CSU Math Day 2004
Team Competition
11:20

1. Assuming $0 < a < b$, express $\frac{a^b b^a}{a^a b^b}$ in terms of one quotient raised to a positive exponent.

   ANS: $\frac{a^b b^a}{a^a b^b} = \left(\frac{a}{b}\right)^b \left(\frac{b}{a}\right)^a = \boxed{\left(\frac{a}{b}\right)^{b-a}}$

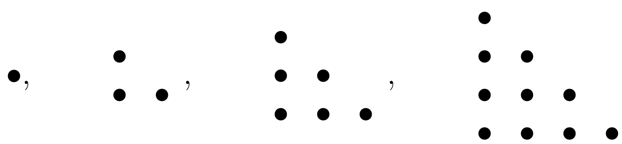
2. What is the area of the triangle with sides 5, 12, and 13?

   ANS: Right triangle with legs 5 and 12: Area $= \frac{1}{2} \cdot 5 \cdot 12 = \boxed{30}$

3. George is a 50% free-throw shooter. What is his expected score if he shoots twice?

   ANS: $\boxed{1.0}$ or $\boxed{1}$

4. What is the 17th term of the sequence that begins $1, 3, 6, 10, \ldots$?

   ANS:  $\ldots,\quad 1+2+3+\cdots+17 = \frac{17\cdot 18}{2} = \boxed{153}$

5. What prime number is nearest to 2004?

   ANS: $\boxed{2003}$

6. A pyramid is built out of blocks by placing 100 blocks on the floor, placing 81 blocks on top of the bottom layer, and so forth. How many cubes are there in the pyramid?

   ANS: $\sum_{i=1}^{10} i^2 = \boxed{385} = \frac{10\cdot 11\cdot 21}{6}$

7. The set S has 8 elements. If a and b are (distinct) elements of S, how many subsets of S contain both a and b?

   ANS: $\boxed{64} = \boxed{2^6}$

8. Visually, an 8-9-12 triangle appears to be a right triangle. Is it actually acute, or is it obtuse?

   ANS: $\boxed{\text{Acute}}$ (largest angle$\approx 89.602109^\circ$ or $145 = 8^2 + 9^2 > 12^2 = 144$)

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9. If 2 is the first prime, what is the seventh prime?

   ANS: $2, 3, 5, 7, 11, 13, \boxed{17}, 19, 23, 29$

10. What positive number is 5 times as big as its reciprocal?

    ANS: $\boxed{\sqrt{5}}$ since $x = \frac{5}{x} \Longrightarrow x^2 = 5$

11. What famous mathematical object was named after the mathematician David Hilbert: A) Hilbert Comma, B) Hilbert Period, or C) Hilbert Space?

    ANS: $\boxed{\text{C) Hilbert Space}}$

12. What is the volume of the tetrahedron whose vertices are $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$?

    ANS: $1/6$

13. Two cards are drawn at random from a standard 52-card deck. What is the probability that both are diamonds?

    ANS: $\frac{\binom{13}{2}}{\binom{52}{2}} = \boxed{\frac{1}{17}}$

14. What is the area of the triangle bounded by the lines $y = 0$, $y = x$, and $y = 5 - x$?

    ANS: The triangle has base 5, altitude $5/2$, and hence the area is $\frac{1}{2} \cdot 5 \cdot \frac{5}{2} = \boxed{\frac{25}{4}} = \boxed{6\frac{1}{4}}$

15. What is the greatest integer in the sum $\frac{19}{5} + \frac{5}{19}$?

    ANS: $\frac{19}{5} + \frac{5}{19} = 4.06315\,78947\,3684 = \boxed{4} + .06315\,78947\,3684$

Tiebreaker Question

What is the prime factorization of 105?

ANS: $105 = \boxed{3 \times 5 \times 7}$

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1. What is the distance between two opposite vertices of a cube of edge $2\sqrt{3}$?

   ANS: $\sqrt{\left(2\sqrt{3}\right)^2 + \left(2\sqrt{3}\right)^2 + \left(2\sqrt{3}\right)^2} = \boxed{6}$

2. The average of your first 5 exams is 92. If you score 68 on your next exam, what is your average for all 6 exams?

   ANS: $\boxed{88} = \frac{5 \cdot 92 + 68}{6}$

3. Four digits are chosen without replacement from the set $\{1, 3, 5, 7, 9\}$. The digits are then used as the numerators and denominators of two fractions, one digit as each numerator and one digit as each denominator. The sum of the two fractions created is less than 1. What is the greatest possible sum of the two fractions?

   ANS: $\frac{3}{7} + \frac{5}{9} = \boxed{\frac{62}{63}}$

4. What is the largest 3-digit prime?

   ANS: $\boxed{997}$

5. State the contrapositive of the statement, "If $x^2 \le 9$, then $x \le 3$."

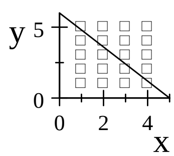
   ANS: $\boxed{\text{If } x > 3, \text{ then } x^2 > 9.}$

6. According to a review, the book, *A New Kind of Science*, forces a whole new way of looking at the operation of our universe. What mathematical software was written by the same author?

   ANS: $\boxed{\text{Mathematica}}$ by Stephen Wolfram

7. A triangle has vertices $(0, 0)$, $(5, 0)$, and $(0, 6)$. How many points with integer coordinates are strictly inside this triangle?

   ANS: $\boxed{10}$

8. List the following three numbers in increasing order: $\sqrt[6]{10}$, $\sqrt{2}$, $\sqrt[3]{3}$.

   ANS: $\boxed{\sqrt{2}} = 1.4142 < \boxed{\sqrt[3]{3}} = 1.4422 < \boxed{\sqrt[6]{10}} = 1.4678$ since $\left(2^{1/2}\right)^6 = 8 < \left(3^{1/3}\right)^6 = 9 < \left(10^{1/6}\right)^6 = 10$

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9. The age of a mathematician and her son added together is 62. In 4 years the mother will be 4 times as old as her son. How old is the mathematician?

   ANS: 52

10. A ball thrown vertically into the air 100 feet, falls and rebounds to a height of 50 feet the first time, rebounds to 25 feet on the second bounce, and so forth. What is the entire distance the ball will have moved when it finally comes to rest?

    ANS: $2\sum_{i=0}^{\infty} 100 \cdot \left(\frac{1}{2}\right)^i = \boxed{400}$ ft

11. Grandma Josephine offers each of her 5 grandchildren the choice of 3 different kinds of cookies. If each grandchild only gets one cookie, in how many ways can the choices be made?

    ANS: $3^5 = \boxed{243}$

12. Expand $(2x^2 + 3y)^2$

    ANS: $\boxed{4x^4 + 12x^2y + 9y^2}$

13. The sum of two positive integers is 21 and their product is 108. What are the two integers?

    ANS: $\boxed{9,\ 12}$

14. Let $x_n = 3^n - n$. What is the last digit of x_8?
    ANS: $\boxed{3}$ since $3^8 - 8 = 6553$

15. Sarah is an 80% free throw shooter. What is her expected score if she shoots a one and one (if she makes the first she gets a second chance)?

    ANS: $\boxed{\frac{36}{25}}$ or $\boxed{1.44}$

Tiebreaker Question

Factor 123 as a product of primes.
ANS: $123 = \boxed{3 \times 41}$

CSU Math Day 2004
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12:00

1. An automobile travels at 90 kilometers per hour for 6 minutes, 15 km/ h for 2 minutes, and then 30 km/ h for 3 minutes. How far did it travel during the 11 minutes?

   ANS: $\frac{6}{60} \cdot 90 + \frac{2}{60} \cdot 15 + \frac{3}{60} \cdot 30 = \boxed{11}$ km

2. Sarah got scores of 87, 91, and 88 on her first three exams. What is the lowest score she can get on the forth exam to give her an average of 90?

   ANS: $\frac{87+91+88+x}{4} = 90$, Solution is: $\boxed{94}$

3. In 1984, Louis de Brange solved which of the following famous problems: A) Bieberbach Conjecture, B) Continuum Hypothesis, or C) Fermat's Last Theorem?
   ANS: A) Bieberbach Conjecture

4. Solve the equation $x\sqrt{0.25} = 6$

   ANS: $x = \boxed{12}$

5. A wooden cube of edge 5 inches is painted red. The cube is then cut into 125 one-inch cubes by making 12 saw cuts. How many of the one-inch cubes have exactly 3 red faces?

   ANS: 8 corners $\times$ 1 cube per corner $= \boxed{8}$ cubes

6. You are given eight points in the plane, no three of which lie on the same line. How many lines are there which pass through exactly two of these points?

   ANS: $\binom{8}{2} = \boxed{28}$

7. A rectangle's length is increased by 30% and its width is decreased by 30%. How much does its area change?

   ANS: $1.3L \times 0.7W = \boxed{0.91LW}$ or $\boxed{\text{decreases by } 9\%}$

8. The interior of a triangle is painted black. The midpoints of the three edges are connected, and the interior of this triangle is painted white. The midpoints of each of the remaining black triangles are connected and the interiors are painted white. Continuing this process leaves a bit of black paint. If the area of the original black triangle is 1, what is the area covered by the remaining set of black paint?

   ANS: $1 - \sum_{n=0}^{\infty} \frac{3^n}{4^{n+1}} = \boxed{0}$ Sierpinski triangle

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9. How many minutes were there in February of this year?

   ANS: $29 \times 24 \times 60 = 41\,760 = \boxed{41\,760}$ minutes

10. Find a solution to the equation $p^a = q^b + 1$ where p and q are prime and a and b are integers greater than one.
    ANS: $\boxed{3^2 = 2^3 + 1}$

11. How many nines are there in the number $10^8 - 111$?
    ANS: $\boxed{6}$ since $10^8 - 111 = 100\,000\,000 - 111 = 99\,999\,889$

12. What is the area of the region bounded by the lines $y = 0$, $y = x$, and $y = 12 - 2x$?

    ANS: Area $= \frac{1}{2} \cdot 6 \cdot 4 = \boxed{12}$

13. Three cards are selected at random from a standard 52-card deck. What is the probability that one is a heart and the other two are diamonds?

    ANS: $\frac{\binom{13}{1}\binom{13}{2}}{\binom{52}{3}} = \boxed{\frac{39}{850}}$

14. Given 5 gallons of a 10% antifreeze/water mixture, how much pure antifreeze must be added to yield a 50% antifreeze/water mixture?

    ANS: $\frac{5 \cdot 0.1 + x}{5 + x} = 0.5$, Solution is: $\boxed{4}$ gallons

15. A triangular section of Old Town is divided into a smaller triangle and two trapezoids by two streets parallel to one of the boundary streets. The heights of the two trapezoids are equal to the height of the small triangle, and the area of the middle trapezoid is 9 acres. How many acres are there in the larger trapezoid?

    ANS: $\boxed{15}$ acres

Tiebreaker Question

What is the greatest common divisor of 399 and 357?
ANS: $\gcd(399, 357) = \boxed{21}$

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1. What is the smallest positive integer with exactly 6 positive divisors?

   ANS: $\boxed{12}$ (divisors are $\{1, 2, 3, 4, 6, 12\}$)

2. How many integers between 1 and 99 are divisible by 3 or by 7?

   ANS: $\left\lfloor \frac{100}{3} \right\rfloor = 33$, $\left\lfloor \frac{100}{7} \right\rfloor = 14$, $\left\lfloor \frac{100}{21} \right\rfloor = 4$, $33 + 14 - 4 = \boxed{43}$

3. Kyle uses pure guessing on a TRUE/FALSE exam. Which of the following options give Kyle the best chance to score (at least) 50%? (A) A seven-question exam (guess correctly on 4, 5, 6, or 7 questions). (B) A five-question exam (guess correctly on 3, 4, or 5 questions). (C) The chances are equal.

   ANS: $\boxed{\text{(C)}}$ Chances are equal (both probabilities are $\frac{1}{2}$)

4. Sometimes, always, or never: The sum of two irrational numbers is irrational.

   ANS: $\boxed{\text{Sometimes}}$ $\sqrt{2} + \sqrt{3}$ is irrational; $\sqrt{2} + \left(3 - \sqrt{2}\right) = 3$ is rational

5. What is the area of the largest rectangle that can fit inside a circle of radius 2?

   ANS: $Area = \left(2\sqrt{2}\right)^2 = \boxed{8}$

6. What is the smallest diameter log needed for cutting a square post of dimensions 4 in by 4 in?

   ANS: $\boxed{4\sqrt{2}}$ in

7. A basketball team has 11 players on the roster. How many different starting lineups are possible?

   ANS: $\binom{11}{5} = \boxed{462}$

8. Find x so that the average of the four numbers 33, 25, 42, and x is 32.

   ANS: $x = \boxed{28}$

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9. How many distinct real roots does the polynomial x^4+1 have?

   ANS: $\boxed{0}$ or $\boxed{\text{none}}$ x^4+1, roots: $\pm\left(\frac{1}{2}\pm\frac{1}{2}i\right)\sqrt{2}$

10. The 8 faces of a regular octahedron are equilateral triangles. How many edges does a regular tetrahedron have?

    ANS: $\dfrac{3\cdot 8}{2}=\boxed{12}$

11. In 1949, Selberg and Erdös found an elementary proof of which of the following theorems: A) Four Color Theorem, B) Fermat's Last Theorem, or C) Prime Number Theorem?
    ANS: $\boxed{\text{C) Prime Number Theorem}}$

12. How many times do the hands of a clock form a right angle between 6pm one day and 6pm the next day?
    ANS: $\boxed{48}$

13. What is the surface area of a spherical raindrop of volume $\frac{32\pi}{3}$?

    ANS: $4\pi\,(2)^2=\boxed{16\pi}$ mm^2

14. What is the perimeter of an isosceles triangle with base 10 and area 60?

    ANS: $\boxed{36}$ $10-13-13$ triangle has height 12

15. What is the maximum number of non-overlapping regions created by three distinct lines in a plane?

    ANS: $\boxed{7}$

Tiebreaker Question

How many vertices does an icosahedron have?
ANS: $\frac{3\times 20}{5}=\boxed{12}$

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1. At 6:00 AM, Chad starts jogging at 5 km/h. At 7:00 AM Marzelle starts jogging from Chads's starting place at 6 km/h. How far behind is Marzelle at 9:00 AM?

   ANS: $\boxed{3}$ km

2. Always, sometimes, or never: The sum of a rational number and an irrational number is irrational.

   ANS: $\boxed{\text{Always}}$

3. How many integer solutions are there to the inequality $1 < x^2 < 26$?

   ANS: $\boxed{8}$ ± 2, ± 3, ± 4, ± 5

4. Use the approximation $2^{10} \approx 10^3$ to approximate 2^{57} as a number in scientific notation $2^{57} \approx c \times 10^n$, where n is an integer and c is an integer between 0 and 10.

   ANS: $2^{57} = 2^7 \times (2^{10})^5 \approx 128 \times (10^3)^5 \approx \boxed{1 \times 10^{17}}$ (In fact, $2^{57} \approx 1.4412 \times 10^{17}$)

5. During a very light thunderstorm, four drops of rain hit a window that has four identical panes of glass. Is the expected number of dry panes A) less than 1, B) greater than 1, or C) exactly equal to 1?

   ANS: $\boxed{\text{B) greater than 1}}$ The expected number of dry panes is $4\left(\frac{3}{4}\right)^4 = \frac{81}{64}$

6. At the heart of a popular play is Catherine's struggle to determine how much of her father's genius or madness she will inherit and whom, if anyone, she can trust with the answer. What is the name of this play?

   ANS: $\boxed{\text{Proof}}$ by David Auburn

7. If the absolute value of $x + 2$ is equal to the absolute value of $x - 4$, what is x?

   ANS: $x = \boxed{1}$

8. What fraction is represented by the repeating decimal $.6666666\cdots$?

   ANS: $\boxed{\frac{2}{3}}$

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9. What is the 17th term of the sequence that starts $2, 5, 8, 11, \ldots$?

   ANS: A) Sequence $\{3n-1\}$, $3 \cdot 17 - 1 = \boxed{50}$

10. To the nearest minute, at what time between 9:30 a.m. and 10:00 a.m. are the minute hand and the hour hand at right angles?

    ANS: $45 + \frac{m}{12} = m + 15$, Solution is: $m = \frac{360}{11} = 32.727$ $\boxed{10{:}33}$ a.m.

11. Sarah's first four exam scores were 88, 93, 96, and 85. What is the lowest score on the forth exam that will give her an average of at least 90 for the four exams?

    ANS: $\frac{88+93+96+85+x}{5} = 90$, Solution is: $\boxed{88}$

12. Find a three-digit number with three distinct digits which has one digit in the same place and one digit in a different place as each of the three numbers 836, 315 and 983 .

    ANS: $\boxed{813}$

13. At what point do the two lines $y = 3x - 1$ and $y = 9 - 2x$ intersect?

    ANS: $\boxed{(2, 5)}$

14. Two circles are mutually tangent at one point, and the smaller circle passes through the center of the larger circle. What is the ratio between the areas of the two circles?

    ANS: $\boxed{4:1}$ or $\boxed{1:4}$

15. What is the sum of the first 100 positive integers?

    ANS: $\sum_{n=1}^{100} n = \boxed{5050}$

Tiebreaker Question

How many edges does a dodecahedron have?
ANS: $\frac{5 \times 12}{2} = \boxed{30}$

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1. What is the area of the triangular region bounded by $y = x$, $y = 0$, and $y = 3 - x$

   ANS: $\frac{1}{2} \cdot 3 \cdot \frac{3}{2} = \boxed{\frac{9}{4}} = \boxed{2.25}$

2. Kyle's average score on the first 3 exams is 88. What is the lowest score he can make on the forth exam so that his average score is at least 90?

   ANS: $\frac{88 \cdot 3 + x}{4} = 90$, Solution is: $\boxed{96}$

3. What is the sum of the binomial coefficients (5 choose zero) plus (5 choose 1) plus $\cdots$ plus (5 choose 5)?

   ANS: $\boxed{32} = 2^5 = (1+1)^5 = \sum_{i=0}^{5} \binom{5}{i}$

4. Sean decides to by a \$10 raffle ticket for a chance to win a \$75,000 Hummer. The sponsor has declared that it will sell 10,000 tickets. What is Sean's expected gain (or loss)?

   ANS: $\frac{1}{10000} \cdot 75000 - 10 = -\frac{5}{2}$ $\boxed{\text{Loss of \$2.50}}$

5. For what choices of a does the polynomial $ax^2 - 6x + 1$ have no real roots?

   ANS: $a > 9$

6. A wooden cube of edge 5 inches is painted red. The cube is then cut into 125 one-inch cubes by making 12 saw cuts. How many of the one-inch cubes have exactly one red face?

   ANS: 6 faces $\times$ 9 cubes per face $= \boxed{54}$ cubes

7. During a recent election, Alfie, Betty, and Gammer received votes for mayor. Alfie received $\frac{1}{3}$ as many votes as Betty and 2 times as many as Gammer. If the total number of votes was 19 800, how many did each person get?

   ANS: $\boxed{\text{Alfie: 4400 Betty: 13 200 Gammer: 2200}}$

8. If the line $y = mx$ touches the curve $y = \frac{1}{2}(x^2 + 1)$ in exactly 1 point, what are the possibilities for m?

   ANS: $m = \boxed{\pm 1}$

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9. What is the sum of all the integers greater than 10 and less than 29?

   ANS: $\sum_{n=11}^{28} n = \boxed{351}$ (18 numbers, average is 19.5, product is $18 \cdot 19.5 = 351$)

10. What mathematician first resolved the Königsberg bridge problem?

    ANS: $\boxed{\text{Euler}}$ ('Oiler')

11. What is the length of the arc on a circle of radius 12 subtended by a central angle of $135\,°$?

    ANS: $\boxed{9\pi}$

12. How many ways can the top 3 finishers be picked in a 8-person race?

    ANS: $8 \times 7 \times 6 = \boxed{336}$

13. Find the area of the circle inscribed in a regular hexagon with sides of length 4.

    ANS: $\boxed{12\pi}$

14. What is the product of the next two primes numbers greater than 32?

    ANS: $37 \times 41 = \boxed{1517}$

15. What is the area of the region bounded by the ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$?

    ANS: $\boxed{6\pi}$ $Area = \pi ab$

Tiebreaker Question

Which of the following three numbers is smallest: e^π, π^e, or 3^3?
ANS: $e^\pi = 23.140\,692\,64$, $\boxed{\pi^e} = 22.459\,157\,71$, $3^3 = 27$

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1. Write the expression $\frac{13x+4}{(2x-1)(3x+2)}$ in partial fraction form (i.e. $\frac{A}{Bx+C}+\frac{D}{Ex+F}$, where A, B, C, D, E, and F are integers).

   ANS: $\boxed{\frac{3}{2x-1}+\frac{2}{3x+2}}$

2. Find three distinct positive integers a, b, and c with $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{2}$.
   ANS: $\boxed{4,\ 6,\ 12}$ since $\frac{1}{4}+\frac{1}{6}+\frac{1}{12}=\frac{1}{2}$

3. What is the area of the parallelogram with vertices $(0,0)$, $(1,2)$, $(2,1)$, and $(3,3)$?

   ANS: $\boxed{3} = \left|\det\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\right| = 9 - 2\left(\frac{1}{2}\cdot 2\cdot 1 + 1\cdot 1 + \frac{1}{2}\cdot 2\cdot 1\right) = \|(2,1,0)\times(1,2,0)\|$

4. If eggs weigh 2 oz each and a dozen eggs cost 90¢, what is the cost of a pound of eggs? (16oz = 1 pound)

   ANS: $\boxed{60}$ ¢

5. Is the number 12 A) Abundant, B) deficient, or C) perfect?

   ANS: The sum of the proper divisors is $1+2+3+4+6=16>12$ so $\boxed{\text{A) Abundant}}$

6. What is the edge length of a cube whose volume (in cubic feet) is the same as its surface area (in square feet)?
   ANS: $\boxed{6}$ feet

7. List the following three numbers in increasing order: 2^7, 5^3, 11^2

   ANS: $\boxed{11^2}=121<\boxed{5^3}=125<\boxed{2^7}=128$

8. John and Jill traded positions several times while rowing a canoe through the Boundary Waters of Minnesota. With Jill at the rear, the canoe went fast enough to complete the entire trip in 8 hours, and with John at the rear the trip would have taken 12 hours. The trip actually took 9 hours. For how many hours did John sit in the back of the canoe?

   ANS: $\boxed{3}$ hours since $\frac{1}{12}\cdot 3+\frac{1}{8}\cdot 6=1$

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9. What is the smallest prime greater than 500?

   ANS: $\boxed{503}$

10. An octahedron is a regular solid whose 8 faces are equilateral triangles. What is the distance between a pair of opposite vertices, assuming each edge of the octahedron is of length 1?

    ANS: $\boxed{\sqrt{2}}$

11. A committee of 3 people is to be chosen from among 5 men and 6 women. How many ways can this be done if the committee must include at least one man and at least one woman?

    ANS: $\boxed{135} = \binom{11}{3} - \binom{5}{3} - \binom{6}{3}$

12. The compact disk UR2gly sells at outlet AC for \$12.95 less a discount of 15%, and at outlet DC for \$14.75 less a discount of 25%. Which outlet has the lower price?

    ANS: $\boxed{AC} = \$12.95 \times .85 = \$11.0075,\ DC = \$14.75 \times .75 = \$11.063$

13. What is the area of an equilateral triangle inscribed in a circle of radius 2?

    ANS: $3\sqrt{3}$

14. The average score on Kyle's first three exams was 94 and the average after four exams was 92. What was his score on the forth exam?

    ANS: $\frac{94 \cdot 3 + x}{4} = 92$, Solution is: $\boxed{86}$

15. Name the Scottish mathematician who invented logarithms.

    ANS: $\boxed{\text{Napier}}$

Tiebreaker Question

Find all solutions to $x^3 = x$.
ANS: $x^3 = x$, Solution is : $\boxed{x = 0,\ x = 1,\ x = -1}$

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1. The RSA algorithm is one of the most popular modern crypto systems. Name at least one of the three mathematicians for whom the algorithm was named.

   ANS: Ronald L. $\boxed{\text{Rivest}}$, Adi $\boxed{\text{Shamir}}$, and Leonard $\boxed{\text{Aldeman}}$ published the algorithm in 1977

2. How far apart are the two points at which the curves $y = x + 5$ and $y = x^2$ intersect?

   ANS: $\left\{ \begin{array}{c} y = x + 2 \\ y = x^2 \end{array} \right\}$, Solution is : $\{(2, 4), (-1, 1)\}$, distance $\boxed{3\sqrt{2}}$

3. The 12 faces of a regular dodecahedron are regular pentagons. How many vertices does a regular dodecahedron have?

   ANS: $\frac{12 \cdot 5}{3} = \boxed{20}$

4. What is the coefficient of xy^2 in the expansion of $(2x + y)^3$?

   ANS: $(2x + y)^3 = 8x^3 + 12x^2y + \boxed{6}xy^2 + y^3$

5. $\frac{1}{3}$ of the air in a container is removed by each cycle of an air pump. What fractional part of the air remains after 2 cycles?

   ANS: $\boxed{\frac{4}{9}}$

6. An ant and a drop of honey are at diagonally opposite corners of a rectangular box of dimensions 1 by 2 by 4. What is the shortest distance the ant can travel to reach the (stationary) drop of honey by crawling along walls?

   ANS: $\sqrt{3^2 + 4^2} = \boxed{5}$

7. The 24 ways to write rearrangements of the letters MATH are listed in alphabetical order. Which rearrangement appears in the list just before MATH and which rearrangement appears just after MATH?

   ANS: AHMT, AHTM, ..., $\boxed{\text{MAHT}}$, MATH, $\boxed{\text{MHAT}}$, ..., TMHA

8. The number of times a pendulum oscillates in a given time varies inversely as the square root of its length. If a 40 inch pendulum oscillates once per second, what is the length of a pendulum that oscillates twice each second?

   ANS: $\boxed{10}$ in

Team Competition
1:35

9. If the first term of a geometric sequence is $\frac{2}{3}$ and the second term is $\frac{1}{2}$, what is the fifth term?

   ANS: $\frac{2}{3} \cdot \left(\frac{3}{4}\right)^4 = \boxed{\frac{27}{128}}$

10. A square is inscribed in a circle, which in turn is inscribed in a square. What percentage of the area of the large square is inside the small square?

    ANS: 50%

11. A golf bag contains 5 yellow golf balls and 8 orange golf balls. Two balls are drawn at random from the bag. What is the probability that one is yellow and the other is orange?

    ANS: $\boxed{\frac{20}{39}}$

12. If $2^x = 7$, then what is 2^{2x+1}?

    ANS: $2^{3x+2} = (2^x)^2\, 2^1 = 7^2 2 = \boxed{98}$

13. How many digits 0–9 must be printed in order to number the pages of a 500-page book?

    ANS: $9 + 90 \cdot 2 + 401 \cdot 3 = \boxed{1392}$

14. Give an equation in the form $ax + by = c$, where a, b, and c are integers, for the line through $(2, -3)$ that is parallel to the line $9x + y = 9$.

    ANS: $\boxed{9x + y = 15}$

15. Sometimes, always, or never: The product of a rational number with an irrational number is irrational.

    ANS: $\boxed{\text{Sometimes}}$ $2 \cdot \pi = 2\pi$ is irrational; $0 \cdot \pi = 0 = 0/1$ is rational.

Tiebreaker Question

Arrange the following three numbers from smallest to largest: $\frac{3}{7}, \frac{4}{9}, \frac{7}{16}$

ANS: $\boxed{\frac{3}{7}} = 0.428\,57 < \boxed{\frac{7}{16}} = 0.437\,5 < \boxed{\frac{4}{9}} = 0.444\,44$

CSU Math Day 2004
Team Competition
1:50

1. A triangle has vertices $(0, 0)$, $(10, 0)$, and $(13, 5)$. Find the area of the triangle.

   ANS: Altitude is 5, so $Area = \frac{1}{2}(10)(5) = \boxed{25}$

2. A computer sequentially computes integers by the following rule: If n is odd then add 3 and divide the result by 2; otherwise add 5 . Starting at $n = 11$, what is the integer after 6 iterations?

   ANS: $11 \rightarrow \frac{11+3}{2} = 7 \rightarrow \frac{7+3}{2} = 5 \rightarrow \frac{5+3}{2} = 4 \rightarrow 9 \rightarrow \frac{9+3}{2} = 6 \rightarrow \boxed{11}$

3. Former CSU Mathematics professor R. C. Bose is credited for helping develop the A) RSA Algorithm, B) BCH Code, or C) FFT Transform.

   ANS: $\boxed{\text{B) BCH Code}}$ an error-correcting code with wide applications

4. A pair of slacks priced at \$20 has been marked down 20% and then marked down 30%. What is the new price?

   ANS: $\$20 \cdot 0.8 \cdot 0.7 = \$\boxed{11.20}$

5. What is x if $512^x = 32^3$?

   ANS: $(2^9)^x = (2^5)^3 = 2^{15} \Rightarrow 9x = 15 \Rightarrow x = \boxed{\frac{5}{3}}$

6. Give the point(s) of intersection of the two curves $y = 3x + 2$ and $y = x^2 + 4x$.

   ANS: $\boxed{(-2, -4),\ (1, 5)}$

7. A rectangle has area 88 and perimeter 38. Find the dimensions of the rectangle.

   ANS: $\boxed{8 \times 11}$

8. The determinant of the matrix of coefficients of a system of two linear equations in two unknowns is 0. What is true about the graphs of the two equations?

   ANS: $\boxed{\text{The lines are parallel}}$

Team Competition
1:50

9. A regular hexagon is inscribed in a circle of radius 7 cm. What is the perimeter of the hexagon?

   ANS: 42 cm

10. What is the base 9 representation of the decimal number 618?

    ANS: $\boxed{(756)_9}$ or $\boxed{756}$ since $7 \cdot 9^2 + 5 \cdot 9 + 6 = 618$

11. A right triangle has legs of length 6 and 8. What is the radius of the circle that circumscribes the triangle?

    ANS: diameter = 10, radius $= \boxed{5}$

12. A positive integer is called *perfectly balanced* if adding and subtracting the same positive integer results in two perfect squares. What is the smallest perfectly balanced integer?

    ANS: $\boxed{2}$ since $2 + 2 = 2^2$ and $2 - 2 = 0^2$

13. Alpher, Beta, and Gamow are now 7, 11, and 13 years old, respectively. How many years will it be until they again have prime-numbered ages?

    ANS: $\boxed{\text{Six years from now}}$ they will have ages 13, 17, and 19.

14. How many ways can you insert a cube with side length two into a square hole with side length two?
    ANS: $\boxed{24}$

15. A shirt has been marked down 35% and then 10% to \$23.40. What was the original price?

    ANS: \$$\boxed{40}$ since $40 \cdot 0.65 \cdot 0.90 = 23.4$

Tiebreaker Question

Of the 1024 subsets of $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, how many contain the element 6?
ANS: $\boxed{512}$ or $\boxed{\text{half}}$

CSU Math Day 2004
Team Competition
2:05

1. If $f(x)$ is a linear function such that $f(3) = 3$ and $f(-4) = -11$, what is $f(-3)$?

   ANS: $f(-3) = \boxed{-9}$

2. In how many ways can 7 boys and 7 girls be paired, if each pair must contain one boy and one girl?

   ANS: $7! = \boxed{5040}$

3. Two baseball teams, the Rockies and the Cardinals, begin a 7-game series. The odds makers give the green Rockies 5 to 9 odds of winning the first game. Assuming the odds makers are good at their job, what is the probability that the Rockies will win the first game?

   ANS: $\boxed{\frac{5}{14}}$

4. The ancient Greek mathematician who developed a formula for the area of a triangle in terms of its side lengths was A) Euclid, B) Hero, or C) Pythagoras.

   ANS: $\boxed{\text{B) Hero}}$ or Heron of Alexandria developed the formula $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$

5. Sometimes, always, or never: The rational number $\frac{n!}{k!(n-k)!}$ is an integer if k and n and positive integers with $k < n$.

   ANS: $\boxed{\text{Always}}$

6. One of two different prizes is included in each box of low-carb Sugar Squares. If the prizes are randomly distributed, how many boxes do you expect to purchase before you get at least one of each of the two prizes?

   ANS: $\boxed{3} = \sum_{n=1}^{\infty} \frac{n+1}{2^n}$

7. What is the largest integer $\leq \sqrt{700}$?

   ANS: $\sqrt{700} = 26.458$, so $\boxed{26}$

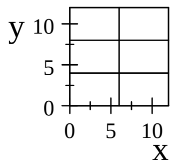
8. What is the smallest perfect cube larger than 700?

   ANS: $\boxed{729} = 9^3$

Team Competition
2:05

9. What is the smallest number of 4-inch by 6-inch rectangular tiles required to form a square with no cutting or gaps?

ANS: $\boxed{6}$



10. The plane can be completely tiled using equilateral triangles, or using squares, or using regular hexagons. These give the three regular tessellations of the plane. A semi-regular tessellation is a tiling that uses at least two different regular polygons, and where every vertex is congruent to every other vertex. How many semi-regular tessellations are there?

ANS: $\boxed{8}$

11. What is the distance between the points of intersection of the two curves $y = 3x - 5$ and $y = x^2 + 2x - 11$?

ANS: Distance between $(-2, -11)$ and $(3, 4)$ is $\sqrt{(3+2)^2 + (4+11)^2} = \boxed{5\sqrt{10}}$

12. The sum of two integers is -14 and their product is 48. What are the two integers?

ANS: $\boxed{-6 \text{ and } -8}$

13. How many positive integral factors does 48 have?

ANS: $\boxed{10}$ $48 = 2^4 3$ $\{1, 2, 3, 4, 6, 8, 12, 16, 24, 48\}$

14. How many primes are there between 200 and 210?

ANS: $\boxed{\text{zero}}$ or $\boxed{\text{none}}$ $(201 = 3 \times 67,\ 203 = 7 \times 29,\ 205 = 5 \times 41,\ 207 = 3^2 23,\ 209 = 11 \times 19)$

15. A wooden cube of edge 5 inches is painted red, then cut into 125 one-inch cubes using 12 saw cuts. How many of the small cubes have exactly 2 red faces?

ANS: 12 edges $\times$ 3 cubes per edge $= \boxed{36}$

Tiebreaker Question

Factor 1001 as a product of primes.
ANS: $1001 = \boxed{7 \times 11 \times 13}$

CSU Math Day 2004
Team Competition
2:15

1. List the following three numbers in increasing order: e^π, π^e, 3^3.

   ANS: $\boxed{\pi^e} = 22.459 < \boxed{e^\pi} = 23.141 < \boxed{3^3} = 27$

2. When an integer is divided by 15, the remainder is 7. Find the sum of the remainders when the same integer is divided by 3 and by 5.

   ANS: $\boxed{3} = (7 \bmod 3) + (7 \bmod 5)$

3. In the geometric progression -54, 18, -6, n, what is the value of n?

   ANS: $\boxed{2}$

4. What is the area of the parallelogram bounded by the lines $y = 3x$, $y = 2x$, $y = 3x - 2$, and $y = 2x + 1$?

   ANS: $\left| \det \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \right| = \boxed{2} = 21 - 2\left(\frac{1}{2} \cdot 2 \cdot 4 + 4 + \frac{1}{2} \cdot 1 \cdot 3\right)$

5. A biological brick grows 5% in length, 10% in width, and shrinks in height by 16%. Is it larger or smaller than when it started out?

   ANS: $\boxed{\text{smaller}}$ ($(1 + 0.05) * (1 + 0.1) * (1 - .16) = 0.9702$)

6. What is the probability that the sum of a pair of rolled dice is either 7 or 11?

   ANS: $\frac{1}{6} + \frac{1}{18} = \boxed{\frac{2}{9}}$

7. Two wheels are connected by a belt. One has a diameter of 45 centimeters and a speed of 300 rpm. The other has a speed of 250 rpm. What is its diameter?

   ANS: $\boxed{54}$ cm

8. Consider all the rational numbers between 0 and 1 whose denominators are ≤ 5. Put these numbers in increasing order. This sequence has the curious property that each member of the sequence is equal to the rational whose numerator is the sum of the numerators of the fractions on either side, and whose denominator is the sum of the denominators of the fractions on either side. This sequence is an example of A) a Goblin sequence, B) an Elf sequence, or C) Farey sequence.

   ANS $\boxed{\text{C) Farey sequence}}$ (named after John Farey (1766–1826))

Team Competition
2:15

9. Grayson is a 70% free throw shooter. What is his expected score if he has a chance to shoot one-and-one (if he hits the first he gets to try a second shot)?

   ANS: $1 \cdot \frac{7}{10} \cdot \frac{3}{10} + 2 \cdot \frac{7}{10} \cdot \frac{7}{10} = \boxed{\frac{119}{100}}$ or $\boxed{1.19}$

10. What is the square root of 1 million?

    ANS: $(10^6)^{1/2} = \boxed{1000}$

11. For what choices of a does the polynomial $ax^2 - 6x + 1$ have exactly one real root?

    ANS: $a = \boxed{9}$

12. What is the smallest positive integer greater than one that is both a perfect cube and a perfect fourth power?
    ANS: $2^{12} = \boxed{4096} = (2^4)^3 = (2^3)^4$

13. The 12 faces of a regular dodecahedron are pentagons. How many edges does a regular dodecahedron have?

    ANS: $\frac{12 \times 5}{2} = \boxed{30}$

14. Bo is going to the store to buy candy that will cost somewhere between 5 cents and 26 cents. What is the fewest number of coins Bo can carry in order to be certain to have exact change to buy the candy?

    ANS: 1¢, 1¢, 1¢, 1¢, 5¢, 10¢, 10¢ $\boxed{7}$ coins

15. What is the equation of the line in the xy plane all of whose points are equidistant from the two points $(1, 1)$ and $(-1, 1)$?

    ANS: $\boxed{x = 0}$

Tiebreaker Question

What is the prime factorization of 91?
ANS: $91 = \boxed{7 \times 13}$