed{20}}$$

28. Find the Greatest Common Divisor of 147 and 77.

$$\boxed{\text{ANS: } \gcd(147, 77) = \boxed{7}}$$

29. A wooden cube of edge 4 inches is painted red. The cube is then cut into 64 one-inch cubes by making 9 saw cuts. How many of the one-inch cubes have exactly 2 red faces?

$$\boxed{\text{ANS: 12 edges} \times \text{2 cubes per edge} = \boxed{24} \text{ cubes}}$$

30. How many possible ways are there to pick the first place, second place, and third place in a 10-person race?

$$\boxed{\text{ANS: } 10 \times 9 \times 8 = \boxed{720}}$$

31. If $f(x)$ is a linear function with $f(2) = 9$ and $f(4) = 13$, then what is $f(1)$?

$$\boxed{\text{ANS: } f(1) = \boxed{7}}$$

32. The first famous woman mathematician was Hypatia, who lived from 370 to 415 A.D. Although none of her writings survive, it is believed that her work included integer solutions to equations. What branch of mathematics deals with such problems?

$$\boxed{\text{ANS: } \boxed{\text{Number Theory}} \text{ (Source: G. F. Simmons, } \textit{Calculus Gems}\text{)}}$$

33. In how many ways can 3 different math books, 2 different stat books, and 1 physics book be arranged on a shelf, assuming the math books must be together and the stat books must be together?

$$\boxed{\text{ANS: } 3!\,(3! \times 2! \times 1!) = \boxed{72}}$$

34. What is the smallest prime larger than 500?

$$\boxed{\text{ANS: nextprime}(500) = \boxed{503}}$$

35. If a single 60-Watt bulb provides sufficient light to read a newspaper 5 feet from the bulb, how many

60-Watt bulbs are required in a light fixture 10 feet from the newspaper in order to provide the same apparent level of brightness?

$$\boxed{\text{ANS: } \left(\tfrac{10}{5}\right)^2 = \boxed{4} \text{ bulbs}}$$

36. A wooden cube of edge 4 inches is painted red. The cube is then cut into 64 one-inch cubes by making 9 saw cuts. How many of the one-inch cubes have exactly 3 red faces?

$$\boxed{\text{ANS: } 8 \text{ corners} \times 1 \text{ cube per corner} = \boxed{8} \text{ cubes}}$$

37. A quadrilateral kite is made with a right angle at the top, angles of 2α left and right, and an angle of α at the bottom. What is α?

$$\boxed{\text{ANS: } 5\alpha = 360^\circ - 90^\circ \Longrightarrow \alpha = \boxed{54^\circ} \text{ or } \boxed{\tfrac{3\pi}{10}}.}$$

38. Find C so that the equation $x^2 - 16x + C = 0$ has exactly one real root.

$$\boxed{\text{ANS: } x = 8 \pm \sqrt{64 - C}, \text{ same when } C = \boxed{64}}$$

39. What is the $10th$ term of the arithmetic sequence that begins 49, 43, 37,. . .?

$$\boxed{\text{ANS: } 49 - 9 \times 6 = \boxed{-5}}$$

40. A shirt has been marked down 20% twice. The current price is \$16. What was the original price?

$$\boxed{\text{ANS: } (.8)\,(.8)\,x = 16, \text{ Solution is : } x = \boxed{\$25.00}}$$

41. An equilateral triangle has vertices at $(0, 0)$ and $(8, 0)$. Give one set of possible coordinates for the third vertex.

$$\boxed{\text{ANS: } (4, y) \text{ where } 4^2 + y^2 = 8^2, \text{ so vertex at } \boxed{(4, 4\sqrt{3})} \text{ or } \boxed{(4, -4\sqrt{3})}}$$

42. The 12 faces of a regular dodecahedron are pentagons. How many vertices does a regular dodecahedron have?

$$\boxed{\text{ANS: } \tfrac{12\times 5}{3} = \boxed{20}}$$

43. Express the volume V of a cube as a function of the area A of one of its faces.

$$\boxed{\text{ANS: } \boxed{V = A^{3/2}} \text{ or } \boxed{V = A\sqrt{A}}}$$

44. Two baseball teams, the Green Socks and the Yellow Elbows, begin a 7-game series. The odds makers give the green Socks 7 to 5 odds of winning the first game. Assuming the odds makers are good at their job, what is the probability that the Green Socks will win the first game?

$$\boxed{\text{ANS: } \boxed{\tfrac{7}{12}}}$$

45. Sarah averages 50% on multiple choice exams. What is the probability that she gets exactly 2 correct on a 3-question exam?

$$\boxed{\text{ANS: } 3\left(\tfrac{1}{2}\right)^3 = \boxed{\tfrac{3}{8}}}$$

46. The 4 faces of a regular tetrahedron are equilateral triangles. How many edges does a regular tetrahedron

have?

ANS: $\frac{4\times 3}{2} = \boxed{6}$

47. Kyle uses pure guessing on a TRUE/FALSE exam. Which of the following options give Kyle the best chance to score (at least) 80%? (A) A ten-question exam (guess correctly on 8, 9, or 10 questions). (B) A five-question exam (guess correctly on 4 or 5 questions). (C) The chances are equal.

ANS: (A) probability of $80\% \approx .054688$ $\boxed{\text{(B)}}$ probability of $80\% \approx .188$

48. Sarah has a license plate with 2 letters and four digits, e.g. $\boxed{\text{LW-0508}}$. What is the probability that at least one of the four digits is a 7?

ANS: $1 - \left(\frac{9}{10}\right)^4 = \boxed{\frac{3439}{10000}} = \boxed{.3439}$

49. Express the perimeter P of a square as a function of its area A.

ANS: $A = \left(\frac{P}{4}\right)^2 \Longrightarrow \boxed{P = 4\sqrt{A}}$

50. Find a point equidistant from the three points $(-2,-2)$, $(2,2)$, and $(2,-2)$.

ANS: $\boxed{(0,0)}$

51. A bag of chicken feed with feed 18 chickens for 72 days. For how many days will it feed 12 chickens?

ANS: $(72)\,\frac{18}{12} = \boxed{108}$ days

52. How many primes are there between 90 and 100?

ANS: $\boxed{\text{One}}$ (it is 97)

53. How many different 6-place licence plates are possible if the first two places are letters and the last 4 places are digits?

ANS: $26^2 \times 10^4 = \boxed{6,760,000}$

54. What Greek philosopher raised paradoxes that argued that motion is impossible?

ANS: $\boxed{\text{Zeno}}$ of Elea

55. How many of the 1024 subsets of the set $\{1,2,3,4,5,6,7,8,9,10\}$ contain the integer 2?

ANS: $\boxed{\text{Half}}$ or $\boxed{512}$

56. If $f(x) = 2x - 3$, what is $f(f(f(4)))$?

ANS: $f(f(f(4))) = \boxed{11}$

57. What is the diameter of a circle with area 25π cm^2?

ANS: $25\pi = (D/2)^2\pi$, $D = \boxed{10}$ cm

58. A 14-inch pizza is cut into 8 congruent slices. What is the perimeter of each slice?

ANS: $C = 14\pi$ so $P = \frac{14\pi}{8} + 2 \times 7 = \boxed{\frac{7}{4}\pi + 14}$ in

59. If $f(x) = x^{5/3}$, what is $f(64)$?

ANS: $f(64) = \boxed{1024}$

60. State the contrapositive of the statement, "If $x^2 \leqq 9$, then $x \leqq 3$."

ANS: $\boxed{\text{If } x > 3, \text{ then } x^2 > 9.}$

61. Sarah is an 80% free throw shooter. What is her expected score if she shoots 2 free throws?

ANS: $2\left(\frac{8}{10}\right)^2 + 1\left(\frac{8}{10}\right)\left(\frac{2}{10}\right) + 1\left(\frac{2}{10}\right)\left(\frac{8}{10}\right) = \boxed{\frac{8}{5}} = \boxed{1.6}$

62. If 2 cards are drawn from a standard deck of 52 cards, what is the probability that both are hearts?

ANS: $\frac{\binom{13}{2}}{\binom{52}{2}} = \frac{13 \cdot 12}{2} \cdot \frac{2}{52 \cdot 51} = \frac{2^2 3 \times 13}{2^2 3 \times 13 \times 17} = \boxed{\frac{1}{17}}$

63. A triangle has sides of length 13, 10, and 13. Find the area of the triangle.

Altitude is 12, so $Area = \frac{1}{2}(10)(12) = \boxed{60}$

64. What is the surface area of a spherical raindrop of diameter 1 millimeter?

ANS: $4\pi\left(\frac{1}{2}\right)^2 = \boxed{\pi}\ \text{mm}^2$

65. If 84 centimeters of wire are used to build the skeleton of edges of a cube, what is the volume of the cube?

ANS: $\left(\frac{84}{12}\right)^3 = 7^3 = \boxed{343}\ \text{cm}^3$

66. An elevator moves at constant speed and it takes 12 seconds for an elevator to go from the first floor to the third floor. How long should it take to go from the first floor to the sixth floor?

ANS: $6 \times 5 = \boxed{30}$ sec

67. What is the prime factorization of 999?

ANS: $999 = \boxed{3^3 37}$ or $\boxed{3 \times 3 \times 3 \times 37}$

68. In how many ways can the U.S. Senate select a committee of 4 from among its 100 members?

ANS: $\binom{100}{4} = \boxed{3,921,225}$

69. A wooden cube of edge 4 inches is painted red. The cube is then cut into 64 one-inch cubes by making 9 saw cuts. How many of the one-inch cubes have no red faces?

ANS: The center of the large cube contains $2 \times 2 \times 2 = \boxed{8}$ one-inch cubes

70. The 20 faces of a regular icosahedron are equilateral triangles. How many edges does a regular icosahedron have?

ANS: $\frac{20 \times 3}{2} = \boxed{30}$

71. What mathematician introduced the dy/dx notation in calculus?

ANS: Gottfried Wilhelm $\boxed{\text{Leibniz}}$

72. A biological brick grows 20% in length, 10% in width, and shrinks in height by 25%. Is its volume larger or smaller than when it started out?

ANS: $(1.2)(1.1)(.75) = .99$ smaller

73. Bo and Jo raise goats and geese. If Bo counts 90 feet and Jo counts 56 eyes, how many goats and how many geese do Bo and Jo have?

ANS: $\left\{ \begin{array}{l} 4a + 2b = 90 \\ 2a + 2b = 56 \end{array} \right\}$, Solution is : $a =$ 17 goats, $b =$ 11 geese

74. Name the smallest integer whose square is less than 1000.

ANS: $(-32)^2 = 1024$, $(-31)^2 = 961$, $(-30)^2 = 900$, etc. -31 is the smallest

75. Bo scored 67 on the first exam and 75 on the second. What is the lowest possible score on the third exam so that Bo's average for the three exams will be 80?

ANS: $\frac{x+67+75}{3} = 80$, Solution is : $x =$ 98

76. What number is halfway between $1/6$ and $1/8$?

ANS: $\left(\frac{1}{6} + \frac{1}{8}\right)/2 = \frac{7}{48}$

77. Expand $(2x^2 + y^3)^3$.

ANS: $(2x^2 + y^3)^3 = 8x^6 + 12x^4y^3 + 6x^2y^6 + y^9$

78. What cubic curve was named after Maria Agnesi?

ANS: Witch of Agnesi

79. Six poker players are seated at a round table. How many rearrangements are possible, if the only considerations are who is seated at each person's left, and at each person's right?

ANS: $\frac{6!}{6} =$ 120

80. A Spanish port was protected by large cannons, and each cannon had a pile of cannon balls nearby stacked neatly in the shape of a tetrahedron. If each bottom layer contained a total of 28 cannon balls, how many cannon balls were there altogether in each pile?

ANS: $28 + 21 + 15 + 10 + 6 + 3 + 1 =$ 84

81. Write 1995 as a product of primes.

ANS: $1995 = 3 \times 5 \times 7 \times 19$

82. The Center Ring Janitorial Supply owns a fleet of 3 vehicles: One car, which gets 25 miles per gallon, and two trucks, each of which gets 10 miles per gallon. If in a typical week the car is driven 450 miles and each truck is driven 240 miles, how many miles per gallon is Center Ring's fleet getting, rounded to the nearest mile per gallon?

ANS: mi $= 930$, gal $= \frac{450}{25} + \frac{480}{10} = 66$, mpg $= \frac{930}{66} = \frac{155}{11} =$ 14.09090909

83. A pair of dice is rolled. What is the probability that the sum is either a 3 or a 7?

$$\boxed{\text{ANS: } \tfrac{2}{36} + \tfrac{6}{36} = \boxed{\tfrac{2}{9}}}$$

84. What is the perimeter of an isosceles triangle with base 8 and area 12?

$$\boxed{\text{ANS: } h = 3,\ 4^2 + 3^2 = s^2 \Longrightarrow s = 5,\ 8 + 2 \times 5 = \boxed{18}}$$

85. What are the dimensions of a rectangle with area 110 and perimeter 42?

$$\boxed{\text{ANS: } \begin{array}{l} ab = 110 \\ 2a + 2b = 42 \end{array} \text{, Solution is : } a = \boxed{11} \text{ by } b = \boxed{10}}$$

86. What is the sum of the binomial coefficients (4 choose zero) plus (4 choose 1) plus (4 choose 2) plus (4 choose 3) plus (4 choose 4)?

$$\boxed{\text{ANS: } \tbinom{4}{0} + \tbinom{4}{1} + \tbinom{4}{2} + \tbinom{4}{3} + \tbinom{4}{4} = \boxed{16} = 2^4}$$

87. A gold watch has been reduced by 10%, then by 20%, and finally sold for $648.00. What was the original price?

$$\boxed{\text{ANS: } (.9)\,(.8)\,x = 648, \text{ Solution is : } x = \boxed{\$900.00}}$$

88. Every diameter of a spherical orange is 80% fruit and 20% peel. What percentage of the volume is the peel?

$$\boxed{\text{ANS: } 1 - \left(\tfrac{4}{5}\right)^3 = \tfrac{61}{125} = .488 = \boxed{48.8\%}}$$

89. What is the maximum number of pieces into which a circular pizza can be cut using 3 chops of a knife (with no intermediate rearrangements of the pieces)?

$$\boxed{\text{ANS: } \boxed{7}}$$

90. The 20 faces of a regular icosahedron are equilateral triangles. How many vertices does a regular icosahedron have?

$$\boxed{\text{ANS: } \tfrac{20 \times 3}{5} = \boxed{12}}$$

91. Factor the polynomial $x^3 - 3x^2 + 2x - 6$.

$$\boxed{\text{ANS: } x^3 - 3x^2 + 2x - 6 = \boxed{\left(x^2 + 2\right)(x - 3)}}$$

92. The 8 faces of a regular octahedron are equilateral triangles. How many vertices does a regular octahedron have?

$$\boxed{\text{ANS: } \tfrac{8 \times 3}{4} = \boxed{6}}$$

93. What is the perimeter of a right triangle with legs 8 and 15?

$$\boxed{\text{ANS: } s^2 = 8^2 + 15^2 = 289 \Longrightarrow s = 17 \Longrightarrow P = 8 + 15 + 17 = \boxed{40}}$$

94. The sum of two numbers is 14 and their product is 48. What are the numbers?

$$\boxed{\text{ANS: } \left\{ \begin{array}{l} a + b = 14 \\ ab = 48 \end{array} \right\} \text{, Solution is : } a = \boxed{8}, b = \boxed{6}}$$

95. Give an equation in the form $ax + by = c$, where a, b, and c are integers, for the line through the point

$(2, -3)$ and perpendicular to the line $3x - 2y = 4$.

ANS: $\boxed{2x + 3y = -5}$ or $\boxed{-2x - 3y = 5}$

96. A pyramid is build out of cubical blocks by placing 64 blocks on the floor, 49 blocks on top of the bottom layer, and so forth. How many cubes are required to build the pyramid?

ANS: $\sum_{i=1}^{8} i^2 = \boxed{204} = \frac{8 \times 9 \times 17}{6}$

97. Find the vertex of the parabola $y = 3x^2 - 12x + 7$.

ANS: $3\left(x^2 - 4x + 4\right) + 7 - 12 = 3\left(x - 2\right)^2 - 5$ vertex $\boxed{(2, -5)}$

98. A bag of Halloween candy contains 6 pieces of chocolate and 3 pieces of fruit bar. What is the probability that two items selected at random are both chocolate?

ANS: $\binom{6}{2} \div \binom{9}{2} = \boxed{\frac{5}{12}}$

99. Sarah is an 80% free throw shooter. What is her expected score if she shoots a one and one (if she makes the first she gets a second chance)?

ANS: $2\left(\frac{8}{10}\right)^2 + 1\left(\frac{8}{10}\right)\left(\frac{2}{10}\right) = \boxed{\frac{36}{25}} = \boxed{1.44}$

100. A ladder leans against a vertical wall. The bottom of the ladder is 5 feet from the wall and the top of the ladder is 12 feet above the floor. How long is the ladder?

ANS: $\sqrt{5^2 + 12^2} = \boxed{13}$ ft

101. If 2 is the first prime, what is the tenth prime?

ANS: $2, 3, 5, 7, 11, 13, 17, 19, 23, \boxed{29}$

102. A committee of 3 people is to be chosen from among 4 men and 5 women. How many ways can this be done if the committee must include at least one man and at least one woman?

ANS: $\binom{9}{3} - \binom{4}{3} - \binom{5}{3} = \boxed{70} = \binom{4}{2}\binom{5}{1} + \binom{4}{1}\binom{5}{2}$

103. What mathematician first resolved the Königsberg bridge problem?

ANS: $\boxed{\text{Euler}}$

104. The compact disk UR2ugly sells at outlet AC for \$13.90 less a discount of 15%, and at outlet DC for \$15.70 less a discount of 25%. Which outlet has the lower price?

ANS: $\boxed{\text{AC}}$ $13.90 \times .85 = \$11.815$, DC $15.90 \times .75 = \$11.925$

105. What is the area of the ellipse $\dfrac{x^2}{36} + \dfrac{y^2}{49} = 1$?

ANS: $6 \cdot 7\pi = \boxed{42\pi}$

106. Factor the polynomial $x^3 - 7x^2 + 14x - 8$.

ANS: $x^3 - 7x^2 + 14x - 8 = \boxed{(x-1)(x-2)(x-4)}$

107. The sum of two numbers is 13 and their product is 48. What are the two numbers?

ANS: $\left\{ \begin{array}{l} x + y = 13 \\ xy = 40 \end{array} \right\}$, Solution is : $y = \boxed{5}$, $x = \boxed{8}$

108. Give an equation in the form $ax + by = c$, where a, b, and c are integers, for the line through the point $(-2, 3)$ and parallel to the line $y = \frac{3}{4}x + 5$.

ANS: $y - 3 = \frac{3}{4}(x + 2)$, Solution is : $\boxed{-3x + 4y = 18}$ or $\boxed{3x - 4y = -18}$

109. What is the area of the triangle bounded by the lines $x = 0$, $y = x$, and $2y + x = 12$?

ANS: Vertices $(0, 0)$, $(0, 6)$, $(4, 4)$, base 6, height 4, area $\boxed{12}$

110. The height of a rectangle is 25% less than its base. The perimeter of the rectangle is 64 in. Find the area of the rectangle.

ANS: $2x + 2\left(\frac{3}{4}\right)x = 56$, Solution is : $x = 16$, $16 \times 12 = \boxed{192}$ in^2

111. A Social Security number has nine digits. Assuming the digits are random, what is the expected number of fives in a social security number?

ANS: $9 \times \frac{1}{10} = \boxed{\frac{9}{10}}$

112. In which quadrant is the center of the circle $x^2 + y^2 + 6x - 8y = 0$?

ANS: $x^2 + 6x + y^2 - 8y + 25 = (x + 3)^2 + (y - 4)^2 = 25$, center $(-3, 4)$ $\boxed{\text{2nd}}$ quadrant

113. Sarah averages 80% on true/false exams. What is the probability that she gets exactly two correct on a three-question true/false exam?

ANS: $3 \times (.8)^2 (.2) = \boxed{.384}$ or $3 \times \left(\frac{8}{10}\right)^2 \left(\frac{2}{10}\right) = \boxed{\frac{48}{125}}$

114. What is the volume of the tetrahedron whose vertices are $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$?

ANS: $\boxed{1/6}$

115. What is the sum of the roots of the polynomial $x^2 - 8x + 13$?

ANS: $(x - a)(x - b) = x^2 - x(a + b) + ab$, so $a + b = \boxed{8}$

116. What is the tenth term of of an arithmetic sequence whose first term is -5 and whose difference between successive terms is 3?

ANS: $f(n) = -8 + 3n$, $f(1) = -5$, $f(10) = \boxed{22}$

117. Solve the equation $x\sqrt{.16} = 5$.

ANS: $x\sqrt{.16} = 5$, Solution is : $x = \boxed{12.5}$

118. If $2^p = 5$ then what is 4^p?

ANS: $4^p = \left(2^2\right)^p = \left(2^p\right)^2 = 5^2 = \boxed{25}$

119. One of the most influential mathematicians of all time was the ninth century Arab named Mohammed ibn-Musa al-Khwarizmi. His last name survives in mathematics today as the term "algorithm". His most important work was *Al-jabr wa'l mugabālah*. What mathematical term was derived from this title?

ANS: Algebra (Source: Boyer and Merzbach, A History of Mathematics

120. The average of 5 numbers is 23. What is the average of these 5 numbers together with 17?

ANS: $\frac{5\times 23+17}{6} = \boxed{22}$

121. 200 cadets throw their hats high into the air and randomly pick up hats afterwards. How many cadets are expected to pick up their own hats?

ANS: $200\left(\frac{1}{200}\right) = \boxed{1}$ cadet

122. You decide to guess on a three-question true/false test. What is the probability that you guess correctly on exactly 2 of the three questions?

ANS: 3/8

123. The average of 5 numbers is 23. The number 3 is discarded from the collection of 5 numbers. What is the average of the remaining 4 numbers?

ANS: $\frac{5\times 23-3}{4} = \boxed{28}$

124. Neglecting order of addition, in how many ways can 24 be written as a sum of 2 primes?

ANS: $24 = 19 + 5 = 17 + 7 = 13 + 11$, 3 ways

125. After what mathematician was the Cartesian coordinate system named?

ANS: (René) Descartes

126. What is the area of the parallelogram whose four vertices are at the points $(0, 0)$, $(5, 8)$, $(6, 9)$, and $(11, 17)$?

ANS: $\det\begin{pmatrix} 6 & 9 \\ 5 & 8 \end{pmatrix} = \boxed{3}$

127. A regular tetrahedron has edges of length 3. What is the total surface area of the tetrahedron?

ANS: $\boxed{9\sqrt{3}}$

128. A circle of radius 1 is divided into 5 pieces. One of the pieces is 1/2 as large as each of the other four pieces. What is the area of the smallest piece?

ANS: $\boxed{\pi/9}$

129. Six people are in the same room. If everyone shakes hands with everyone else in the room, how many

handshakes take place?

$$\boxed{\text{ANS: } 5+4+3+2+1 = \boxed{15}}$$

130. In which quadrant do the two lines $y = 26 + 2x$ and $y = 6 - 3x$ intersect?

$$\boxed{\text{ANS: } \left\{ \begin{array}{l} y = 26 + 2x \\ y = 6 - 3x \end{array} \right\}, \text{ Solution is : } (-4, 18) \boxed{\text{second quadrant}}}$$

131. If a cube has a volume of 343, what is its surface area?

$$\boxed{\text{ANS: } 7^3 = 343, 7^2 \times 6 = \boxed{294}}$$

132. A car travels 30 miles per hour for 1 mile and 60 miles per hour for 1 mile. What is the average speed of the car for the 2 miles?

$$\boxed{\text{ANS: } d = rt \Longrightarrow t = \tfrac{1}{30} + \tfrac{1}{60} \Longrightarrow r = \tfrac{2}{\frac{1}{30}+\frac{1}{60}} = \boxed{40} \text{ mph}}$$

133. How far apart are the two points of intersection of the two curves $y = x^2 + 1$ and $y = x + 7$?

$$\boxed{\text{ANS: } \left\{ \begin{array}{l} y = x^2 + 1 \\ y = x + 7 \end{array} \right\}, \text{ Solution is : } (-2, 5), (3, 10), \sqrt{(3+2)^2 + (10-5)^2} = \boxed{5\sqrt{2}}}$$

134. What is the converse of the statement, "If roses are red, then violets are blue"?

$$\boxed{\text{ANS: } \boxed{\text{If violets are blue, then roses are red.}}}$$

135. Bo has \$6.00 in quarters and dimes. If Bo has twice as many quarters as dimes, how many dimes does Bo have?

$$\boxed{\text{ANS: } \left\{ \begin{array}{l} 25q + 10d = 600 \\ q = 2d \end{array} \right\}, \text{ Solution is : } d = \boxed{10}, q = 20}$$

136. What is the smallest perfect cube larger than 500?

$$\boxed{\text{ANS: } 6^3 = 216, 7^3 = 343, 8^3 = \boxed{512}}$$

137. What is the sum of the roots of the polynomial $x^2 - 13x + 7$?

$$\boxed{\text{ANS: } (x-a)(x-b) = x^2 - (b+a)\,x + ab, \text{ so } a + b = \boxed{13}}$$

138. An automobile is driven 15,000 miles with 5 tires rotated often for even wear. How many miles are there on each tire?

$$\boxed{\text{ANS: } \tfrac{15000 \times 4}{5} = \boxed{12,000} \text{ miles}}$$

139. Out of a group of ten members, in how many different ways can a president, vice-president, and treasurer be elected?

$$\boxed{\text{ANS: } 10 \cdot 9 \cdot 8 = \boxed{720}}$$

140. How many different strings of length 7 can be formed using the letters in ELLIPSE?

$$\boxed{\text{ANS: } \tfrac{7!}{2!2!} = \boxed{1260}}$$

141. What is the area of the trapezoid bounded by the x-axis, the vertical lines $x = 1$ and $x = 3$, and the line

$y = -3x + 21$?

ANS: $-3 + 21 = 18$, $-3\,(3) + 21 = 12$, $\frac{(3-1)(18+12)}{2} = \boxed{30}$

142. What is the least common denominator of the two fractions $\frac{1}{63}$ and $\frac{1}{56}$.

ANS: $\mathrm{lcm}(63, 56) = \boxed{504} = 2^3 3^2 7$

143. A computer sequentially computes integers by the following rule: If n is a square then multiply by 2, otherwise subtract 2. Starting at $n = 13$, what is the integer after 6 iterations?

ANS: $13 \to 11 \to 9 \to 18 \to 16 \to 32 \to \boxed{30}$

144. What is the surface area of a sphere of volume 972π?

ANS: $\frac{4}{3}\pi r^3 = 972\pi$, Solution is : $r = 9$, $4\pi r^2 = \boxed{324\pi}$

145. A solid statue is made by melting 9 cm^3 of metal and pouring it into a mold. A larger model needs to be constructed by increasing each of its linear dimensions by a factor of 4. How much metal will the new statue require?

ANS: $9 \times 4^3 = \boxed{576}$ cm^3

146. How many vertices does an n-dimensional cube have?

ANS: $\boxed{2^n}$

147. What is the area of a right triangle having a leg of length ℓ and hypotenuse of length 25?

ANS: $A = \boxed{\frac{\ell\sqrt{25-\ell^2}}{2}}$

148. Bertrand Russell and Alfred North Whitehead wrote the monumental *Principia mathematica*. What was the subject of this publication?

ANS: Mathematical Foundations or Logic or Set Theory

149. What is the greatest integer less than the sum $\frac{17}{3} + \frac{3}{17}$?

ANS: $\frac{17}{3} + \frac{3}{17} = 5.843137255 = \boxed{5} + .843137255$

150. 200 cadets throw their hats high into the air and randomly pick up hats afterwards. How many cadets are expected to pick up their own hats?

ANS: $200\left(\frac{1}{200}\right) = \boxed{1}$ cadet

151. You currently earn \$5.00 per hour delivering pizza. You are due for a raise, and you figure the probability of a \$.50 raise is 70% and the probability of a \$1.00 raise is 30%. What is your expected new salary?

ANS: $5.00 + .7(.50) + .3(1.00) = \boxed{\$5.65}$

152. A square is inscribed in a circle, which in turn is inscribed in a square. What percentage of the area of the large square is inside the small square?

ANS: $\boxed{50\%}$

153. The probability of picking a dog to finish in the top 3 at the dog track is $\frac{1}{4}$. What is the probability of picking 2 straight losers?

ANS: $\left(\frac{3}{4}\right)^2 = \boxed{\frac{9}{16}}$

154. The polynomial equation $x^5 + x^2 + 1 = 0$ has one real root. How many imaginary roots does it have?

ANS: four

155. The number 12 is called *abundant* because the sum of the proper divisors $1 + 2 + 3 + 4 + 6 = 16$ is greater than 12. What is the next abundant number?

ANS: $\boxed{18} < 1 + 2 + 3 + 6 + 9 = 21$

156. Andrew Wiles has claimed to have proven something that had remained an open problem for over 350 years. Either state the problem or name the person who claimed over 350 years ago to have a solution to the problem.

ANS: $x^n + y^n = z^n$ has no positive integer solutions for $n > 2$.

or Fermat's Last Theorem

157. The diameter of a square of edge a is $a\sqrt{2}$. The diameter of a cube of edge a is $a\sqrt{3}$. That is the diameter of a 4-dimensional cube of edge a?

ANS: $2a$

158. What is the perimeter of a regular hexagon inscribed in a circle of radius 3?

ANS: 18

159. If it takes two liters of paint to paint a two-meter model of a battleship, how much paint is required to paint a three-meter model of the same ship?

ANS: $2\left(\frac{3}{2}\right)^2 = \frac{9}{2} = 4\frac{1}{2} = 4.5$ liters

160. In the early 1900s a self-taught Indian mathematician sent some of his unusual mathematical formulas to the English mathematician G. H. Hardy, who recognized the depth of the formulas and invited this mathematician to England. This exceptional Indian mathematician was the subject of a public television documentary. A biography, *The man Who Knew Infinity*, was published in 1991. Name this mathematician.

ANS: Srinivasa Ramanujan

161. What is the area of one face of a diamond-shaped kite whose two sticks are 20 and 26 inches long?

ANS: 260 in^2

162. Three line segments joining the midpoints of the sides of a triangle determine a smaller triangle whose sides lie inside the larger triangle. What is the ratio of the area of the larger triangle to the area of the smaller triangle?

ANS: 4 : 1

163. In how many ways can the U.S. Senate select a committee of 4 from among its 100 members?

ANS: $\binom{100}{4} = \boxed{3,921,225}$

164. Bo is going to the store to buy candy that will cost somewhere between 5 cents and 26 cents. What is the fewest number of coins Bo can carry in order to be certain to have exact change to buy the candy?

ANS: penny, penny, penny, penny, nickel, dime, dime $\boxed{7}$ coins

165. The formula $e^{i\pi} + 1 = 0$ relates five of the most popular numbers in mathematics. What is e rounded to 6 significant digits?

ANS: $e \approx 2.718281828 \approx \boxed{2.71828}$

166. Three positive integers have a sum of 10. What is the maximum possible value for the sum of their squares?

ANS: $1^2 + 1^2 + 8^2 = \boxed{66}$

167. A contemporary, prolific, Hungarian mathematician is reputed to have said, "A mathematician is a machine for converting coffee into theorems." Name this mathematician.

ANS: Paul $\boxed{\text{Erdös}}$

168. Alice Longwalker plans to walk around the Earth at the equator. Assuming this is possible (she walks on water) and that the Earth is a perfect sphere, how much farther will her nose travel than her feet? Alice holds her nose 5 feet above the ground when walking.

ANS: $2\pi\,(r+5) - 2\pi r = 2\pi\,(r+5) - 2\pi r = \boxed{10\pi \text{ ft}}$

169. It is well known that the Hindus introduced zero into our number system, but that concept was also developed independently by other civilizations. What Central American civiliation used a symbol for zero that resembled a half-open eye?

ANS: $\boxed{\text{The Mayans}}$ (Source: Boyer and Merzbach, *A History of Mathematics*)

170. It takes 1000 square tiles to tile a room, or 1440 smaller tiles whose edge is 1 inch less. How large is the room in square feet?

$1000x^2 = 1440\left(x - \frac{1}{12}\right)^2$, Solution is : $x = \frac{1}{2}$, $1000\left(\frac{1}{2}\right)^2 = \boxed{250}$ ft^2

171. What is the highest power of 10 dividing 20!?

ANS: $20! = \underline{20} \cdot 19 \cdots \underline{15} \cdots \underline{10} \cdots \underline{5} \cdots 2 \cdot 1 = 2432\,90200\,81766\,40000 = 10^4 \cdot m$ so $\boxed{4}$ or $\boxed{10^4}$

172. A triangular section of Old Town is divided into a smaller triangle and two trapezoids by two streets parallel to one of the boundary streets. The heights of the two trapezoids are equal to the height of the small triangle, and the area of the middle trapezoid is 12 acres. How many acres are there in the larger trapezoid?

ANS: $\boxed{20}$ acres

173. A triangle has 3 vertices and 3 edges. A tetrahedron has 4 vertices and 6 edges. How many vertices and

edges does an n-dimensional tetrahedron have?

ANS: $\boxed{n+1 \text{ vertices}}$ and $\boxed{n(n+1)/2 \text{ edges}}$

174. Solve the system of equations

$$\begin{aligned} -x+y+z &= 1 \\ x-y+z &= 1 \\ x+y-z &= 1 \end{aligned}$$

ANS: By symmetry, $\boxed{x=y=z=1}$

175. According to Descartes' Rule of Signs, how many positive real roots does the polynomial equation $3x^4+10x^2+5x-4=0$ have?

ANS: $\boxed{\text{one}}$ (since there is one sign change)

176. The plane can be completely tiled using equilateral triangles, or using squares, or using regular hexagons. These give the three *regular tessellations* of the plane. A *semi-regular tessellation* is a tiling that uses at least two different regular polygons, and where every vertex is congruent to every other vertex. How many semi-regular tessellations are there?

ANS: $\boxed{8}$

177. What is the smallest positive integer with exactly 5 positive divisors?

ANS: $\boxed{16}$ divisors are {1,2,4,8,16}

178. A right triangle has legs of length 6 and 8. What is the radius of the circle that circumscribes the triangle?

ANS: diameter $= 10$, radius $= \boxed{5}$

179. A pair of dice is rolled 30 times. What is the expected number of doubles?

ANS: $30\left(\frac{1}{6}\right) = \boxed{5}$

180. Twenty students in an algebra class take a five-question true/false test. None of the students had studied for the test, so all of them used pure guessing. What is the probability that at least 10 students score 60% or higher on the test?

ANS: $\boxed{50\%}$

181. Given 5 gallons of a 20% antifreeze/water mixture, how much pure antifreeze must be added to yield a 50% antifreeze/water mixture?

ANS: $\frac{1+x}{5+x} = \frac{1}{2}$, Solution is : $x = \boxed{3}$ gal

182. If g is a function such that $g(1)=2$, $g(2)=-1$, and

$$g(n+1) = g(n) + 2g(n-1)$$

for $n \geq 2$, what is $g(4)$?

ANS: $g(3) = g(2) + 2g(1) = -1+4 = 3$, $g(4) = g(3) + 2g(2) = 3-2 = \boxed{1}$

183. Write the expression $\frac{8x-1}{(x-2)(x+3)}$ in the form $\frac{a}{x-2}+\frac{b}{x+3}$.

ANS: $\frac{8x-1}{(x-2)(x+3)} = \boxed{\frac{3}{x-2}+\frac{5}{x+3}}$

184. A golf bag contains 13 balls, some yellow and the rest orange. Two balls are drawn at random from the bag and one is yellow, the other orange. What is the probability that exactly half of the balls in the bag are yellow?

ANS: $\boxed{0}$

185. Two cars start 2 miles apart and drive toward each other. One car goes 35 mph, the other 45 mph. After how many seconds do the two cars meet?

ANS: $t = \frac{d}{r} = \frac{2}{35+45} = \frac{1}{40}$ hr $= 1.5$ min $= \boxed{90 \text{ sec}}$

186. Cubic polynomials have three zeros, which in general are complex numbers. The polynomial $p(x) = x^3 - 7x^2 + x + 5$ has the number 1 as one of its zeros. What is the product of the other two zeros?

ANS: $(x-1)\left(x^2-6x-5\right)$, roots: $\left\{\begin{array}{c} 1 \\ 3+\sqrt{14} \\ 3-\sqrt{14} \end{array}\right\}$, $\left(3+\sqrt{14}\right)\left(3-\sqrt{14}\right) = \boxed{-5}$

187. A multiple-choice exam has 12 questions with 4 choices per question. If you answer the questions randomly, what is the expected number of correct responses?

ANS: $12 \times \frac{1}{4} = \boxed{3}$

188. How many integers between 50 and 450 are divisible by 17?

ANS: $17 \times 3 = 51$, $17 \times 26 = 442$, $26 - 3 + 1 = \boxed{24}$

189. What is the area of the smallest right triangle with all sides positive integers?

$3-4-5$ right triangle has area $3 \cdot 2 = \boxed{6}$

190. According to the Rational Root Theorem, what are all the possible rational roots of the polynomial $3x^3 - 4x^2 + 5x - 6$?

ANS: $\boxed{\pm\frac{1}{3}, \pm\frac{2}{3}, \pm 1, \pm 2, \pm 3, \pm 6}$

191. How many edges does an n-dimensional cube have?

ANS: $\boxed{n2^{n-1}}$

192. You are given seven points in the plane, no three of which lie on the same line. How many lines are there which pass through exactly two of these points?

ANS: $\binom{7}{2} = \boxed{21}$

193. Name at least two mathematicians who made early contributions to differential calculus.

ANS:Pierre $\boxed{\text{Fermat}}$ or John $\boxed{\text{Wallis}}$ or Isaac $\boxed{\text{Newton}}$ or Gottfried $\boxed{\text{Leibniz}}$ or $\boxed{\text{Cavalieri}}$ or $\boxed{\text{Barrow}}$

194. To the nearest minute, at what time between 10:30 a.m. and 11:00 a.m. are the minute hand and the hour hand at right angles?

ANS: $50 + \frac{m}{12} = m + 15$, Solution is : $m = \frac{420}{11} = 38.18181818$ 10:38

195. Find an integer between 10 and 100 that is both a perfect square and a perfect cube.

ANS: $64 = 4^3 = 8^2$

196. A 3-dimensional cube has 8 vertices and 12 edges. How many vertices and how many edges does a 4-dimensional cube have?

ANS: 16 vertices and 32 edges

197. What is x if $128^x = 32^6$?

ANS: $\left(2^7\right)^x = \left(2^5\right)^6 \Rightarrow 7x = 30 \Rightarrow x = \frac{30}{7}$

198. Given an isosceles right triangle with a hypotenuse of length $\sqrt{50}$, what is its area?

ANS: $\frac{25}{2} = 12\frac{1}{2} = 12.5$

199. If the probability that the Nuggets beat the Suns is 0.3, what is the probability that the Nuggets beat the Suns 4 times in a row?

ANS: $(0.3)^4 = .0081$

200. In how many ways can ALLAN misspell his name, assuming he uses all the right letters (the right number of times)? (Spell out A-L-L-A-N.)

ANS: $\frac{5!}{2!2!} - 1 = 29$

201. What is the greatest integer in $\frac{7}{26} + \frac{26}{7}$?

ANS: $\frac{7}{26} + \frac{26}{7} = \frac{725}{182} = 3.983516484\boxed{3}$

202. A popular novel has 342 pages and 97470 words. What is the average number of words per page?

ANS: $\frac{97470}{342} = 285$

203. Bo plants a vine that is intended to cover one outside wall on the south side of a house. The area that is covered with leaves doubles every month. After 11 months the entire wall is covered. When was exactly half of the wall covered with leaves?

ANS: One month earlier or after 10 months

204. A spherical balloon's diameter increases by 30%. By what percentage does the surface area change?

ANS: $(1.3)^2 = 1.69$ 69% increase

205. What is the contrapositive of the statement, "If n is a multiple of 6, then n is a multiple of 3."

ANS: If n is not a multiple of 3, then n is not a multiple of 6.

206. The average of the four numbers 11, 15, 17, and x is 23. What is x?

ANS: $\frac{11+15+17+x}{4} = 23$, Solution is : $x = \boxed{49}$

207. How many feet per second is 60 miles per hour?

ANS: $60 \cdot \frac{5280}{3600} = \boxed{88}$ ft/sec

208. What is the base 8 representation of the number 65?

ANS: $\boxed{101}$ or $\boxed{101_8}$

209. The 2×2 matrix A is given by $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$. What is A^{-1}?

ANS: $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}^{-1} = \boxed{\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}}$