

CSU Math Day 2002
Team Competition
11:20

1. If 84 centimeters of wire are used to build the skeleton of edges of a cube, what is the volume of the cube?

ANS: 343

2. To the nearest one-hundredth of an inch, what is the smallest diameter log needed for cutting a square post of dimensions 4 in by 4 in?

ANS: $4\sqrt{2} = 5.65685424949238 \approx \boxed{5.66}$ in

3. What is the largest perfect cube smaller than 2000?

ANS: $11^3 = 1331$, $12^3 = \boxed{1728}$, $13^3 = 2197$

4. A basketball team has 12 players on the roster. How many different starting lineups are possible?

ANS: $\binom{12}{5} = \boxed{792}$

5. At 6:00 AM, Chad starts jogging at 5 km/h. At 6:00 AM Marzelle starts jogging from Chad's starting place at 7 km/h. How far behind is Marzelle at 11:00 AM?

ANS: 11 km

6. How many rearrangements of the letters a, b, c, d, e, f have a listed before b and b listed before c ?

ANS: $\frac{6!}{3!} = \boxed{120}$

7. Name the smallest integer whose fourth power is less than 1000.

ANS: $(-5)^4 = 625$, $(-6)^4 = 1296$, etc. $\boxed{-5}$ is the smallest

8. How many committees of 4 men and 2 women can be formed from a group of 5 men and 7 women?

ANS: 105

Team Competition

11:20

9. What former University of Colorado student went on to help found Apple Computer?

ANS: Steve Wozniak

10. For what choices of a does the polynomial $ax^2 - 4x + 2$ have exactly one real root?

ANS: $a = 2$

11. What is the area of the largest octagon that can be inscribed in a square of side 1?

ANS:

$$\begin{aligned}x + 2\left(\frac{x}{\sqrt{2}}\right) &= 1, \text{ Solution is: } x = \frac{1}{\sqrt{2} + 1} \\ \text{area} &= 1 - x^2 = 1 - \frac{1}{(\sqrt{2} + 1)^2} \\ &= \boxed{\frac{2}{1 + \sqrt{2}}} = \boxed{2(\sqrt{2} - 1)} = \boxed{2\sqrt{2} - 2}\end{aligned}$$

12. What is the area of a right triangle having a leg of length ℓ and hypotenuse of length 25?

ANS: $A = \frac{\ell\sqrt{25 - \ell^2}}{2}$

13. A triangle has sides of length 13, 10, and 13. Find the area of the triangle.

ANS: Altitude is 12, so $Area = \frac{1}{2}(10)(12) = \boxed{60}$

14. What is the smallest value of the expression $3x + \frac{1}{3x}$ if x is a positive real number?

ANS: $\left[3x + \frac{1}{3x}\right]_{x=1/3} = \boxed{2}$

15. What is the 13th term of the arithmetic sequence that begins 5, 10, 15■?

ANS: 65

Tiebreaker Question

What is the smallest four-digit prime?

ANS: $1001 = 7 \times 11 \times 13$, $1003 = 17 \times 59$, $1007 = 19 \times 53$, $\boxed{1009}$

CSU Math Day 2002
Team Competition
11:40

1. What is the sum of the first 88 positive integers?

ANS: 3916

2. Bo is going to the store to buy candy that will cost somewhere between 5 cents and 26 cents. What is the fewest number of coins Bo can carry in order to be certain to have exact change to buy the candy?

ANS: 1¢, 1¢, 1¢, 1¢, 5¢, 10¢, 10¢ 7 coins

3. What is the area of the right triangle whose hypotenuse is 17 if one of the legs has length 8?

ANS: $x^2 + 8^2 = 17^2$, Solution is : $x = 15$, $Area = \frac{1}{2} \times 8 \times 15 =$ 60

4. Let S be the set of the first 6 natural numbers. How many of the 64 subsets of S contain the number 2?

ANS: 32 or Half

5. What is the volume of the tetrahedron whose vertices are $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$?

ANS: $1/6$

6. What is the sum of the first six odd positive integers?

ANS: $1 + 3 + 5 + 7 + 9 + 11 =$ 36 $= 6^2$

7. Find the area of the circle inscribed in a regular hexagon with sides of length 1.

ANS: $\frac{3}{4}\pi$

8. How many positive divisors (including itself) does the number 81 have?

ANS: 5 divisors are $\{1, 3, 9, 27, 81\}$

Team Competition

11:40

9. A baseball manager has selected 9 starters for a game. If the pitcher must bat last and the second baseman must bat first, how many different batting line-ups are possible?

ANS: $7! = \boxed{5040}$

10. What Greek philosopher raised paradoxes that argued that motion is impossible?

ANS: Zeno of Elea

11. The graph of a cubic polynomial has x -intercepts 0 and 1 (only). What is a possible expression for the polynomial?

ANS: $\boxed{x^2(x-1)}$ = $\boxed{x^3 - x^2}$ or $\boxed{x(x-1)^2}$ = $\boxed{x^3 - 2x^2 + x}$ (or a nonzero multiple)

12. What is the area of one face of a diamond-shaped kite whose two sticks are 32 and 56 inches long?

ANS: 896

13. What is the area of the triangle bounded by the lines $x = 0$, $y = x$, and $y + \frac{2}{5}x = 7$?

ANS: $\frac{35}{2}$ since $(0, 0)$, $(0, 7)$, $(5, 5)$ are the vertices \implies base = 7 and height = 5

14. Two baseball teams, the Green Socks and the Yellow Elbows, start a 7-game series. For each game, the odds are 6 to 1 in favor of the Green Socks. What is the probability that the Green Socks sweep the series in four games?

ANS: $\frac{1296}{2401}$

15. What is the smallest perfect cube larger than 51?

ANS: $4^3 = 64$

Tiebreaker Question

What integer is closest to $\sqrt{42}$?

ANS: $\sqrt{42} \approx 6.480740698 = \boxed{6} + .480740698$

CSU Math Day 2002
Team Competition
12:00

1. Two circles are mutually tangent at one point, and the smaller circle passes through the center of the larger circle. What is the ratio between the areas of the two circles?

ANS: $\boxed{4 : 1}$ or $\boxed{1 : 4}$

2. The formula $e^{i\pi} + 1 = 0$ relates five of the most popular numbers in mathematics. What is π rounded to 10 significant digits?

ANS: $\pi = 3.14159265358979\dots \approx \boxed{3.141592654}$

3. The number 6 is perfect because $6 = 1 + 2 + 3$ is the sum of the proper divisors. Name the next perfect number.

ANS: $\boxed{28} = 1 + 2 + 4 + 7 + 14$

4. A baseball manager has selected 9 starters for a game. If the pitcher must bat last, how many different batting line-ups are possible?

ANS: $8! = \boxed{40320}$

5. Sarah averages 90% on multiple choice exams. What is the probability that she gets exactly 2 correct on a 3-question exam?

ANS: .243 or $\frac{243}{1000}$

6. A square is inscribed in a circle, which in turn is inscribed in a square. What percentage of the area of the large square is inside the small square?

ANS: 50%

7. Express the surface area S of a cube as a function of its volume V .

ANS: $S = 6x^2 = 6(x^3)^{2/3} = \boxed{6V^{2/3}}$

8. How many diagonals are there in a regular 15-sided polygon?

ANS: 90

Team Competition

12:00

9. According to Descartes' Rule of Signs, how many positive roots are there of the polynomial equation $2x^3 + 5x^2 - x - 8 = 0$?

ANS: 1 (One sign change in the sequence $++--$)

10. A city lot is twice as long as it is wide. By increasing its length 20 yards and its width 30 yards, the area will be increased by 2200 square yards. What are its dimensions?

ANS: $(2x + 20)(x + 30) - 2x^2 = 2200$, 20 yards by 40 yards

11. What is the smallest number greater than 50 has the property that when divided by 10 the remainder is 1 and when divided by 12 the remainder is also 1?

ANS: 121, since $121 = 10 \times 12 + 1$

12. What is the coefficient of x^7 in the expansion of $(x - 1)^8$?

ANS: -8

13. If it takes 16 minutes to inflate a large spherical balloon to a radius of 2 meters, how long will it take to inflate a large spherical balloon to a radius of 5 meters?

ANS: 250 min

14. If a third degree polynomial has leading coefficient of 1 and roots -2 , -3 , and 4 , what is its constant term?

ANS: 24

15. What is the converse of the statement, "If roses are red, then violets are blue"?

ANS: If violets are blue, then roses are red.

Tiebreaker Question

Name the largest integer whose 5th power is < 1000 .

ANS: 3 $3^5 = 243$, $4^5 = 1024$

CSU Math Day 2002
Team Competition
12:20

1. What is the area of the smallest right triangle with all sides positive integers?

ANS: 3 – 4 – 5 right triangle has area $3 \cdot 2 = \boxed{6}$

2. A computer sequentially computes integers by the following rule: If n is odd then replace n by $3n + 1$; otherwise replace n by $n/2$. If n starts at 7, what is n after 5 iterations?

ANS: $7 \rightarrow 22 \rightarrow 11 \rightarrow 34 \rightarrow 17 \rightarrow \boxed{52}$

3. How many primes are there between 90 and 100?

ANS: One (it is 97)

4. Paul Erdős (pronounced “Air’-dish”) wrote hundreds of papers in mathematics, many of which were coauthored by various mathematicians representing nearly every country on Earth. In which country was Paul Erdős born?

ANS: Hungary

5. If $\sin x = \frac{1}{3}$ and $0 < x < \pi/2$, what is $\sin 2x$?

ANS: $\sin 2x = 2 \sin x \cos x = 2 \cdot \frac{1}{3} \sqrt{1 - \frac{1}{9}} = \boxed{\frac{4}{9}\sqrt{2}}$

6. If $f(x) = 3x + 2$, what is $f(f(-\frac{1}{2}))$?

ANS: -1

7. Which regular polyhedron has the same number of faces as vertices?

ANS: Tetrahedron 4 faces and 4 vertices

8. Use the approximation $2^{10} \approx 10^3$ to estimate $\log_2(10^{42})$.

ANS: $\log_2(10^{42}) = \log_2((10^3)^{14}) \approx \log_2((2^{10})^{14}) = \log_2(2^{140}) = \boxed{140}$

Team Competition

12:20

9. A mathematician named Wolfram started a company named Wolfram Research. What is its primary product?

ANS: Mathematica

10. What is the area of a regular hexagon of edge 1?

ANS: 6 triangles of base 1, height $\sqrt{3}/2 = 6 \times (\frac{1}{2}) \times 1 \times \frac{\sqrt{3}}{2} = \boxed{\frac{3}{2}\sqrt{3}}$

11. What is the area of a regular hexagon with sides of length 2?

ANS: $6\sqrt{3}$

12. Give an equation in the form $ax + by = c$, where a , b , and c are integers, for the line through $(0, -4)$ that is parallel to the line $4x = -1$.

ANS: $x = 0$

13. A regular icosahedron has 20 faces, each of which is an equilateral triangle. If the midpoint of each face is connected with an edge to the midpoint of each adjacent face, what solid do these new edges determine?

ANS: Dodecahedron (12 faces, each face is a pentagon.)

14. Divide 52 into 3 parts, such that $\frac{1}{10}$ of the first, $\frac{1}{7}$ of the second, and $\frac{1}{9}$ of the third are equal.

ANS: 20, 14, and 18

15. Brigitte plants a pumpkin seed. The area that is covered by the vine doubles every month. After 5 months the entire garden is covered. When was exactly half of the garden covered with the vine?

ANS: or

Tiebreaker Question

Arrange the following three numbers from smallest to largest: $\frac{5}{7}$, $\frac{2}{3}$, $\frac{3}{4}$.

$$\boxed{\frac{2}{3}} = .666\ 666\ 666\ 7 < \boxed{\frac{5}{7}} = .714\ 285\ 714\ 3 < \boxed{\frac{3}{4}} = .75$$

CSU Math Day 2002
Team Competition
12:40

1. Given an integer n with $0 < n < 31$, what is the remainder when n^{30} is divided by 31?

ANS: 1 (Fermat's Little Theorem)

2. What are the next two prime numbers greater than 50?

ANS: $51 = 3 \times 17$, $\boxed{53}$, $55 = 5 \times 11$, $57 = 3 \times 19$, $\boxed{59}$

3. Find all the roots of the equation $x^3 - 9x^2 - 81x + 729 = 0$?

ANS: 9, 9, -9.

4. How many different 6-place license plates are possible if the first two places are letters and the last 4 places are digits?

ANS: $26^2 \times 10^4 = \boxed{6,760,000}$

5. In how many ways can one arrange the letters in OBOE?

ANS: $\frac{4!}{2!} = \boxed{12}$

6. If a regular polygon has 30 sides, what is the size, in degrees, of one of its interior angles?

ANS: 168°

7. The 8 faces of a regular octahedron are equilateral triangles. How many vertices does a regular octahedron have?

ANS: $\frac{8 \times 3}{4} = \boxed{6}$

8. According to Descartes' Rule of Signs, how many positive real roots does the polynomial equation $3x^4 + 10x^2 + 5x + 4 = 0$ have?

ANS: $\boxed{\text{None}}$ (since there are no sign changes)

Team Competition

12:40

9. A solid statue is made by melting 9 cm^3 of metal and pouring it into a mold. A larger model needs to be constructed by increasing each of its linear dimensions by a factor of 5. How much metal will the new statue require?

ANS: 1125 cm^3

10. Find a point equidistant from the points $(-2, -2)$, $(2, 2)$, and $(2, -2)$.

ANS: $(0, 0)$

11. How bits are there in a nibble?

ANS: 4 (a nibble is half a byte, which is 8 bits)

12. Solve the equation $x\sqrt{.16} = 6$

ANS: $x = 15$

13. Completely factor the polynomial $2x^2 - 2$.

ANS: $2(x + 1)(x - 1)$

14. Archie, Bob, and Claire are now 11, 17, and 19 years old, respectively. In how many years will they again have prime-numbered ages?

ANS: Twelve years from now they will have ages 23, 29, and 31.

15. A circular pizza is diameter 12 inches is cut into 6 congruent slices. What is the perimeter of each slice?

ANS: $12 + 2\pi$

Tiebreaker Question

Which has the larger surface area: A cube of volume 1 or a sphere of volume 1.

ANS: cube area 6, sphere with surface area approximately 4.835 975 864

CSU Math Day 2002
Team Competition
1:00

1. A 3-dimensional cube has 8 vertices and 12 edges. How many vertices and how many edges does a 4-dimensional cube have?

ANS: 16 vertices and 32 edges

2. A pyramid is built out of blocks by placing 100 blocks on the floor, placing 81 blocks on top of the bottom layer, and so forth. How many cubes are there in the pyramid?

ANS: $\sum_{i=1}^{10} i^2 = \boxed{385} = \frac{10 \cdot 11 \cdot 21}{6}$

3. The compact disk UR2ugly sells at outlet AC for \$15.49 less a discount of 20%, and at outlet DC for \$16.95 less a discount of 30%. Which outlet has the lower price?

ANS: AC, $15.49 \times .8 = \$12.39$; $\boxed{\text{DC}}$, $16.95 \times .7 = \$11.86$

4. In which quadrant is the center of the circle $x^2 + y^2 + 6x - 8y = 0$?

ANS: $x^2 + 6x + y^2 - 8y + 25 = (x + 3)^2 + (y - 4)^2 = 25$, center $(-3, 4)$ $\boxed{\text{2nd}}$ quadrant

5. A regular tetrahedron has edges of length 3. What is the total surface area of the tetrahedron?

ANS: $9\sqrt{3}$

6. A bag contains 4 white balls, 3 red balls, and 4 green balls. If 3 balls are selected at random from the bag, what is the probability that they are white?

ANS: $\frac{4}{165}$

7. What is the prime factorization of $4! = 24$?

ANS: $2^3 \cdot 3$

8. In a river that flows at $\frac{8}{3}$ miles per hour, it takes a certain swimmer 5 times as long to swim a certain distance up stream as it does to swim the same distance down stream. What is her speed in still water?

ANS: 4

Team Competition

1:00

9. This British mathematician led a team of mathematicians and cryptologists during World War II that broke ciphertext generated by the German Enigma machine. Name this person.

ANS: Alan Turing

10. What is the area of the triangle bounded by the x -axis, the y -axis, and the line $y = 4x + 8$?

ANS: 8

11. List the following three numbers in increasing order: π , $22/7$, and 3.14 ?

ANS: $\boxed{3.14} < \boxed{\pi} \approx 3.141592654 < \boxed{22/7} \approx 3.1429$

12. The 24 ways to write rearrangements of the letters MATH are listed in alphabetical order. Where in this list does MATH appear?

ANS: 14th on the list right after MAHT and before MHAT (1-6 start with A, 7-12 with H, and MAHT is 13th.)

13. The probability of picking a dog to finish in the top 3 at the dog track is $\frac{1}{5}$. What is the probability of picking 3 straight losers?

ANS: $\frac{64}{125}$

14. How many 2-person groups can be made out of a group of 2 people?

ANS: 1

15. What is the volume of a spherical raindrop of diameter 2 millimeters?

ANS: $\frac{4\pi}{3} (1)^3 = \boxed{\frac{4\pi}{3}}$ mm³

Tiebreaker Question

Factor 123 as a product of primes.

ANS: $123 = \boxed{3 \times 41}$

CSU Math Day 2002
Team Competition
1:20

1. If the absolute value of $x^2 + 4$ is equal to the absolute value of $x^2 - 6$, what is x ?

ANS: $|x^2 + 4| = |x^2 - 6|$, Solution is : $x = 1, x = -1$

2. What did the Norwegian mathematician Niels Henrik Abel prove about general fifth-degree polynomials?

ANS: $\boxed{\text{Cannot be solved}}$ in terms of radicals involving the coefficients

3. Which real numbers are equal to their cubes?

ANS: $x = x^3$, Solution is : $x = \boxed{0}, x = \boxed{1}, x = \boxed{-1}$

4. If the interior angle of a regular polygon is 108° , how many sides does it have?

ANS: 5 sides

5. The number 12 is called abundant because the sum of the proper divisors $1 + 2 + 3 + 4 + 6 = 16$ is greater than 12. What is the next abundant number?

ANS: ANS: $\boxed{18} < 1 + 2 + 3 + 6 + 9 = 21$

6. In which quadrant do the two lines $y = 4 - 2x$ and $y = 18 + 5$ intersect?

ANS: Point $(-2, 8)$ is in the $\boxed{\text{second quadrant}}$

7. Who proved Fermat's Last Theorem?

ANS: Andrew Wiles

8. Express the perimeter P of a square as a function of its area A .

ANS: $A = \left(\frac{P}{4}\right)^2$, Solution is : $\boxed{P = 4\sqrt{A}}$

Team Competition

1:20

9. What is the maximum number of pieces into which a circular pizza can be cut using 4 chops of a knife (with no intermediate rearrangements of the pieces)?

ANS: 11

10. In how many ways can ALLAN (spelled A-L-L-A-N) misspell his name, assuming he uses all the right letters (the right number of times)?

ANS: $\frac{5!}{2!2!} - 1 = \boxed{29}$

11. How many different strings of length 7 can be formed using the letters in ELLIPSE?

ANS: $\frac{7!}{2!2!} = \boxed{1260}$

12. The first famous woman mathematician was Hypatia, who lived from 370 to 415 A.D. Although none of her writings survive, it is believed that her work was included integer solutions to equations. What branch of mathematics deals with such problems?

ANS: Number Theory or Diophantine Analysis (Simmons, *Calculus Gems*)

13. The polynomial equation $x^5 + x^2 + 1 = 0$ has one real root. How many imaginary roots does it have?

ANS: four

14. How many distinct complex roots does the polynomial $x^5 - 1$ have?

ANS: 5

15. When this Hungarian mathematician died recently, the New York Times acknowledged his contributions to mathematics that included some 1500 published papers. Name this person.

ANS: Paul Erdős

Tiebreaker Question

What is the largest two-digit prime?

ANS: $\boxed{97}$

CSU Math Day 2002
Team Competition
1:35

1. What is the vertex of the parabola with equation $y = x^2 - 8x + 11$?

ANS: $(4, -5)$ since $y = x^2 - 8x + 11 = (x - 4)^2 - 5$

2. A computer sequentially computes integers by the following rule: If n is a square then multiply by 2; otherwise subtract 2. Starting at $n = 13$, what is the integer after 6 iterations?

ANS: $13 \rightarrow 11 \rightarrow 9 \rightarrow 18 \rightarrow 16 \rightarrow 32 \rightarrow \boxed{30}$

3. What is the area of the largest square that can be inscribed in the ellipse $\frac{x^2}{4} + \frac{y^2}{2} = 1$?

ANS: $x = y \implies \frac{3x^2}{4} = 1 \implies 4x^2 = \boxed{\frac{16}{3}}$

4. What cubic curve was named after Maria Agnesi?

ANS: Witch of Agnesi

5. If a right triangle has hypotenuse c and legs a and b , then $c^2 = a^2 + b^2$. After whom is this result named?

ANS: Pythagoras

6. If the absolute value of $x + 1$ is equal to the absolute value of $x - 1$, what is x ?

ANS: $x = 0$

7. Two positive integers have a sum of 9. What is the smallest possible value for the sum of their cubes?

ANS: $4^3 + 5^3 = \boxed{189}$

8. The equation $x_{n+1} = \frac{1}{2} \left(x_n + \frac{3}{x_n} \right)$ provides an algorithm for approximating $\sqrt{3}$. Starting with $x_1 = 1$, what is x_3 (as a rational number)?

ANS: $x_2 = \frac{1}{2}(1 + 3) = 2$, $x_3 = \frac{1}{2} \left(2 + \frac{3}{2} \right) = \boxed{\frac{7}{4}} = \boxed{1\frac{3}{4}}$

Team Competition

1:35

9. During the 1970's Steve Jobs and Steve Wozniak starting selling electronic equipment out of a garage. What company did they start?

ANS: Apple Computer

10. If a stone falls 64 feet in 2 seconds, how far will it fall in 5 seconds?

ANS: $16 \cdot 5^2 = \boxed{400}$ ft

11. Bo has \$6.0 in quarters and dimes. If Bo has 5 times as many dimes as quarters, how many of each type of coin does Bo have?

ANS: 8 quarters and 40 dimes.

12. List the following three numbers in increasing order: $\sqrt[6]{10}$, $\sqrt{2}$, $\sqrt[3]{3}$.

ANS: $\boxed{\sqrt{2}} = 1.4142 < \boxed{\sqrt[3]{3}} = 1.4422 < \boxed{\sqrt[6]{10}} = 1.4678$

13. For whom was the computer language Ada named?

ANS: Ada Lovelace

14. Three unit circles are mutually tangent and enclose a triangular region R . Find the area of R .

ANS: $\sqrt{3} - \frac{\pi}{2}$

15. What is the statement of Fermat's Last Theorem?

ANS: The equation $x^n + y^n = z^n$ has no positive integer solutions for $n > 2$.

Tiebreaker Question

What is the sum of the first ten odd integers?

ANS: $\sum_{n=1}^{10} (2n - 1) = \boxed{100}$

CSU Math Day 2002
Team Competition
1:50

1. An octahedron is a regular solid whose 8 faces are equilateral triangles. What is the distance between a pair of opposite vertices, assuming each edge of the octahedron is of length 1?

ANS: $\sqrt{2}$

2. Use the approximation $2^{10} \approx 10^3$ to approximate 2^{43} as a number in scientific notation $2^{43} \approx c \times 10^n$, where n is an integer and c is a number between 1 and 10.

ANS: $2^{43} = 8 \times (2^{10})^4 \approx 8 \times (10^3)^4 = \boxed{8 \times 10^{12}} = 8000000000000$

3. How many ways can the top 3 finishers be picked in a(n) 10-person race?

ANS: 720

4. What are the next two prime numbers greater than 90?

ANS: $91=7 \times 13$, $93=3 \times 31$, $\boxed{97, 101}$

5. What is the smallest positive integer whose square is larger than 688?

ANS: 27

6. What is the largest prime less than 500?

ANS: 499

7. CAD describes a type of computer software. What do the letters stand for?

ANS: Computer-Aided Design

8. In which quadrant do the two lines $y = 26 + 2x$ and $y = 6 - 3x$ intersect?

ANS: $\left\{ \begin{array}{l} y = 26 + 2x \\ y = 6 - 3x \end{array} \right\}$, Solution is : $(-4, 18)$ $\boxed{\text{second quadrant}}$

Team Competition

1:50

9. If a sequence is defined by $x_1 = 1$, $x_2 = -4$, and $x_{n+1} = x_n - x_{n-1}$, what is x_4 ?

ANS: 1, -4, $(-4) - 1 = -5$, $-5 - (-4) = \boxed{-1}$

10. What is the length of the arc on a circle of radius 9 subtended by a central angle of 120° ?

ANS: 6π

11. The average of 10 numbers is 24. What is the average of these 10 numbers together with -20 ?

ANS: 20

12. Express the volume V of a cube as a function of the area A of one of its faces.

ANS: $V = x^3 = (x^2)^{3/2} = A^{3/2}$, so $\boxed{V = A^{3/2}}$ or $\boxed{V = A\sqrt{A}}$

13. Maple is a widely-used computer algebra system. In what country was it developed?

ANS: Canada

14. Give the slope-intercept equation for a line through the origin that contains no other points with integer coordinates.

ANS: $y = \alpha x$, α any irrational number

15. Which of the following best describes how many 6-symbol license plates are possible if the symbols come from the letters A-Z together with the digits 0-9? (a) 2-3 million (b) 20-30 million (c) 200-300 million (d) 2-3 billion

ANS: $36^6 = 2176782336 \approx \boxed{2-3 \text{ billion}}$ or $\boxed{(d)}$

Tiebreaker Question

Which of the following three numbers is smallest: e^π , π^e , or 3^3 ?

ANS: $e^\pi = 23.14069264$, $\boxed{\pi^e = 22.45915771}$, $3^3 = 27$

CSU Math Day 2002
Team Competition
2:05

1. If g is a function such that $g(1) = 2$, $g(2) = 1$, and

$$g(n+1) = g(n) - 2g(n-1)$$

for $n \geq 3$, what is $g(4)$?

ANS: $g(3) = g(2) - 2g(1) = 1 - 4 = -3$, $g(4) = g(3) - 2g(2) = -3 - 2 = \boxed{-5}$

2. What is the volume of a sphere of surface area 400π ?

ANS: $\frac{4000}{3}\pi$

3. What is the area of the region bounded by the x -axis, the vertical lines $x = 0$, $x = 4$, and the line $y = -4x + 15$?

ANS: 28

4. For what choices of a does the polynomial $ax^2 - 14x + 2$ have no real roots?

ANS: $a > \frac{49}{2}$

5. How many distinct complex roots does the polynomial $x^6 + 1$ have?

ANS: 6

6. What is the sum of the first five odd positive integers?

ANS: $1 + 3 + 5 + 7 + 9 = \boxed{25} = 5^2$

7. A ladder leans against a vertical wall. The bottom of the ladder is 8 feet from the wall and the top of the ladder is 10 feet above the floor. How long is the ladder?

ANS: $2\sqrt{41}$ feet

8. A computer sequentially computes integers by the following rule: If n is a square then multiply by 2; otherwise subtract 3. Starting at $n = 12$, what is the integer after 6 iterations?

ANS: $12 \rightarrow 9 \rightarrow 18 \rightarrow 15 \rightarrow 12 \rightarrow 9 \rightarrow \boxed{18}$

Team Competition

2:05

9. What is the contrapositive of the statement, “If $x > 4$ then $x^2 > 16$ ”?

ANS: If $x^2 \leq 16$, then $x \leq 4$.

10. Bertrand Russell and Alfred North Whitehead wrote the monumental *Principia mathematica*. The subject of this publication was

(a)

- (b) Who wrote the monumental *Principia mathematica*?

ANS: Bertrand Russell and Alfred North Whitehead

- (c) Sarah averages 50% on multiple choice exams. What is the probability that she gets at least 3 correct on a 5-question exam?

ANS: $(\frac{1}{2})^5 + 5(\frac{1}{2})^4(\frac{1}{2}) + 10(\frac{1}{2})^3(\frac{1}{2})^2 = \boxed{\frac{1}{2}}$

- (d) An orange has a diameter that is 75% fruit and 25% peel. To the nearest percent, what percentage of the volume is the peel?

ANS: 58% ($\approx 57.8125\%$)

- (e) What can you say about a parallelogram whose diagonals have equal length?

ANS: It is a rectangle.

- (f) In data processing terminology, what does FIFO mean?

ANS: First In First Out

Tiebreaker Question

What is the least common multiple of 33 and 77?

ANS: $\text{lcm}(33, 77) = \boxed{231}$

CSU Math Day 2002
Team Competition
2:15

1. Allison scored 66 on the first exam and 58 on the second exam. What must she average on the next two exams to bring her average for the four exams up to 75?

ANS: 88

2. It takes 1000 square tiles to tile a room, or 1440 smaller tiles whose edge is 1 inch less. How large is the room in square feet?

ANS: $1000x^2 = 1440(x - \frac{1}{12})^2$, Solution is : $x = \frac{1}{2}$, $1000(\frac{1}{2})^2 = \boxed{250}$ ft²

3. Given an isosceles right triangle with a hypotenuse of length $\sqrt{93}$, what is its area?

ANS: $\frac{93}{4}$

4. How many edges does an n -dimensional cube have?

ANS: $n2^{n-1}$

5. What complex numbers are equal to their squares?

ANS: $x = x^2$, Solution is: $x = \boxed{0}$, $x = \boxed{1}$

6. What is the largest integer that can be stored in a 6-bit computer word?

ANS: 63

7. A pair of dice is rolled. What is the probability that the total is 6 or 9?

ANS: $\frac{1}{4}$

8. What is the distance between two opposite vertices of a cube of edge $2\sqrt{3}$?

ANS: 6

Team Competition

2:15

9. The set S has 10 elements. If a and b are (distinct) elements of S , how many subsets of S contain both a and b ?

ANS: $256 = 2^8$

10. State the contrapositive of the statement, "If $x^2 \leq 9$, then $x \leq 3$."

ANS: If $x > 3$, then $x^2 > 9$.

11. A deck of 52 cards is thoroughly shuffled and the cards are turned over two at a time. How many pairs (two cards of the form $\boxed{A} \boxed{A}$ or $\boxed{5} \boxed{5}$) do you expect to see?

ANS: $26 \left(\frac{3}{51} \right) = \boxed{\frac{26}{17}}$

12. An automobile travels at 15 kilometers per hour for 6 minutes, 60 km/h for 1 minute, and then 30 km/h for 6 minutes. How far did it travel during the 13 minutes?

ANS: $\frac{11}{2}$ kilometers

13. Solve the system of equations

$$y + z = 1$$

$$x + z = 1$$

$$x + y = 1$$

ANS: By symmetry, $x = y = z = \frac{1}{2}$

14. Find the fourth vertex of a rectangle, three of whose vertices are $(3, -5)$, $(7, -1)$, and $(7, -9)$.

ANS: $(11, -5)$

15. Farmer Jill raises goats and geese. If she counts 32 eyes and 44 feet, how many goats and how many geese does Jill have?

ANS: 10 geese and 6 goats.

Tiebreaker Question

Which cylindrical glass hold more water: One of diameter 4 and height 3, or one of diameter 3 and height 4?

ANS: $\boxed{\text{Diameter 4 height 3}}$ volume = $\pi 2^2 3 = 12\pi$ or diameter 3 height 4, volume = $\pi \left(\frac{3}{2}\right)^2 4 = 9\pi$