Math Day 1994
Team Competition
Colorado State University

1. At 10:00 a.m., Bob starts jogging at 8 km/hr. At noon, Kate starts jogging from Bob's starting place at 10 km/hr. How far behind will she be at 2:00 p.m.?

$$\boxed{16\text{-}2\times 2=\boxed{12 \text{ km}}}$$

2. What is the surface area of a spherical raindrop of diameter 0.5 millimeters?

$$\boxed{4\pi\,(.5/2)^2=\boxed{.25\pi}\ \text{mm}^2}$$

3. How many possible ways are there to pick the top 4 finishers in a 9-person race?

$$\boxed{9\cdot 8\cdot 7\cdot 6=\boxed{3024}}$$

4. How many minutes were there in February of 1994?

$$\boxed{28\times 24\times 60=\boxed{40320}\ \text{(not a leap year)}}$$

5. Find C so that the equation $x^2-12x+C=0$ has exactly one real root.

$$\boxed{x^2-12x+C=0, \text{ Solution is: } \left\{x=6\pm\sqrt{(36-C)}\right\} \text{Same when } C=\boxed{36}}$$

6. A shirt has been marked down 10% to \$31.50. What was the original price?

$$\boxed{31.5=.9x, \text{ Solution is:} \left\{x=\boxed{\$35.00}\right\}}$$

7. What is the 15th term of the arithmetic sequence that begins 42,35,28,. . .?

$$\boxed{f(n)=49-7n,\ f(1)=42,\ f(2)=35,\ f(3)=28,\ f(15)=\boxed{-56}}$$

8. What is the smallest positive integer with exactly 6 positive divisors?

$$\boxed{\boxed{12}\ \text{(divisors are } \{1,2,3,4,6,12\})}$$

9. What is the diameter of a circle with area 36π cm^2?

$$\boxed{36\pi=(D/2)^2\pi, \text{ Solution is:} D=\boxed{12}\ \text{cm}}$$

10. Two wheels are connected by a belt. One has a diameter of 25 centimeters and a speed of 350 rmp. The other has a speed of 75 rpm. What is its diameter?

$$\boxed{24\times 350=75x, \text{ Solution is: } x=\boxed{112}\ \text{cm}}$$

11. If $f(x)=3x-2$, what is $f(f(f(2)))$?

$$\boxed{f(f(f(2)))=\boxed{28}}$$

12. Neglecting the order of addition, in how many ways can 30 be written as the sum of two primes?

$$\boxed{30=7+23=11+19=13+17 \qquad \boxed{3 \text{ ways}}}$$

13. Find x so that the average of the four numbers 23, 14, x, and 29 have an average of 25.

$$\boxed{\frac{23+14+x+29}{4} = 25, \text{ Solution is: } \left\{x = \boxed{34}\right\}}$$

14. What positive number is twice as large as its reciprocal?

$$\boxed{x = 2 \times \frac{1}{x}, \text{ Solution is: } \left\{x = \boxed{\sqrt{2}}\right\}}$$

15. What is the cube root of 1 million?

$$\boxed{\boxed{100} \qquad 100^3 = 1000000}$$

16. Which is largest: π, 355/113, or 3.1416?

$$\boxed{355/113 \approx 3.14159292, \pi \approx 3.141592654, \boxed{3.1416}}$$

17. What is the coefficient of x^2 in the expansion of $(3x+2)^4$?

$$\boxed{(3x+2)^4 = 81x^4 + 216x^3 + \boxed{216}x^2 + 96x + 16}$$

$$\boxed{\binom{4}{2}3^2 2^2 = \boxed{216}}$$

18. What is the perimeter of an isosceles triangle with base 10 and area 60?

$$\boxed{h = 12,\ s^2 = h^2 + 25,\ s = 13,\ P = 10 + 2 \times 13 = \boxed{36}}$$

19. In how many ways can the US Senate pick a committee of 3 from among its 100 members?

$$\boxed{\binom{100}{3} = \boxed{161700}}$$

20. What is the reduced form of the fraction $\frac{169}{260}$?

$$\boxed{\frac{169}{260} = \boxed{\frac{13}{20}}}$$

21. Factor the polynomial $2x^3 - 5x^2 + 2x - 5$.

$$\boxed{2x^3 - 5x^2 + 2x - 5 = \boxed{\left(x^2+1\right)(2x-5)}}$$

22. If 132 feet of wire are used to build a skeleton of a cube, what is the volume of the cube?

$$\boxed{\left(\frac{132}{12}\right)^3 = (11)^3 = \boxed{1331}\ \text{ft}^3}$$

23. What is the perimeter of a right triangle with legs 8 and 15?

$$\boxed{s^2 = 8^2 + 15^2,\ s = 17,\ P = 8 + 15 + 17 = \boxed{40}}$$

24. The sum of two number is 13 and their product is 40. What are the numbers?

$$\boxed{\left\{\begin{array}{c} a+b=13 \\ ab=40 \end{array}\right\}, \text{ Solution is : } \boxed{\{b=5, a=8\}}}$$

25. A pyramid is built out of blocks by placing 100 blocks on the floor, placing 81 blocks on top of the

bottom layer, and so forth. How many cubes are there in the pyramid?

$$\boxed{\sum_{i=1}^{10} i^2 = \boxed{385}}$$

$$\boxed{\frac{10 \cdot 11 \cdot 21}{6} = \boxed{385}}$$

26. What are the dimensions of a rectangle with area 96 and perimeter 40?

$$\boxed{\left\{ \begin{array}{c} ab = 96 \\ 2a + 2b = 40 \end{array} \right\}, \text{ Solution is : } \boxed{\{a = 8, b = 12\}}}$$

27. Given the equation $x^3 - 17x^2 + 90x - 144 = 0$, what is the product of the roots?

$$\boxed{x^3 - 17x^2 + 90x - 144 = (x-3)(x-6)(x-8), 3 \cdot 6 \cdot 8 = \boxed{144}}$$

28. Art, Betty and Claude are now 7, 11, and 13 years old. In what year will they again all have prime numbered ages?

$$\boxed{\text{In the year } \boxed{2000}, \text{ they will have ages 13, 17, and 19.}}$$

29. What is the greatest integer in the sum $\frac{11}{3} + \frac{3}{11}$?

$$\boxed{\frac{11}{3} + \frac{3}{11} = \frac{130}{33} = 3.939393939, \text{ so greatest integer is } \boxed{3}}$$

30. A circular pizza is diameter 16 inches is cut into 8 congruent slices. What is the perimeter of each slice?

$$\boxed{c = 16\pi, p = \frac{16\pi}{8} + 2 \cdot 8 = \boxed{2\pi + 16} \text{ inches}}$$

31. How many strings of length 8 can be made with the letters in parabola?

$$\boxed{8!/3! = \boxed{6720}}$$

32. What number is halfway between $\frac{1}{5}$ and $\frac{1}{7}$?

$$\boxed{\left(\frac{1}{5} + \frac{1}{7}\right)/2 = \boxed{\frac{6}{35}}}$$

33. What is $\binom{5}{0} + \binom{5}{1} + \binom{5}{2} + \binom{5}{3} + \binom{5}{4} + \binom{5}{5}$? (Sum of binomial coefficients.)

$$\boxed{\binom{5}{0} + \binom{5}{1} + \binom{5}{2} + \binom{5}{3} + \binom{5}{4} + \binom{5}{5} = \boxed{32} = 2^5}$$

34. How many edges does an octahedron have?

$$\boxed{\frac{8 \cdot 3}{2} = \boxed{12}}$$

35. Completely factor the polynomial $18x^2 + 33x - 30$.

$$\boxed{18x^2 + 33x - 30 = \boxed{3(3x-2)(2x+5)}}$$

36. Give an equation in the form $ax + by = c$, where a, b, and c are integers, for the line through $(-2, 1)$ that is perpendicular to the line $2x + 3y = 7$.

$$\boxed{\boxed{-3x + 2y = 8} \text{ or } \boxed{3x - 2y = -8}}$$

37. Name the smallest integer whose fourth power is less than 100.

$(-4)^4 = 256$, $(-3)^4 = 81$, $(-2)^4 = 16$, etc. $\boxed{-3}$ is the smallest

38. Allison scored 74 on the first exam and 83 on the second exam. What must she average on the next two exams to bring her average for the four exams up to 85?

$\frac{74+82+2x}{4} = 85$, Solution is : $\left\{x = \boxed{92}\right\}$

39. How many strings of length 6 can be made from the letters in CONOCO?

$\frac{6!}{3!2!} = \boxed{60}$

40. What is the length of the arc on a circle of radius 10 subtended by a central angle of 144°?

$\frac{144}{360} \cdot 10 \cdot 2\pi = \boxed{8\pi}$

41. Given 4 gallons of a 25% antifreeze/water mixture, how much pure antifreeze must be added to yield a 50% antifreeze/water mixture?

$a = 1$ gal, $w = 3$ gal, need $\boxed{2}$ gal antifreeze

42. What is the prime factorization of 1001?

$1001 = \boxed{7 \times 11 \times 13}$

43. What society first developed the concept of zero?

The Hindus.

44. Seven poker players are seated at a round table. How many rearrangements are possible, if the only considerations are who is seated at each person's left, and at each person's right? (Because of the betting order Arlene on your left is different from Arlene on your right.)

$\frac{7!}{7} = \boxed{720}$

45. What is the area of the right triangle whose hypotenuse is 17 if one of the legs has length 8?

$x^2 + 8^2 = 17^2$, Solution is : $x = 15$, $Area = \frac{1}{2} \cdot 8 \cdot 15 = \boxed{60}$

46. What is the sum of the first five odd integers?

$1 + 3 + 5 + 7 + 9 = \boxed{25}$

47. Farmer Jill raises goats and geese. If she counts 130 eyes and 214 feet, how many goats and how many geese does Jill have?

$\begin{matrix} 2a + 2b = 130 \\ 4a + 2b = 214 \end{matrix}$, Solution is : $\boxed{a = 42 \text{ goats}, b = 23 \text{ geese}}$

48. What is the area of the parallelogram whose four vertices are at $(0, 0)$, $(1, 3)$, $(2, 5)$, and $(3, 8)$?

$\det \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} = \boxed{1}$

49. If the first term of a geometric sequence is $\frac{1}{32}$ and the second term is $\frac{1}{48}$, what is the fifth term?

$$\boxed{\frac{1}{32}\left(\frac{2}{3}\right)^4 = \boxed{\frac{1}{162}}}$$

50. If two cards are drawn from a standard deck of 52 cards, what is the probability that both are kings?

$$\boxed{\frac{\binom{4}{2}}{\binom{52}{2}} = \frac{6}{1326} = \boxed{\frac{1}{221}}}$$

51. The quartic polynomial $6x^4 - 11x^3 - 66x^2 + 59x - 12$ has four real roots. What is the product of these four roots?

$$\boxed{6\left(x+3\right)\left(x-4\right)\left(x-\tfrac{1}{2}\right)\left(x-\tfrac{1}{3}\right), -3\cdot 4\cdot\tfrac{1}{2}\cdot\tfrac{1}{3} = \boxed{-2}}$$

52. The distance between the points $(2, 4)$ and $(7, c)$ is 13. Find all the possible values for c.

$$\boxed{(2-7)^2 + (4-c)^2 = 13^2, \text{ Solution is : } \boxed{\{c = 16\}, \{c = -8\}}}$$

53. What is the contrapositive of the statement, "If $x > 4$ then $x^2 > 16$"?

$$\boxed{\text{If } x^2 \leqq 16, \text{ then } x \leqq 4.}$$

54. Give the points of intersection of the two curves $y = 20 - 6x$ and $y = 8 - 6x + 3x^2$.

$$\boxed{\left\{\begin{array}{c} y = 8 - 6x + 3x^2 \\ y = 20 - 6x \end{array}\right\}, \text{ Solution is : } \boxed{(2, 8), (-2, 32)}}$$

55. Which ancient civilization is responsible for dividing the circle into 360 equal parts (which we now call *degrees*)?

$$\boxed{\text{The Babylonians.}}$$

56. A Spanish port was protected by large cannons, and each cannon had a pile of cannon balls nearby stacked neatly in the shape of a tetrahedron. If each bottom layer contained a total of 21 cannon balls, how many cannon balls were in each pile?

$$\boxed{21 + 15 + 10 + 6 + 3 + 1 = \boxed{56}}$$

57. What is the smallest positive integer divisible by all of the integers 1, 2, 3, 4, 5, 6, and 7?

$$\boxed{\text{lcm}(1, 2, 3, 4, 5, 6, 7) = \boxed{420}}$$

58. What is the smallest 4-digit prime?

$$\boxed{1001 = 7 \times 11 \times 13, 1003 = 17 \times 59, 1007 = 19 \times 53, \boxed{1009}}$$

59. For what choices of c does the polynomial $cx^2 - 6x + 1$ have two real roots?

$$\boxed{36 - 4c > 0, \text{ Solution is : } \boxed{c < 9}}$$

60. Find a point equidistant from the points $(-1, -1)$, $(1, 1)$, and $(1, -1)$.

$$\boxed{(0, 0)}$$

61. What is the coefficient of y^3 in the expansion of $(x+y)^5$?

$$(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + \boxed{10x^2}y^3 + 5xy^4 + y^5$$

62. If $f(x) = 4x - 3$ what is $f(f(f(-1)))$?

$$f(f(f(-1))) = \boxed{-127}$$

63. How far apart are the two points with the curves $y = x + 6$ and $y = x^2$ intersect?

$\left\{ \begin{array}{c} y = x+6 \\ y = x^2 \end{array} \right\}$, Solution is : $\{(-2,4),(3,9)\}$, distance $\boxed{5\sqrt{2}}$

64. What is the converse of the statement, "If roses are red, then violets are blue"?

$$\boxed{\text{If violets are blue, then roses are red.}}$$

65. Sam has \$12.00 in quarters and dimes. If he has twice as many quarters as dimes, how many dimes does Sam have?

$25 \cdot 2d + 10d = 1200$, Solution is : $d = \boxed{20}$

66. If $\sin x = \frac{1}{3}$, what is $\sin 2x$?

$$\sin 2x = 2\sin x \cos x = 2 \cdot \tfrac{1}{3}\sqrt{1 - \tfrac{1}{9}} = \boxed{\tfrac{4}{9}\sqrt{2}}$$

67. What is the smallest perfect cube larger than 100?

$4^3 = 64$, $5^3 = \boxed{125}$

68. What is the greatest common divisor of 368 and 667?

$$\gcd(368, 667) = \boxed{23}$$

69. In which quadrant do the two lines $y = 4 - 2x$ and $y = 18 + 5$ intersect?

Point $(-2, 8)$ is in the $\boxed{\text{second quadrant}}$

70. A biological brick grows 10% in length, 5% in width, and shrinks in height by 14%. Is it larger or smaller than when it started out?

$1.1 \cdot 1.05 \cdot (.86) = .9933$ $\boxed{\text{smaller}}$

71. A pair of dice is rolled. What is the probability that the sum is either a 7 or an 11?

$$\tfrac{6}{36} + \tfrac{2}{36} = \boxed{\tfrac{2}{9}}$$

72. A computer sequentially computes integers by the following rule: If n is a square then multiply by 2. Otherwise, subtract 2. Starting at $n = 9$, what is the integer after 6 iterations?

$9, 18, 16, 32, 30, 28, \boxed{26}$

73. What is the sum of the roots of the polynomial $x^2 - 7x + 3$?

$(x-a)(x-b) = x^2 - x(a+b) + ab$ so $\boxed{7}$ is the sum of the roots

74. A committee of 3 people is to be chosen from among 3 men and 3 women. How many ways can this be done if the committee must include at least one man and at least one woman?

$$\boxed{\binom{6}{3} - \binom{3}{3} - \binom{3}{3} = \boxed{18}}$$

75. What is the area of the trapezoid bounded by the x -axis, the vertical lines $x = 1$, $x = 4$, and the line $y = -3x + 15$.

$$\boxed{-3 + 15 = 12, -12 + 15 = 3, 3(12 + 3)/2 = \boxed{\tfrac{45}{2}}}$$

76. If g is a function such that $g(1) = 1$, $g(2) = 1$, and $g(n) = g(n-2) + 3g(n-1)$ for $n \geq 2$, what is $g(4)$?

$$\boxed{g(3) = g(1) + 3g(2) = 4, g(4) = g(2) + 3g(3) = 1 + 3 \cdot 4 = \boxed{13}}$$

77. A bag of Halloween candy contains 5 pieces of chocolate and 4 pieces of fruit bar. What is the probability that two items selected at random are both chocolate?

$$\boxed{\frac{\binom{5}{2}}{\binom{9}{2}} = \frac{10}{36} = \boxed{\frac{5}{18}}}$$

78. A punch bowl at a Halloween party is the shape of a truncated cone of height 1 foot, base diameter 1 foot, and top diameter 18 inches. The orange plastic punch cups are also truncated cones of height 2 inches, base diameter 2 inches, and top diameter 3 inches. How many cups of punch will the punch bowl hold?

$$\boxed{6^3 = \boxed{216} \text{ cups}}$$

79. If eggs weigh 1.5 ounces each and a dozen eggs cost 90 cents, what is the cost of a pound of eggs?

$$\boxed{\tfrac{90 \cdot 16 \cdot 2}{12 \cdot 3} = \boxed{80} \text{ cents}}$$

80. What number greater than 50 has the property that when divided by 23 the remainder is 1 and when divided by 13 the remainder is also 1?

$$\boxed{23 \cdot 13 + 1 = \boxed{300}}$$

81. What is the area of the ellipse $\frac{x^2}{25} + \frac{y^2}{144} = 1$?

$$\boxed{5 \cdot 12 \cdot \pi = \boxed{60\pi}}$$

82. A gold watch has been reduced 10%, then 20%, and then 30% and finally sold for $\$504$. What was the original price?

$$\boxed{x\tfrac{9}{10} \cdot \tfrac{8}{10} \cdot \tfrac{7}{10} = \$504, \text{ Solution is : } x = \boxed{\$1000}}$$

83. An orange has a diameter that is 90% fruit and 10% peel. What percentage of the volume is the peel?

$$\boxed{1 - \left(\tfrac{9}{10}\right)^3 = \tfrac{271}{1000} = \boxed{27.1\%}}$$

84. What is the area of the triangle bounded by the lines $x = 0$, $y = x$, and $3y + x = 12$?

$$\boxed{\text{Vertices } (0,0), (0,4), (3,3), \text{ base 4, height 3, area } \boxed{6}}$$

85. If $f(x) = x^{3/2}$, what is $f(4)$?.

$$\boxed{f(x) = x^{3/2}, f(4) = \boxed{8}}$$

86. In an algebra class with 30 students, each person shakes hands with all the other students. How many handshakes are there?

$$\boxed{\frac{30\cdot 29}{2} = \boxed{435},\ \binom{30}{2} = \boxed{435},\ \sum_{i=1}^{29} i = \boxed{435}}$$

87. Find the vertex of the parabola$y = 3x^2 - 4x - 2$.

$$\boxed{3\left(x - \tfrac{2}{3}\right)^2 - \tfrac{10}{3},\ \boxed{\left(\tfrac{2}{3}, -\tfrac{10}{3}\right)}}$$

88. What is the partial fraction expansion of $\frac{9x-24}{(x-4)(x-2)}$?

$$\boxed{\frac{9x-24}{(x-4)(x-2)} = \boxed{\frac{6}{x-4} + \frac{3}{x-2}}}$$

89. The probability of picking a dog to finish in the top 3 at the dog track is $\frac{1}{4}$. What is the probability of picking 3 straight losers?

$$\boxed{\left(\tfrac{3}{4}\right)^3 = \boxed{\tfrac{27}{64}}}$$

90. What are all the possible rational roots of $2x^3 - 5x^2 - x + 8 = 0$, as limited by the Rational Root Theorem?

$$\boxed{\pm\tfrac{1}{2}, \pm 1, \pm 2, \pm 4, \pm 8}$$

91. The compact disk UR2gly sells at outlet AC for \$12.95 less a discount of 15%, and at outlet DC for \$14.65 less a discount of 25%. Which outlet has the lower price?

$$\boxed{AC = 12.95 \times .85 = 11.0075,\ \boxed{DC} = 14.65 \times .75 = 10.9875}$$

92. The sum of 4 consecutive integers is 98. What is the largest of the 4 integers?

$$\boxed{\text{Average is } \tfrac{49}{2}\text{, add } \tfrac{3}{2} \text{ to get } \boxed{26}}$$

93. How far apart are the two points at which the curves $x = 5$ and $x^2 + y^2 = 169$ intersect?

$$\boxed{\left\{ \begin{array}{c} x = 5 \\ x^2 + y^2 = 169 \end{array} \right\}\text{, Solution is : } (5, 12)\,, (5, -12)\,\text{, distance} = \boxed{24}}$$

94. If the line $y = mx$ touches the curve $y = x^2 + 1$ in exactly 1 point, what are the possibilities for m?

$$\boxed{m = \boxed{\pm 2}}$$

95. A computer sequentialy computes integers by the following rule: If n is odd then replace n by $3n + 1$, otherwise replace n by $n/2$. If n starts at 3, what is n after 5 iterations?

$$\boxed{3 \to 10 \to 5 \to 16 \to 8 \to \boxed{4}}$$

96. If a cube has a volume of 512, what is its surface area?

$$\boxed{8^3 = 512, 8^2 \cdot 6 = \boxed{384}}$$

97. How many rearrangements of the letters a, b, c, d, e, f have a listed before b and b listed before c?

$$\boxed{\tfrac{6!}{3!} = \boxed{120}}$$

98. If $\binom{n}{1} + \binom{n}{2} = 36$, what is n?

$\boxed{\binom{n}{1} + \binom{n}{2} = 36, \text{ Solution is : } \boxed{n = 8}}$

99. In how many ways can 6 boys and 6 girls be teamed into pairs, if each pair must contain one girl?

$\boxed{6! = \boxed{720}}$

100. What is the surface area of a sphere of volume 288π?

$\boxed{\frac{4}{3}\pi r^3 = 288\pi, \text{ Solution is : } r = 6, 4\pi r^2 = \boxed{144\pi}}$

101. What is the area of the triangle bounded by the x-axis, the y-axis, and the line $y = 4x + 8$?

$\boxed{\frac{1}{2} \cdot 2 \cdot 8 = \boxed{8}}$

102. What is the sum of the roots of the polynomial $x^2 - 6x - 9$?

$\boxed{\text{Solution is : } x = 3 \pm 3\sqrt{2}, \text{ sum is } \boxed{6}}$

103. On a four-question true/false exam, correct answers are worth 3 points, wrong answers 0, and blanks count 1 point. How many different responses will result in a total score of 4?

$\boxed{\text{Form 1111 or 3100, so } 1 + \frac{4!}{2!} = \boxed{13}}$

104. What is the largest integer $\leqq \sqrt{700}$?

$\boxed{\sqrt{700} = 26.45751311, \text{ so } \boxed{26}}$

105. How many subsets of $\{1, 2, 3, 4, 5, 6, 7\}$ contain both 2 and 4?

$\boxed{2^5 = \boxed{32}}$

106. How many committees of 2 men and 2 women can be formed from a group of 4 men and 5 women?

$\boxed{\binom{4}{2}\binom{5}{2} = 6 \cdot 10 = \boxed{60}}$

107. What is the prime factorization of 127?

$\boxed{\boxed{127} \text{ (prime)}}$

108. What is the area of an equilateral triangle inscribed in a circle of radius 2 inches?

$\boxed{\boxed{3\sqrt{3}} \text{ in}^2}$

109. Name the five Platonic solids (regular polyhedra).

$\boxed{\text{Tetrahedron, cube (or hexahedron), octahedron, dodecahedron, icosahedron}}$

110. A British mathematician who teaches at Princeton University has recently announced a proof of Fermat's Last Theorem. Name the mathematician.

$\boxed{\text{Andrew Wiles}}$

111. Which is larger, 8^{-9} or 9^{-8}?

$$\boxed{8^{-9} = 7.450580597 \times 10^{-9}, \boxed{9^{-8}} = 2.323057313 \times 10^{-8}}$$

112. What is the tenth term of an arithmetic sequence whose first term is -1 and whose difference between successive terms is 2?

$$\boxed{f(n) = -3 + 2n,\, f(1) = -1,\, f(10) = \boxed{17}}$$

113. How many integers between 50 and 350 are divisible by 13?

$$\boxed{4 \cdot 13 = 52,\, 27 \cdot 13 = 351,\, 27 - 4 = \boxed{23}}$$

114. If a mantel clock strikes the hours, how many times will it strike during a 24-hour period?

$$\boxed{2\sum_{i=1}^{12} i = \boxed{156}}$$

115. One third of the air in a container is removed by each cycle of an air pump. What fractional part of the air remains after 5 cycles?

$$\boxed{\left(\tfrac{2}{3}\right)^5 = \boxed{\tfrac{32}{243}}}$$

116. The height of a rectangle is 25% less than its base. The perimeter of the rectangle is 42 inches. Find the area of the rectangle.

$$\boxed{2x + 2 \cdot \tfrac{3}{4}x = 42, \text{ Solution is : } x = 12,\, Area = 12 \cdot 9 = \boxed{108}\,\text{in}^2}$$

117. Write 1994 as a product of primes.

$$\boxed{1994 = \boxed{2 \times 997}}$$

118. What positive number is twice as big as its reciprocal?

$$\boxed{x = 2\tfrac{1}{x}, \text{ Solution is : } x = \boxed{\sqrt{2}}}$$

119. If 2 is the first prime, what is the eleventh prime?

$$\boxed{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, \boxed{31}}$$

120. What is π rounded to 10 significant digits?

$$\boxed{\pi = \boxed{3.141592654}}$$

121. Which of the following best describes how many 6-symbol license plates are possible if the symbols come from the letters A-Z together with the digits 0-9? (a) 2–3 million (b) 20–30 million (c) 200–300 million (d) 2–3 billion

$$\boxed{36^6 = 21767\,82336 \approx \boxed{\text{2–3 billion}} \text{ or } \boxed{\text{(d)}}}$$

122. If a 100-Watt sound system can break a glass goblet placed 1 foot away from the speaker, how powerful a sound system would it take to break a similar glass goblet placed 10 feet from the speaker?

$$\boxed{100 \times 10^2 = \boxed{10000} \quad \text{Watts}}$$

123. What is the remainder if 3^{18} is divided by 19?

$$\boxed{3^{18} \bmod 19 = \boxed{1} \text{ (Fermat's Little Theorem)}}$$

124. The average of 3 numbers is 19. What is the average of these 3 numbers together with 27?

$$\boxed{\frac{3 \cdot 19 + 27}{4} = \boxed{21}}$$

125. What was the license plate number the Indian mathematician Ramanujan referred to when he said, "On the contrary, that's a very interesting number. It's the first number that can be written as the sum of 2 cubes in 2 different ways."

$$\boxed{12^3 + 1^3 = 10^3 + 9^3 = \boxed{1729}}$$

126. What is the sum of the first 100 positive integers?

$$\boxed{\sum_{i=1}^{100} i = \boxed{5050}}$$

127. What is the smallest positive integer n such that $n^2 - n + 11$ is *not* prime?

n	1	2	3	4	5	6	7	8	9	10	$\boxed{11}$
$n^2 - n + 11$	11	13	17	23	31	41	53	67	83	101	$121 = 11^2$

128. The number 6 is perfect because $6 = 1 + 2 + 3$ is the sum of the proper divisors. Name the next perfect number.

$$\boxed{\boxed{28} = 1 + 2 + 4 + 7 + 14}$$

129. How many edges does a regular tetrahedron have?

$$\boxed{\frac{3 \cdot 4}{2} = \boxed{6}}$$

130. Use the approximation $2^{10} \approx 10^3$ to estimate $\log_2\left(10^{42}\right)$.

$$\boxed{\log_2\left(10^{42}\right) = \log_2\left(\left(10^3\right)^{14}\right) \approx \log_2\left(\left(2^{10}\right)^{14}\right) = \log_2\left(2^{140}\right) = \boxed{140}}$$

131. Express the perimeter P of a square as a function of its area A.

$$\boxed{A = \left(\frac{P}{4}\right)^2, \text{ Solution is : } \boxed{P = 4\sqrt{A}}}$$

132. At the local Dairy Queen, the "Monster Sundae" can be ordered with any of six flavors of ice cream plus any or all of the following topings: Nuts, whip creme, cherries, hot fudge, banana slices, gum balls, and oreo cookies. If you order one such sundae every Saturday, how many weeks will it be before you must order the same sundae twice?

$$\boxed{6 \cdot 2^7 = \boxed{768} \text{ weeks}}$$

133. Give the slope-intercept equation for a line through the origin that contains so other points with integer coordinates.

$$\boxed{y = \alpha x, \, \alpha \text{ any irrational number}}$$

134. How many 5-digit numbers can be constructed entirely out of 3's and 7's?

$$\boxed{2^5 = \boxed{32}}$$

135. A regular tetrahedron has edges of length 3. What is the total surface area of the tetrahedron?

$$\boxed{9\sqrt{3}}$$

136. If a sequence is defined by $x_1 = 1$, $x_2 = -4$, and $x_{n+1} = x_n - x_{n-1}$, what is x_5?

$$\boxed{1, -4, (-4) - 1 = -5, -5 - (-4) = -1, -1 - (-5) = \boxed{4}}$$

137. On January 1, 1994, a savings account contained \$1000. If it earns 4% compounded annually, what will the balance be January 1, 1996?

$$\boxed{(1.04)^2\, 1000 = \boxed{\$1081.60}}$$

138. Farmer Cornpea plowed $3/8$ of her field in 10 hours. At that rate, how much longer will it take her to plow the remainder of her field? (Give your answer to the nearest hours and minutes.)

$$\boxed{10/(3/8) = \tfrac{80}{3} = 26.66666667 \text{ or another } \boxed{\text{10 hours 40 minutes}}}$$

139. How many distinct complex roots does the polynomial $x^5 - 1$ have?

$$\boxed{5}$$

140. The determinant of the matrix of coefficients of a system of two linear equations in two unknows in 0. What is true about the graphs of the two equations?

$$\boxed{\text{The lines are parallel}}$$

141. The Center Ring Janitorial Supply owns a fleet of 3 vehicles: One car, which gets 20 miles per gallon, and two trucks, which each get 12 miles per gallon. If in a typical week the car is driven 400 miles and each truck is driven 240 miles, how many miles per gallon is Center Ring's fleet getting?

$$\boxed{\text{Total miles} = 880, \text{ total gallons} = 20 + 40 = 60, \text{ mileage } \tfrac{880}{60} = \boxed{14\tfrac{2}{3}} \text{ mi/gal}}$$

142. A circle of radius 1 is divided into 5 pieces. One of the peices is 1/2 as large as each of the other four. What is the area of the smallest piece?

$$\boxed{\tfrac{\pi}{9}}$$

143. A solid statue is made by melting 10 cm^3 of metal and pouring it into a mold. A larger model needs to be constructed by increasing each of its linear dimensions by a factor of 3. How much metal will the new statue require?

$$\boxed{10 \cdot 3^3 = \boxed{270} \text{ cm}^3}$$

144. A 6-foot man casts a 4-foot shadow. A flag pole next to him casts a 50-foot shadow. How tall is the flag pole?

$$\boxed{\boxed{75} \text{ feet}}$$

145. Solve the equation $x\sqrt{.09} = 3$.

$$\boxed{\text{Solution is : } \boxed{x = 10.0}}$$

146. The diagonal of a table with a square top is 6 feet. What is the area of the table top?

$$\boxed{\boxed{18} \text{ ft}^2}$$

147. A bag of chicken feed will feed 18 chickens for 54 days. How many days will it feed 12 chickens?

$$\boxed{54 \cdot \tfrac{18}{12} = \boxed{81} \text{ days}}$$

148. Express the volume V of a cube as a function of the area A of one of its faces.

$$\boxed{V = x^3 = \left(x^2\right)^{3/2} = A^{3/2}, \text{ so } \boxed{V = A^{3/2}}}$$

149. Assuming $0 < y < x$, express $\frac{x^y y^x}{y^y x^x}$ in terms of one quotient raised to a positive exponent.

$$\boxed{\frac{x^y y^x}{y^y x^x} = \left(\frac{x}{y}\right)^y \left(\frac{y}{x}\right)^x = \boxed{\left(\frac{y}{x}\right)^{x-y}}}$$

150. If $m > 0$ and the points $(m, 3)$ and $(1, m)$ lie on a line with slope m, find m.

$$\boxed{\frac{3-m}{m-1} = m, \text{ Solution is : } \boxed{m = \sqrt{3}}}$$

151. Let $y = mx + b$ be the image when the line $x - 3y + 11 = 0$ is reflected across the x-axis. What is $m + b$?

$$\boxed{y = -\tfrac{1}{3}x - \tfrac{11}{3}, m + b = -\tfrac{1}{3} - \tfrac{11}{3} = \boxed{-4}}$$

152. In how many ways can one arrange the letters in OBOE?

$$\boxed{\tfrac{4!}{2!} = \boxed{12}}$$

153. The age of a mathematician and her son added together is 44. In two years the mother will be three times as old as her son. How old is the mathematician?

$$\boxed{\left\{ \begin{array}{c} m + s = 44 \\ m + 2 = 3(s + 2) \end{array} \right\}, \text{ Solution is : } s = 10, \boxed{m = 34}}$$

154. A baseball manager has selected 9 starters for a game. If the pitcher must bat last and the second baseman must bat first, how many different batting line-ups are possible?

$$\boxed{7! = \boxed{5040}}$$

155. A golf bag contains 2 white golf ball, 4 yellow balls, and 4 orange balls. Two golf balls are selected at random. What is the probability that both are white?

$$\boxed{\frac{\binom{2}{2}}{\binom{10}{2}} = \boxed{\frac{1}{45}}}$$

156. What are the next 3 terms in the sequence that begins $1, 1, 2, 3, 5, 8$?

$$\boxed{13, 21, 34}$$

157. An automobile travels at 30 kilometers per hour for 1 minute, 20 kph for 3 minutes, and then 10 kph for another 3 minutes. How far did it travel during the 7 minutes?

$$\boxed{30 \cdot \tfrac{1}{60} + 20 \cdot \tfrac{3}{60} + 10 \cdot \tfrac{3}{60} = \boxed{2} \text{ kilometers}}$$

158. Store A sells candy bars 3 for \$1.00. Store B sells candy bars individually for 40 cents, but you get 5 for the price of 4. On Monday John bought some candy bars at store A. On Tuesday Jill bought some candy bars at store B. They compared notes and found that they had gotten the same number of candy bars, and each had paid the same amount of money. What is the least amount of money that each of them could

have spent?

They each bought 6 candy bars and spend $\boxed{\$2.00}$

159. How many different 6-letter words can be formed by rearranging the letters in SCHOOL?

$$\frac{6!}{2!} = \boxed{360}$$

160. What are the next two prime numbers after 90?

$91 = 7 \times 13$, $93 = 3 \times 31$, $\boxed{97, 101}$

161. What is the base 10 value of the base 3 number 222_3?

$$2 + 2 \cdot 3 + 2 \cdot 3^2 = \boxed{26}$$

162. How many 3-member teams can be formed from a group of 5 women and 2 men?

$$\binom{7}{3} = \boxed{35}$$

163. Find the sum of the next two terms of the sequence that begins $0, 3, 8, 15, 24, 35$.

$$48 + 63 = \boxed{111}$$

164. Find the fourth vertex of a rectangle, three of whose vertices are $(-2, 1)$, $(-1, -1)$, and $(1, 0)$.

The fourth vertex is $\boxed{(0, 2)}$

165. What is the midpoint of the line segment joint $(2, 8)$ and $(-4, 6)$?

$$\left(\frac{2-4}{2}, \frac{6+8}{2}\right) = \boxed{(-1, 7)}$$

166. A particle, initially at $(1, 2)$, moves along a line of slope 3 to a new position (x, y). Find y if $x = 5$.

$y - 2 = 3(5 - 1)$, Solution is : $\boxed{y = 14}$

167. Where does the circle of radius 5 centered at the origin intersect the line passing through the origin with slope $-3/4$?

$\left\{ \begin{array}{c} x^2 + y^2 = 25 \\ y = -\frac{3}{4}x \end{array} \right\}$, Solution is : $\boxed{\{y = 3, x = -4\}, \{y = -3, x = 4\}}$

168. Three unit circles are mutually tangent and enclose a triangular region R. Find the area of R.

$$\boxed{\sqrt{3} - \frac{\pi}{2}}$$

169. What prime number is nearest to 1994?

$\boxed{1993}$

170. For which ancient Greek mathematician is the formula for the area of a triangle in terms of its side lengths named?

$\boxed{\text{Hero}}$

171. Five straight lines are drawn in the plane. What is the largest possible number of points of intersection?

$$\boxed{\binom{5}{2} = \boxed{10}}$$

172. What is the largest integer that can be stored in a 6-bit computer word?

$$\boxed{63}$$

173. What is the next term in the sequence that begins $0, 2, 6, 12, 20$?

$$\boxed{30}$$

174. What is the smallest prime greater than 50?

$$\boxed{53}$$

175. In data processing terminology, what does FIFO mean?

$$\boxed{\text{First In First Out}}$$

176. Which is larger, 1 cubic inch or 16 cubic centimeters?

$$\boxed{2.54^3 = 16.387064 \text{ cm}^3 = \boxed{1 \text{ in}^3}}$$

177. What is 72° equal to in radians?

$$\boxed{\frac{2\pi}{5}}$$

178. What is the area of a regular hexagon with sides of length 2?

$$\boxed{6\sqrt{3}}$$

179. Factor the polynomial $x^2 + 4x - 77$.

$$\boxed{x^2 + 4x - 77 = \boxed{(x + 11)(x - 7)}}$$

180. Find the area of the circle inscribed in a regular hexagon with sides of length 2.

$$\boxed{3\pi}$$

181. What is the minimum value of the function $f(x) = x^2 + 2x - 4$?

$$\boxed{x^2 + 2x - 4 = (x + 1)^2 - 5 \text{ has minimum value } \boxed{-5}}$$

182. If the absolute value of $x + 1$ is equal to the absolute value of x, what is x?

$$\boxed{|x + 1| = |x|, \text{ Solution is : } \boxed{x = -\tfrac{1}{2}}}$$

183. What is the greatest common divisor of 91 and 221?

$$\boxed{\gcd(91, 221) = \boxed{13}}$$

184. The wholesale cost of slacks is \$40. The price is marked up 50%, then reduced 20% on sale. What is the sale price?

$$\boxed{40 \cdot (1.5) \cdot (.8) = \boxed{\$48.00}}$$

185. Ann keeps flies and spiders in a box in her dorm room during the Halloween season. There are a total of

15 creatures with 106 legs. How many flies and how many spiders does she have?

$$\boxed{\left\{\begin{array}{c} f+s=15 \\ 6f+8s=106 \end{array}\right\}, \text{ Solution is : } \boxed{f=7, s=8}}$$

186. What is the sum of all the integers greater than 10 and less than 100?

$$\boxed{\sum_{i=11}^{99} i = \boxed{4895} = \frac{99\cdot 100}{2} - \frac{10\cdot 11}{2}}$$

187. Completely factor 720.

$$\boxed{720 = 6! = 6\cdot 5\cdot 4\cdot 3\cdot 2 = \boxed{2^4 3^2 5}}$$

188. Nine liters of paint are required to paint the outside of a cubic box of volume 125 cubic meters. How much paint is needed to paint the outside of a cubic box of volume 1 m^3 if the box has no lid?

$$\boxed{\frac{5}{6}\cdot\frac{1}{25}\cdot 9 = \boxed{\frac{3}{10}} \text{ liter}}$$

189. John and Jill traded positions several times while rowing a canoe through the Boundary Waters of Minnesota. With Jill at the rear, the canoe went fast enough to complete the entire trip in 10 hours, and with John at the rear the trip would have taken 14 hours. The trip actually took 12 hours. For how many hours did John sit in the back of the canoe?

$$\boxed{\text{7 hours}}$$

190. Marty bought a farm, a house, and a barn for \$126,000. If the house cost twice as much as the barn, and the farm twice as much as the house and barn together, how much did each cost?

$$\boxed{barn = \$14,000,\ house = \$28,000,\ farm = \$84,000}$$

191. Henry explained his age by saying, "$\frac{2}{5}$ of my age less $\frac{1}{9}$ of what it will be a year from now is equal to $\frac{1}{3}$ of what my age was 5 years ago." What is his age now?

$$\boxed{\frac{2}{5}a - \frac{1}{9}(a+1) = \frac{1}{3}(a-5), \text{ Solution is : } a = \boxed{35}}$$

192. A city lot is twice as long as it is wide. By increasing its length 20 yards and its width 30 yards, the area will be increased by 2200 square yards. What are its dimensions?

$$\boxed{(2x+20)(x+30) - 2x^2 = 2200, \boxed{\text{20 yards by 40 yards}}}$$

193. Adam can frame a house in 10 days, Burl can do it in 12 days, and Clark can do it in 8 days. Working together, how many days would it take for the three of them to frame a house?

$$\boxed{x\left(\frac{1}{10}+\frac{1}{12}+\frac{1}{8}\right) = 1, \text{ Solution is : } x = \boxed{\frac{120}{37}} \text{ days}}$$

194. Factor $x^2 + 6x(x-y) + 9(x-y)^2$.

$$\boxed{x^2 + 6x(x-y) + 9(x-y)^2 = \boxed{(4x-3y)^2}}$$

195. What is the size of an interior angle of an octagon?

$$\boxed{135^\circ = \frac{3}{4}\pi}$$

196. What fraction is represented by the repeating decimal $0.\overline{63}$?

$$\boxed{\frac{63}{99} = \boxed{\frac{7}{11}}}$$

197. The 24 ways to write rearrangements of the letters MATH are listed in alphabetical order. Where in this list does MATH appear?

$$\boxed{\boxed{\text{14th on the list}} \text{ right after MAHT and before MHAT}}$$

198. Two sides of a nontrivial triangle have lengths 5 and 8. What is the smallest integer length for the third side?

$$\boxed{4}$$

199. A square has area α in^2 and has perimeter α in. What is α?

$$\boxed{\alpha = x^2 = 4x \text{ so } x = 4, \alpha = \boxed{16}}$$

200. What is the area of the parallelogram with vertices $(0,0)$, $(5,4)$, $(6,5)$, and $(11,9)$?

$$\boxed{\det \begin{pmatrix} 5 & 4 \\ 6 & 5 \end{pmatrix} = \boxed{1}}$$

201. What is the reduced form of the fraction $\frac{343}{378}$?

$$\boxed{\frac{343}{378} = \boxed{\frac{49}{54}}}$$

202. In how many ways can you have \$10 worth of dimes and quarters?

$$\boxed{21}$$

203. Two dice are rolled. What is the probability that the sum is a 3, a 6, or an 8?

$$\boxed{\frac{2}{36} + \frac{5}{36} + \frac{5}{36} = \boxed{\frac{1}{3}}}$$

204. Bonnie gets a salary of \$32,000 with a 5% yearly raise. To the nearest \$1000, what will her salary be after 3 years?

$$\boxed{32000 \cdot (1.05)^3 = \$37044.00 \approx \boxed{\$37000}}$$

205. How many 3-digit numbers can be made (with no repetitions) using only the digits 1, 3, 5, 7, 9?

$$\boxed{5 \cdot 4 \cdot 3 = \boxed{60}}$$

206. The sum of the squares of two positive integers is 394, and the difference of their squares is 56. What are the numbers?

$$\boxed{\left\{ \begin{array}{c} x^2 + y^2 = 394 \\ x^2 - y^2 = 56 \end{array} \right\}, \text{ Solution is : } \boxed{y = 13, x = 15}}$$

207. Partition 24 into two parts whose product is 143.

$$\boxed{\left\{ \begin{array}{c} x + y = 24 \\ xy = 143 \end{array} \right\}, \text{ Solution is : } \boxed{y = 11, x = 13}}$$

208. A shelf will hold 20 calculus textbooks and 24 algebra textbooks, or 15 calculus textbooks and 36 algebra

textbooks. How many calculus books alone will the shelf hold?

$$\left\{ \begin{array}{l} 20c + 24a = 1 \\ 15c + 36a = 1 \end{array} \right\}, a = \tfrac{1}{72}, c = \tfrac{1}{30}$$ Shelf will hold 30 calculus books

209. Sally invested \$4,000, part at 5% and the rest at 4%. If the annual interest income from both investments was \$175, how much was invested at 5%?

$$.05x + .04(4000 - x) = 175, \text{Solution is : } x = \$1500.00$$

210. A fox is 70 leaps ahead of a hound and takes 5 leaps while the hound takes 3, but 3 of the hound's leaps equal 7 of the fox's. How many leaps must the hound take to catch the fox?

$$105$$

211. To the nearest minute, at what time after 9:00 a.m. and before 10:00 a.m. are the minute hand and the hour hand at right angles?

$$45 + \frac{m}{12} = m + 15, \text{Solution is: } m = 32.72727273$$ 9:33 a.m.

212. Art spent half his money plus 50 cents, then half the remainder plus 50 cents, then half of what he had left plus 50 cents, when he found that he had \$2.00 remaining. How much did he start with?

$$\left(\left(\left(2 + \tfrac{1}{2}\right) 2 + \tfrac{1}{2}\right) 2 + \tfrac{1}{2}\right) 2 = \$23.00$$

213. Divide 40 into 3 parts, such that $\frac{1}{2}$ of the first, $\frac{1}{3}$ of the second, and $\frac{1}{5}$ of the third are equal.

Solution is : $10, 15, 25$

214. During a recent election, Allen, Betty, and Carla received votes for mayor. Allen received half as many votes as Betty and 4 times as many as Carla. If the total vote lacted 250 votes of being 23000, how many votes did each receive?

Allen 7000,Betty 14000,Carla 1750

215. Barbara got a 73 on the first exam and 81 on the second. What must she average on the next two exams to have an overall average of 85 on the four exams?

$$\frac{73+81+2x}{4} = 85, \text{Solution is : } x = 93$$

216. A ball thrown vertically into the air 100 feet, falls and rebounds to a height of 40 feet the first time, rebounds to 16 feet on the second bounce, and so forth. What is the entire distance the ball will have moved when it finally comes to rest?

$$2\sum_{i=0}^{\infty} 100 \cdot \left(\tfrac{2}{5}\right)^i = \tfrac{1000}{3} = 333\tfrac{1}{3} \text{ ft}$$

217. The number of times a pendulum oscillates in a given time varies inversely as the square root of its length. If a 40 inch penculum oscillates once per second, what is the length of a pendulum that oscillates twice each second?

10 in

218. If a stone falls 64 feet in 2 seconds, how far will it fall in 5 seconds?

$$16 \cdot 5^2 = 400 \text{ ft}$$

219. In a river that flows at $1\frac{1}{2}$ miles per hour, it takes a certain swimmer 3 times as long to swim a certain distance up stream as it does to swim the same distance down stream. What is her speed in still water?

$\boxed{\frac{1}{r-3/2} = \frac{3}{r+3/2}, \text{ Solution is : } r = \boxed{3} \text{ mi/hr}}$

220. From what language was the term 'algebra' derived?

$\boxed{\text{Arabic}}$

221. The difference between two positive numbers is 6, and their product is twice the cube of the smaller number. What are the numbers?

$\boxed{\text{Solution is : } \boxed{8, 2}}$

222. It takes 1000 square tiles to tile a room, or 1440 smaller tiles whose edge is 1 inch less. How large is the room in square feet?

$\boxed{1000x^2 = 1440\left(x - \frac{1}{12}\right)^2, \text{ Solution is : } x = \frac{1}{2}, 1000\left(\frac{1}{2}\right)^2 = \boxed{250} \text{ ft}^2}$

223. Partition 25 into two parts such that the difference of their square roots is 1.

$\boxed{\text{Solution is : } \boxed{9, 16}}$

224. In 1962 a mathematician named Edward O. Thorp wrote a book entitled *Beat the Dealer* that claimed to give a winning strategy, based upon large-scale computer simulations, for a certain card game. What is the name of that card game?

$\boxed{\boxed{\text{Blackjack}} \text{ or } \boxed{21}}$

225. The plane can be completely tiled using equilateral triangles, or using squares, or using regular hexagons. These give the three *regular tessellations* of the plane. A *semi-regular tessellation* is a tiling that uses at least two different regular polygons, and where every vertex is congruent to every other vertex. How many semi-regular tessellations are there?

$\boxed{8}$

226. A 3-dimensional cube has 8 vertices and 12 edges. How many vertices and how many edges does a 4-dimensional cube have?

$\boxed{\text{16 vertices and 32 edges}}$

227. What is x if $128^x = 32^6$?

$\boxed{\left(2^7\right)^x = \left(2^5\right)^6 \Rightarrow 7x = 30 \Rightarrow \boxed{x = \frac{30}{7}}}$

228. Given an isosceles right triangle with a hypotenuse of length $\sqrt{50}$, what is its area?

$\boxed{\frac{25}{2}}$

229. If the probability that the Nuggets beat the Suns is 0.3, what is the probability that the Nuggets beat the Suns 4 times in a row?

$\boxed{(0.3)^4 = \boxed{.0081}}$

230. In how many ways can ALLAN misspell his name, assuming he uses all the right letters (the right number of times)? (Spell out A-L-L-A-N.)

$\boxed{\frac{5!}{2!2!} - 1 = \boxed{29}}$

231. What is the greatest integer in $\frac{7}{26} + \frac{26}{7}$?

$$\boxed{\frac{7}{26} + \frac{26}{7} = \frac{725}{182} = 3.983516484\boxed{3}}$$

232. A popular novel has 342 pages and 97470 words. What is the average number of words per page?

$$\boxed{\frac{97470}{342} = \boxed{285}}$$

233. What is the area of the smallest right triangle with all sides positive integers?

$$\boxed{3-4-5 \text{ right triangle has area } 3 \cdot 2 = \boxed{6}}$$

234. A spherical balloon's diameter increases by 30%. By what percentage does the surface area change?

$$\boxed{(1.3)^2 = 1.69 \ \boxed{69\% \text{ increase}}}$$

235. What is the contrapositive of the statement, "If n is a multiple of 6, then n is a multiple of 3."

$$\boxed{\text{If } n \text{ is not a multiple of 3, then } n \text{ is not a multiple of 6.}}$$

236. The average of the four numbers 11, 15, 17, and x is 23. What is x?

$$\boxed{\frac{11+15+17+x}{4} = 23, \text{ Solution is : } \boxed{x = 49}}$$

237. How many feet per second is 60 miles per hour?

$$\boxed{60 \cdot \frac{5280}{3600} = \boxed{88 \text{ ft/sec}}}$$

238. What is the base 8 representation of the number 65?

$$\boxed{\boxed{101} \text{ or } \boxed{101_8}}$$

239. The 2×2 matrix A is given by $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$. What is A^{-1}?

$$\boxed{\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}^{-1} = \boxed{\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}}}$$