

CSU Math Day 2003  
Team Competition  
11:20

1. What is the area of the smallest right triangle with all sides positive integers?

ANS: 3 – 4 – 5 right triangle has area  $3 \cdot 2 = \boxed{6}$

2. A computer sequentially computes integers by the following rule: If  $n$  is a square then multiply by 2; otherwise subtract 2. Starting at  $n = 9$ , what is the integer after 6 iterations?

ANS: 9, 18, 16, 32, 30, 28,  $\boxed{26}$

3. Pierre de Fermat was not a professional mathematician, although such famous results as Fermat's Last Theorem and Fermat's Little Theorem were named in his honor. What was his profession?

ANS: Lawyer

4. 8000 raffle tickets are to be sold for \$1 each. The winner receives \$2000. If you purchase 1 ticket, how much are your expected earnings?

ANS: – \$0.75

5. A pair of dice is rolled. What is the probability that the total is 8 or 6?

ANS:  $\frac{5}{18}$

6. John got a 10% raise last year and another 15% raise this year and his current salary is \$44275.00. What was his salary before last year's raise?

ANS: \$35000.00

7. On January 1, 1998, an investment bond was purchased for \$1000. If it earns 10% compounded annually, what will the balance be on January 1, 2000?

ANS: \$1210.00

8. How many positive divisors (including itself) does the number 81 have?

ANS: 5 divisors are  $\{1,3,9,27,81\}$

# Team Competition

## 11:20

9. Solve the system of equations

$$y + z = 1$$

$$x + z = 1$$

$$x + y = 1$$

ANS: By symmetry,  $x = y = z = \frac{1}{2}$

10. In which quadrant do the two lines  $y = 4 - 2x$  and  $y = 18 + 5x$  intersect?

ANS: Point  $(-2, 8)$  is in the second quadrant

11. The sum of the squares of two positive integers is 394, and the difference of their squares is 56. What are the numbers?

ANS:  $\left\{ \begin{array}{l} x^2 + y^2 = 394 \\ x^2 - y^2 = 56 \end{array} \right\}$ , Solution is :  $y = 13, x = 15$

12. Expand  $(4x^2 + 4y^3)^3$

ANS:  $64x^6 + 192x^4y^3 + 192x^2y^6 + 64y^9$

13. What is the midpoint of the line segment joint  $(-4, -5)$  and  $(-8, -3)$ ?

ANS:  $(-6, -4)$

14. A pair of dice is rolled. What is the probability that the total is 2, 11 or 9?

ANS:  $\frac{7}{36}$

15. A pyramid is build out of cubical blocks by placing 36 blocks on the floor, 25 blocks on top of the bottom layer, and so forth. How many cubes are required to build the pyramid?

ANS:  $\sum_{i=1}^6 i^2 = \boxed{91}$ ,  $\frac{6 \times 7 \times 13}{6} = \boxed{91}$

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Tiebreaker Question

What is the greatest common divisor of 187 and 209?

ANS:  $\gcd(187, 209) = \boxed{11}$

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1. A 6-foot man casts a 12-foot shadow. A flag pole next to him casts a 54-foot shadow. How tall is the flag pole?

ANS: 27 ft

2. At the local Dairy Queen, the “Monster Sundae” can be ordered with any of 7 flavors of ice cream plus any or all of 7 toppings. If you order one such sundae every Saturday, how many weeks will it be before you must order the same sundae twice?

ANS: 896

3. What is the maximum number of pieces into which a circular pizza can be cut using 3 chops of a knife (with no intermediate rearrangements of the pieces)?

ANS: 7

4. If it takes 1 liter of paint to paint a 3-meter model of a battleship, how much paint is required to paint a 2-meter model of the same ship?

ANS:  $\frac{4}{9}$  liters

5. In an algebra class of 26 students, each student shakes hands with each of the other students exactly once. How many handshakes are there?

ANS: 325

6. The average of your first 5 exams is 92. If you score 68 on your next exam, what is your average for all 6 exams?

ANS: 88

7. The vertices of a quadrilateral are at the points  $(0, 0)$ ,  $(3, 1)$ ,  $(4, 0)$ , and  $(2, -4)$ . What is the area of the quadrilateral?

ANS:  $Area = \frac{1}{2} \cdot 4 \cdot 1 + \frac{1}{2} \cdot 4 \cdot 4 = \boxed{10}$

8. Bo has \$2.20 in quarters and dimes. If Bo has 3 times as many dimes as quarters, how many of each type of coin does Bo have?

ANS: 4 quarters and 12 dimes.

# Team Competition

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9. Sarah averages 50% on multiple choice exams. What is the probability that she gets at least 3 correct on a 5-question exam?

ANS:  $\left(\frac{1}{2}\right)^5 + 5\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right) + 10\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 = \boxed{\frac{1}{2}}$

10. What is  $120^\circ$  equal to in radians?

ANS:  $\frac{2}{3}\pi$

11. Bonnie gets a salary of \$40,000 with a 10% yearly raise. What will her salary be after 2 years?

ANS: \$48400.00

12. If 2 cards are drawn from a standard deck of 52 cards, what is the probability that one is a heart and the other is a diamond?

ANS:  $\frac{\binom{13}{1}\binom{13}{1}}{\binom{52}{2}} = \boxed{\frac{13}{102}}$

13. If a single 60-Watt bulb provides sufficient light to read a newspaper 3 feet from the bulb, how many 60-Watt bulbs are required in a light fixture 6 feet from the newspaper in order to provide the same apparent level of brightness?

ANS: 4

14. Given an isosceles right triangle with a hypotenuse of length  $\sqrt{93}$ , what is its area?

ANS:  $\frac{93}{4}$

15. A right triangle has legs of length 6 and 8. What is the radius of the circle that circumscribes the triangle?

ANS: diameter = 10, radius =  $\boxed{5}$

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## Tiebreaker Question

Which of the following three numbers is smallest:  $e^\pi$ ,  $\pi^e$ , or  $3^3$ ?

ANS:  $e^\pi = 23.14069264$ ,  $\boxed{\pi^e = 22.45915771}$ ,  $3^3 = 27$

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12:00

1. Who wrote the monumental *Principia mathematica*?

ANS: Bertrand Russell and Alfred North Whitehead

2. What complex numbers are equal to their squares?

ANS:  $x = x^2$ , Solution is:  $x = \boxed{0}$ ,  $x = \boxed{1}$

3. Which ancient civilization is responsible for dividing the circle into 360 equal parts (which we now call *degrees*)?

ANS: The Babylonians.

4. A quadrilateral kite is made with a right angle at the top, angles of  $2\alpha$  left and right, and an angle of  $\alpha$  at the bottom. What is  $\alpha$ ?

ANS:  $5\alpha = 360^\circ - 90^\circ \implies \alpha = \boxed{54^\circ}$  or  $\boxed{\frac{3\pi}{10}}$

5. A cube has pyramids cut and discarded from each corner by passing planes through the midpoints of the edges adjacent to each of its vertices. How many edges does the new solid have?

ANS: ANS: 8 vertices  $\times$  3 edges/vertex = 24 edges

6. List the following solids in order increasing volume: A sphere of radius 3, a cube of edge 5, and a pyramid whose height is 6 and whose base is a square of edge 8.

ANS: Sphere < Cube < Pyramid since  $\frac{4}{3}\pi 3^3 = 113.1 < 5^3 = 125 < \frac{1}{3}8^2 6 = 128$

7. If the surface area of a cube is equal to 150, what is its volume?

ANS: 125

8. What is the smallest positive integer with exactly 6 positive divisors?

ANS: 12 (divisors are  $\{1, 2, 3, 4, 6, 12\}$ )

# Team Competition

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9. Find all positive roots of the equation  $x^4 - 5x^2 + 4 = 0$ .

ANS:  $x^4 - 5x^2 + 4 = 0$ , Solution is :  $x = \boxed{1}$ ,  $x = \boxed{2}$ ,  $x = -2$ ,  $x = -1$

10. Two positive integers have a sum of 9. What is the smallest possible value for the sum of their cubes?

ANS:  $4^3 + 5^3 = \boxed{189}$

11. A set of points in the complex plane is determined by iteration of the function  $z \rightarrow z^2 - \lambda$ , where  $z$  and  $\lambda$  are complex numbers. What is the name of this set?

ANS: Mandelbrot set named after Benoit B. Mandelbrot

12. A pair of dice is rolled. What is the probability that the total is 5?

ANS:  $\frac{1}{9}$

13. A regular hexagon is inscribed in a circle of radius 5 cm. What is the perimeter of the hexagon?

ANS: 30 cm

14. An octahedron is a regular solid whose 8 faces are equilateral triangles. What is the distance between a pair of opposite vertices, assuming each edge of the octahedron is of length 1?

ANS:  $\sqrt{2}$

15. What is the least common denominator of  $\frac{2}{39}$  and  $\frac{5}{44}$ ?

ANS: 3432

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## Tiebreaker Question

What is the prime factorization of 105?

ANS:  $105 = \boxed{3 \times 5 \times 7}$

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1. Let  $S$  be the set of the first 7 natural numbers. How many of the 128 subsets of  $S$  contain the number 2?

ANS: 64 or Half

2. Who wrote the monumental *Principia mathematica*?

ANS: Bertrand Russell and Alfred North Whitehead

3. Allison scored 66 on the first exam and 58 on the second exam. What must she average on the next two exams to bring her average for the four exams up to 75?

ANS: 88

4. What is the base 8 representation of the decimal number 371?

ANS:  $563_8$

5. From what language was the term 'algebra' derived?

ANS: Arabic

6. A wheel of radius 1 foot rolls without slipping around the outside of a stationary wheel of radius 2 feet. Exactly how many rotations does the small wheel make?

ANS: 3

7. A regular icosahedron has 20 faces, each of which is an equilateral triangle. If the midpoint of each face is connected with an edge to the midpoint of each adjacent face, what solid do these new edges determine?

ANS: Dodecahedron (12 faces, each face is a pentagon.)

8. According to the Rational Root Theorem, what are all the possible rational roots of the polynomial  $2x^3 - 5x^2 - 11x - 4$ ?

ANS:  $\pm\frac{1}{2}, \pm 1, \pm 2, \pm 4$

# Team Competition

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9. One of the most influential mathematicians of all time was the ninth century Arab named Mohammed ibn-Musa al-Khwarizmi. His last name survives in mathematics today as the term “algorithm”. His most important work was *Al-jabr wa'l mugabālah*. What mathematical term was derived from this title?

ANS: Algebra (Source: Boyer and Merzbach, *A History of Mathematics*)

10. If a cube has a volume of  $64 \text{ cm}^3$ , what is its surface area?

ANS:  $96 \text{ cm}^2$

11. A basketball team has 12 players on the roster. How many different starting lineups are possible?

ANS:  $\binom{12}{5} = \boxed{792}$

12. The plane can be completely tiled using equilateral triangles, or using squares, or using regular hexagons. These give the three regular tessellations of the plane. A semi-regular tessellation is a tiling that uses at least two different regular polygons, and where every vertex is congruent to every other vertex. How many semi-regular tessellations are there?

ANS: 8

13. Given 3 gallons of a 15% antifreeze/water mixture, how much pure antifreeze must be added to yield a 50% antifreeze/water mixture?

ANS: 2.1

14. What positive number is 5 times as big as its reciprocal?

ANS:  $\sqrt{5}$  since  $x = \frac{5}{x} \implies x^2 = 5$

15. According to Descartes' Rule of Signs, how many negative real roots does the polynomial equation  $x^4 - 8x^2 + 6x - 7 = 0$  have?

ANS:  $\boxed{\text{one}}$  (since there is one sign change if  $x$  is replaced by  $-x$ )

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## Tiebreaker Question

What is the least common multiple of 27 and 74?

ANS:  $\text{lcm}(27, 74) = \boxed{1998}$

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1. What is the diameter of a circle with area  $169\pi$  cm<sup>2</sup>?

ANS: 26

2. Find  $x$  so that the average of the four numbers 35, 27, 47, and  $x$  is 32.

ANS:  $x = 19$

3. What is the area of a regular hexagon with sides of length 2?

ANS:  $6\sqrt{3}$

4. A spherical balloon's diameter increases by 30%. By what percentage does the surface area change?

ANS: 69%

5. What is the area of the largest square that can be inscribed in the ellipse  $\frac{x^2}{4} + \frac{y^2}{2} = 1$ ?

ANS:  $x = y \implies \frac{3x^2}{4} = 1 \implies 4x^2 = \boxed{\frac{16}{3}}$

6. Given the circle  $x^2 - 2x + y^2 = 0$  in the  $xy$  plane, what are the equations of the two vertical tangent lines to this circle?

ANS:  $\boxed{x = 0}$  and  $\boxed{x = 2}$

7. Alpher, Beta, and Gamov are now 7, 11, and 13 years old, respectively. How many years will it be until they again have prime-numbered ages?

ANS:  $\boxed{\text{Six years from now}}$  they will have ages 13, 17, and 19.

8. What was the license plate number the Indian mathematician Ramanujan referred to when he said, "On the contrary, that's a very interesting number. It's the first number that can be written as the sum of 2 cubes in 2 different ways."

ANS: 1729 ( $= 12^3 + 1^3 = 10^3 + 9^3$ )

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9. If  $f(x) = -2x - 1$ , what is  $f(f(f(-1)))$ ?

ANS: 5

10. To the nearest minute, at what time between 10:30 a.m. and 11:00 a.m. are the minute hand and the hour hand at right angles?

ANS:  $50 + \frac{m}{12} = m + 15$ , Solution is :  $m = \frac{420}{11} = 38.18181818$  10:38

11. What is the minimum value of the function  $x^2 - 2x - 3$ ?

ANS:  $-4$ , since  $x^2 - 2x - 3 = (x - 1)^2 - 4$

12. Find all the roots of the equation  $x^3 - x^2 - 72x - 180 = 0$  ?

ANS: 10,  $-3$ ,  $-6$ .

13. What is the area of the parallelogram with vertices  $(0, 0)$ ,  $(4, 6)$ ,  $(7, 5)$ , and  $(11, 11)$ ?

ANS: 22

14.  $\frac{1}{3}$  of the air in a container is removed by each cycle of an air pump. What fractional part of the air remains after 2 cycles?

ANS:  $\frac{4}{9}$

15. The graph of a cubic polynomial has  $x$ -intercepts 0 and 1 (only). What is a possible expression for the polynomial?

ANS:  $x^2(x - 1)$  =  $x^3 - x^2$  or  $x(x - 1)^2$  =  $x^3 - 2x^2 + x$  (or a nonzero multiple)

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## Tiebreaker Question

A rectangle has perimeter 36 and area 80. What are the dimensions of the rectangle?

ANS: 8 by 10

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1. The graph of a cubic polynomial crosses the  $x$ -axis at  $x = -2$ ,  $x = 2$ , and  $x = 7$ . In expanded form, what is one such polynomial?

ANS:  $x^3 - 7x^2 - 4x + 28$  or some nonzero multiple thereof

2. What is the sum of the roots of the polynomial  $x^3 + 16x^2 + 73x + 90$  ?

ANS:  $-16$

3. What is the smallest positive integer with exactly 5 positive integer divisors?

ANS: 16 (divisors are  $\{1, 2, 4, 8, 16\}$ )

4. Brigitte plants a pumpkin seed. The area that is covered by the vine doubles every month. After 5 months the entire garden is covered. When was exactly half of the garden covered with the vine?

ANS:  or

5. Farmer Cornpea plowed  $3/8$  of her field in 10 hours. At that rate, how much longer will it take her to plow the remainder of her field? (Give your answer to the nearest hours and minutes.)

ANS:  $10/(3/8) = \frac{80}{3} = 26.66666667$  or another

6. Give the points of intersection of the two curves  $y = 20 - 6x$  and  $y = 8 - 6x + 3x^2$ .

ANS:  $\left\{ \begin{array}{l} y = 8 - 6x + 3x^2 \\ y = 20 - 6x \end{array} \right\}$ , Solution is :

7. List the following three numbers in increasing order:  $x = 1 + 2 + 3 + \dots + 38$ ,  $y = 1 \times 2 \times 3 \times 4 \times 5 \times 6$ , and  $z = 3^6$ .

ANS:  $y = 6! = 720 < z = 3^6 = 729 < x = 1 + 2 + 3 + \dots + 38 = 741$

8. A wooden cube of edge 4 inches is painted red. The cube is then cut into 64 one-inch cubes by making 9 saw cuts. How many of the one-inch cubes have exactly one red face?

ANS: 6 faces  $\times$  4 cubes per face =  cubes

# Team Competition

1:00

9. Name the smallest integer whose fourth power is less than 100.

ANS:  $(-4)^4 = 256$ ,  $(-3)^4 = 81$ ,  $(-2)^4 = 16$ , etc.  $\boxed{-3}$  is the smallest

10. What is the radius of the sphere whose volume is 19 times its surface area?

ANS: 57

11. It takes 1000 square tiles to tile a room, or 1440 smaller tiles whose edge is 1 inch less. How large is the room in square feet?

ANS:  $1000x^2 = 1440(x - \frac{1}{12})^2$ , Solution is :  $x = \frac{1}{2}$ ,  $1000(\frac{1}{2})^2 = \boxed{250}$  ft<sup>2</sup>

12. How far is the point (3,0) from the line  $3y = 4x$ ?

ANS:  $\frac{x}{3} = \frac{4}{5} \implies x = \frac{12}{5} = \boxed{2.4}$

13. The probability of picking a dog to finish in the top 3 at the dog track is  $\frac{1}{3}$ . What is the probability of picking 3 straight losers?

ANS:  $\frac{8}{27}$

14. Factor the polynomial  $x^3 + 4x^2 + 6x + 4$  as a product of a linear and a quadratic, using the fact that  $-2$  is a root.

ANS:  $(x + 2)(x^2 + 2x + 2)$

15. The formula  $e^{i\pi} + 1 = 0$  relates five of the most popular numbers in mathematics. What is  $\pi$  rounded to 10 significant digits?

ANS:  $\pi = 3.14159265358979\dots \approx \boxed{3.141592654}$

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## Tiebreaker Question

What is the greatest common divisor of 165 and 147?

ANS:  $gcd(165, 147) = \boxed{3}$

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1. If a sequence is defined by  $x_1 = 1$ ,  $x_2 = -4$ , and  $x_{n+1} = x_n - x_{n-1}$ , what is  $x_4$ ?

ANS: 1, -4,  $(-4) - 1 = -5$ ,  $-5 - (-4) = \boxed{-1}$

2. If 2 is the first prime, what is the fifteenth prime?

ANS: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43,  $\boxed{47}$

3. How many vertices does an  $n$ -dimensional cube have?

ANS:  $2^n$

4. Who proved Fermat's Last Theorem?

ANS: Andrew Wiles

5. How many different 6-letter words can be formed by rearranging the letters in SCHOOL?

ANS:  $\frac{6!}{2!} = \boxed{360}$

6. Cubic polynomials have three zeros, which in general are complex numbers. Knowing that  $-2$  is a zero of the polynomial  $x^3 - 8x^2 - 8x + 24$ , what is the product of the two remaining real or complex zeros?

ANS: 12

7. How many distinct complex roots does the polynomial  $x^5 - 1$  have?

ANS: 5

8. What is the area of the triangle bounded by the  $x$ -axis, the  $y$ -axis, and the line  $y = x + 3$ ?

ANS:  $\frac{9}{2}$

# Team Competition

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9. After what mathematician was the Cartesian coordinate system named?

ANS: (René) Descartes

10. The diameter of a square of edge  $a$  is  $a\sqrt{2}$ . The diameter of a cube of edge  $a$  is  $a\sqrt{3}$ . That is the diameter of a 4-dimensional cube of edge  $a$ ?

ANS:  $2a$

11. A computer sequentially computes integers by the following rule: If  $n$  is a square then multiply by 2; otherwise subtract 3. Starting at  $n = 12$ , what is the integer after 6 iterations?

ANS:  $12 \rightarrow 9 \rightarrow 18 \rightarrow 15 \rightarrow 12 \rightarrow 9 \rightarrow \boxed{18}$

12. What is the coefficient of  $xy^2$  in the expansion of  $(x + y)^3$ ?

ANS: 3

13. Give the prime factorization of the smallest integer divisible by 5, 1, 3, and 7.

ANS:  $3 \times 5 \times 7$

14. In how many ways can ALLAN (spelled A-L-L-A-N) misspell his name, assuming he uses all the right letters (the right number of times)?

ANS:  $\frac{5!}{2!2!} - 1 = \boxed{29}$

15. Give the points of intersection of the two curves  $y = 20 - 6x$  and  $y = 8 - 6x + 3x^2$ .

ANS:  $\left\{ \begin{array}{l} y = 8 - 6x + 3x^2 \\ y = 20 - 6x \end{array} \right\}$ , Solution is :  $\boxed{(2, 8), (-2, 32)}$

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## Tiebreaker Question

If  $n$  and  $n+2$  are both primes, then the pair  $n, n+2$  is called a pair of twin primes. Examples include 17, 19 and 29, 31. What is the smallest pair of three-digit twin primes?

ANS: 101, 103

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1. What is the area of an equilateral triangle inscribed in a circle of radius 4?  
ANS:  $12\sqrt{3}$
  
2. For what choices of  $a$  does the polynomial  $ax^2 - 4x + 2$  have exactly one real root?  
ANS:  $a = 2$
  
3. CAS describes a type of computer software. What do the letters stand for?  
ANS: Computer Algebra Systems
  
4. What is the prime factorization of  $4! = 24$ ?  
ANS:  $2^3 \cdot 3$
  
5. Three positive integers have a sum of 10. What is the minimum possible value for the sum of their squares?  
ANS:  $3^2 + 3^2 + 4^2 = \boxed{34}$
  
6. What is the smallest prime larger than 500?  
ANS: 503
  
7. The sum of two integers is  $-9$  and their product is 20. What are the two integers?  
ANS:  $-5$  and  $-4$ .
  
8. What Greek philosopher raised paradoxes that argued that motion is impossible?  
ANS: Zeno of Elea

# Team Competition

1:35

9. Factor completely the integer  $2^5 + 3^5$ .

ANS:  $2^5 + 3^5 = 275 = \boxed{5^2 11}$

10. What is the vertex of the parabola with equation  $y = x^2 - 8x + 11$ ?

ANS:  $(4, -5)$  since  $y = x^2 - 8x + 11 = (x - 4)^2 - 5$

11. What is the area of the largest octagon that can be inscribed in a square of side 1?

ANS:

$$\begin{aligned}x + 2\left(\frac{x}{\sqrt{2}}\right) &= 1, \text{ Solution is: } x = \frac{1}{\sqrt{2} + 1} \\ \text{area} &= 1 - x^2 = 1 - \frac{1}{(\sqrt{2} + 1)^2} \\ &= \boxed{\frac{2}{1 + \sqrt{2}}} = \boxed{2(\sqrt{2} - 1)} = \boxed{2\sqrt{2} - 2}\end{aligned}$$

12. What did the Norwegian mathematician Niels Henrik Abel prove about general fifth-degree polynomials?

ANS:  $\boxed{\text{Cannot be solved}}$  in terms of radicals involving the coefficients

13. The compact disk UR2gly sells at outlet AC for \$12.95 less a discount of 15%, and at outlet DC for \$14.65 less a discount of 25%. Which outlet has the lower price?

ANS:  $AC = 12.95 \times .85 = 11.0075$ ,  $\boxed{DC} = 14.65 \times .75 = 10.9875$

14. Partition 25 into two parts such that the difference of their square roots is 1.

ANS: 9, 16

15. What is the perimeter of a right triangle with legs 8 and 15?

ANS:  $s^2 = 8^2 + 15^2$ ,  $s = 17$ ,  $P = 8 + 15 + 17 = \boxed{40}$

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## Tiebreaker Question

Which is larger;  $\frac{468}{936}$  or  $\frac{378}{648}$ ?

ANS:  $\frac{468}{936} = \frac{1}{2} < \boxed{\frac{378}{648}} = \frac{7}{12}$

CSU Math Day 2003  
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1:50

1. What is the smallest value of the expression  $2x + \frac{1}{2x}$  if  $x$  is a positive real number?

ANS:  $\left[2x + \frac{1}{2x}\right]_{x=1/2} = \boxed{2}$

2. A ladder leans against a vertical wall. The bottom of the ladder is 7 feet from the wall and the top of the ladder is 12 feet above the floor. How long is the ladder?

ANS:  $\sqrt{193}$  feet

3. A particle, initially at  $(-5, -2)$ , moves along a line of slope  $\frac{3}{4}$  to a new position  $(x, y)$ . Find  $y$  if  $x = -1$ .

ANS:  $y = 1$

4. What mathematician popularized the use of  $\delta$  and  $\epsilon$  in proofs involving limits?

ANS: Augustin-Louis Cauchy

5. A 3-dimensional cube has 8 vertices and 12 edges. How many vertices and how many edges does a 4-dimensional cube have?

ANS: 16 vertices and 32 edges

6. A rectangle's length is increased by 30% and its width is decreased by 50%. How does its area change?

ANS: decreases by 35%

7. What is the smallest positive integer  $n$  such that  $n^2 - n + 11$  is not prime?

ANS: 11, since 
$$n^2 - n + 11 \quad \begin{matrix} n & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & \boxed{11} \\ & 11 & 13 & 17 & 23 & 31 & 41 & 53 & 67 & 83 & 101 & 121 = 11^2 \end{matrix}$$

8. List the following three numbers in increasing order:  $2^8$ ,  $3^5$ ,  $6^3$ .

ANS:  $\boxed{6^3} = 216 < \boxed{3^5} = 243 < \boxed{2^8} = 256$

# Team Competition

1:50

9. What is the smallest integer whose square is less than 383?

ANS:  $-19$

10. A circle of diameter 4 contains a circle of diameter 1 in its interior. What is the area contained in the larger circle that is exterior to the smaller circle?

ANS:  $\pi 2^2 - \pi \left(\frac{1}{2}\right)^2 = \boxed{\frac{15\pi}{4}}$

11. For what choices of  $a$  does the polynomial  $ax^2 - 8x + 2$  have two real roots?

ANS:  $a < 8$

12. Ann keeps flies and spiders in a box in her dorm room during the Halloween season. There are a total of 16 creatures with 112 legs. How many flies and how many spiders does she have? (Flies have 6 legs and spiders have 8)

ANS: 8 and 8.

13. The quartic polynomial  $6x^4 - 11x^3 - 66x^2 + 59x - 12$  has four real roots. What is the product of these four roots?

ANS:  $6(x+3)(x-4)\left(x-\frac{1}{2}\right)\left(x-\frac{1}{3}\right), -3 \cdot 4 \cdot \frac{1}{2} \cdot \frac{1}{3} = \boxed{-2}$

14. What cubic curve was named after Maria Agnesi?

ANS: Witch of Agnesi

15. How many primes are there between 90 and 100?

ANS: One (it is 97)

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## Tiebreaker Question

Name the largest integer whose square root is  $< 50$ .

ANS:  $\boxed{2499}$   $\sqrt{2499} = 49.989999, \sqrt{2500} = 50$

CSU Math Day 2003  
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2:05

1. A pyramid is build out of cubical blocks by placing 64 blocks on the floor, 49 blocks on top of the bottom layer, and so forth. How many cubes are required to build the pyramid?

ANS:  $\sum_{i=1}^8 i^2 = \boxed{204} = \frac{8 \times 9 \times 17}{6}$

2. The polynomial equation  $x^5 + x^2 + 1 = 0$  has one real root. How many imaginary roots does it have?

ANS: four

3. On a four-question true/false exam, correct answers are worth 3 points, wrong answers 0, and blanks count 1 point. How many different responses will result in a total score of 4?

ANS: Form 1111 or 3100, so  $1 + \frac{4!}{2!} = \boxed{13}$

4. Visually, an 8-9-12 triangle appears to be a right triangle. Is it actually acute, or is it obtuse?

ANS:  $\boxed{\text{Acute}}$  (largest angle  $\approx 89.602109^\circ$  or  $145 = 8^2 + 9^2 > 12^2 = 144$ )

5. Andrew Wiles has proven something that had remained an open problem for over 350 years. Either state the problem or name the person who claimed over 350 years ago to have a solution to the problem.

ANS:  $x^n + y^n = z^n$  has no positive integer solutions for  $n > 2$  OR Fermat

6. For whom was the computer language Ada named?

ANS: Ada Lovelace

7. A mathematician named Wolfram started a company named Wolfram Research. What is its primary product?

ANS: Mathematica

8. The 8 faces of a regular octahedron are equilateral triangles. How many vertices does a regular octahedron have?

ANS:  $\frac{8 \times 3}{4} = \boxed{6}$

# Team Competition

## 2:05

9. A rectangle's length is increased by 20% and its width is decreased by 20%. How much does its area change?

ANS:  $1.2L \times .8W = \boxed{.96LW}$  or  $\boxed{\text{decreases by 4\%}}$

10. This British mathematician lead a team of mathematicians and cryptologists during World War II that broke ciphertext generated by the German Enigma machine. Name this person.

ANS: Alan Turing

11. The equation  $x_{n+1} = \frac{1}{2} \left( x_n + \frac{3}{x_n} \right)$  provides an algorithm for approximating  $\sqrt{3}$ . Starting with  $x_1 = 1$ , what is  $x_3$  (as a rational number)?

ANS:  $x_2 = \frac{1}{2}(1 + 3) = 2$ ,  $x_3 = \frac{1}{2} \left( 2 + \frac{3}{2} \right) = \boxed{\frac{7}{4}} = \boxed{1\frac{3}{4}}$

12. The graph of the polynomial equation  $y = x^4 - x^3 - 9x^2 + 7x + 6$  crosses the  $x$ -axis at  $x = 3$ . Factor the polynomial  $x^4 - x^3 - 9x^2 + 7x + 6$ .

ANS:  $(x - 3)(x^3 + 2x^2 - 3x - 2)$

13. Who designed, but never build, an early "analytic engine" that led eventually to the Automatic Sequence Controlled Calculator developed jointly by IBM and Harvard University in 1944?

ANS: Charles Babbage

14. What is the coefficient of  $x$  in the expansion of  $(3x + 4)^4$ ?

ANS: 768

15. List the following three numbers in increasing order:  $\sqrt[6]{10}$ ,  $\sqrt{2}$ ,  $\sqrt[3]{3}$ .

ANS:  $\boxed{\sqrt{2}} = 1.4142 < \boxed{\sqrt[3]{3}} = 1.4422 < \boxed{\sqrt[6]{10}} = 1.4678$

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### Tiebreaker Question

A rectangle has perimeter 28 and area 49. What are the dimen sions of the rectangle?

ANS:  $\boxed{7 \text{ by } 7}$

CSU Math Day 2003  
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2:15

1. Neglecting order of addition, in how many ways can 25 be written as a sum of 3 distinct primes?

ANS:  $\boxed{2 \text{ ways}}$   $25 = 3 + 5 + 17 = 5 + 7 + 13$

2. Store A sells candy bars 3 for \$1.00. Store B sells candy bars individually for 40¢, but you get 5 for the price of 4. On Monday John bought some candy bars at store A. On Tuesday Jill bought some candy bars at store B. They compared notes and found that they had gotten the same number of candy bars, and each had paid the same amount of money. What is the least amount of money that each of them could have spent?

ANS: They each bought 6 candy bars and spent  $\boxed{\$2.00}$

3. A computer sequentially computes integers by the following rule: If  $n$  is odd then replace  $n$  by  $3n + 1$ ; otherwise replace  $n$  by  $n/2$ . If  $n$  starts at 3, what is  $n$  after 5 iterations?

ANS:  $3 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow \boxed{4}$

4. What are the next two prime numbers greater than 90?

ANS:  $91=7 \times 13$ ,  $93=3 \times 31$ ,  $\boxed{97, 101}$

5. If a stone falls 64 feet in 2 seconds, how far will it fall in 5 seconds?

ANS:  $16 \cdot 5^2 = \boxed{400}$  ft

6. A ball thrown vertically into the air 100 feet, falls and rebounds to a height of 40 feet the first time, rebounds to 16 feet on the second bounce, and so forth. What is the entire distance the ball will have moved when it finally comes to rest?

ANS:  $2 \sum_{i=0}^{\infty} 100 \cdot \left(\frac{2}{5}\right)^i = \frac{1000}{3} = \boxed{333\frac{1}{3}}$  ft

7. What is the cube root of 1 million?

ANS: 100

8. A multiple-choice exam has 15 questions with 3 choices per question. If you answer the questions randomly, what is the expected number of correct responses?

ANS: 5

# Team Competition

## 2:15

9. A square is inscribed in a circle, which in turn is inscribed in a square. What percentage of the area of the large square is inside the small square?

ANS: 50%

10. How bits are there in a nibble?

ANS: 4 (a nibble is half a byte, which is 8 bits)

11. 6 liters of paint are required to paint the outside of a cubic box of volume  $1 \text{ m}^3$ . How much paint is needed to paint the outside of a cubic box of volume  $8 \text{ m}^3$ ?

ANS: 24

12. Three unit circles are mutually tangent and enclose a triangular region  $R$ . Find the area of  $R$ .

ANS:  $\sqrt{3} - \frac{\pi}{2}$

13. Find an integer between 100 and 1000 that is both a perfect square and a perfect cube.

ANS:  $\boxed{729} = 9^3 = 27^2$

14. How many committees of 2 men and 2 women can be formed from a group of 8 men and 7 women?

ANS: 588

15. How many different 6-place license plates are possible if the first two places are letters and the last 4 places are digits?

ANS:  $26^2 \times 10^4 = \boxed{6,760,000}$

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### Tiebreaker Question

Which has the larger surface area: A cube of volume 1 or a sphere of volume 1.

ANS:  $\boxed{\text{cube}}$  area 6, sphere with surface area approximately 4.835 975 864