

CSU Math Day 2001  
Team Competition  
11:20

1. Bonnie gets a salary of \$50,000 with a 15% yearly raise. What will her salary be after 2 years?

ANS:  $\boxed{\$66125.00}$

2. What fact about the number 1729 did the Indian mathematician Ramanujan consider interesting?

ANS: Smallest positive integer = sum of cubes in 2 ways:  $12^3 + 1^3 = 10^3 + 9^3$

3. What is the base 3 representation of the decimal number 13?

ANS:  $\boxed{111_3}$

4. Which real numbers are equal to their cubes?

ANS:  $x = x^3$ , Solution is :  $x = \boxed{0}, x = \boxed{1}, x = \boxed{-1}$

5. Give an equation in the form  $ax + by = c$ , where  $a$ ,  $b$ , and  $c$  are integers, for the line through  $(5, 1)$  that is parallel to the line  $y = \frac{1}{2}x + 4$ .

ANS:  $\boxed{x - 2y = 3}$  or  $\boxed{-x + 2y = -3}$

6. A right triangle has legs of length 6 and 8. What is the radius of the circle that circumscribes the triangle?

ANS: diameter = 10, radius =  $\boxed{5}$

7. Bertrand Russell and Alfred North Whitehead wrote the monumental *Principia mathematica*. The subject of this publication was a) ALGEBRA b) LOGIC AND SET THEORY or c) NUMBER THEORY?

ANS:  $\boxed{\text{b) LOGIC AND SET THEORY}}$

(a)

8. 8000 raffle tickets are to be sold for \$1 each. The winner receives \$2000. If you purchase 1 ticket, how much are your expected earnings?

(a) ANS:  $\boxed{-\$0.75}$

# Team Competition

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9. A pair of slacks priced at \$30 has been marked up 10% and then marked down 50%. What is the new price?

(a) ANS:  $\boxed{\$16.50}$

10. If  $\sin x = \frac{1}{3}$  and  $0 < x < \pi/2$ , what is  $\sin 2x$ ?

(a) ANS:  $\sin 2x = 2 \sin x \cos x = 2 \cdot \frac{1}{3} \sqrt{1 - \frac{1}{9}} = \boxed{\frac{4}{9}\sqrt{2}}$

11. Neglecting the order of addition, in how many ways can 30 be written as the sum of two primes?

(a) ANS:  $\boxed{3 \text{ ways}}$   $30 = 7 + 23 = 11 + 19 = 13 + 17$

12. What is the largest prime less than 500?

(a) ANS:  $\text{factor}(499) = \boxed{499}$

13. The interior of a triangle is painted black. The midpoints of the three edges are connected, and the interior of this triangle is painted white. The midpoints of each of the remaining black triangles are connected and the interiors are painted white. Continuing this process leaves a bit of black paint. What is the name of the resulting set of black points?

(a) ANS:  $\boxed{\text{Sierpinski triangle}}$

14. Partition 25 into two parts such that the difference of their square roots is 1.

(a) ANS:  $\boxed{9, 16}$

15. What is the volume of a spherical raindrop of diameter 2 millimeters?

(a) ANS:  $\frac{4\pi}{3} (1)^3 = \boxed{\frac{4\pi}{3}}$  mm<sup>3</sup>

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### Tiebreaker Question

What integer is closest to  $12.8 \times 23.5$ ?

ANS:  $12.8 \times 23.5 = 300.8 = \boxed{301} - .2$

CSU Math Day 2001  
Team Competition  
11:40

1. Adam can frame a house in 15 days, Burl can do it in 8 days, and Clark can do it in 12 days. Working together, how many days would it take for the three of them to frame a house?

ANS:  $\frac{40}{11}$  days

2. Maple is a widely-used computer algebra system. In what country was it developed?

ANS: Canada

3. Archie, Bob, and Claire are now 11, 17, and 19 years old, respectively. In how many years will they again have prime-numbered ages?

ANS: Twelve years from now they will have ages 23, 29, and 31.

4. What is the surface area of a spherical raindrop of diameter 0.5 millimeters?

ANS:  $4\pi (.5/2)^2 =$  $.25\pi$  $\text{mm}^2$

5. For what choices of  $a$  does the polynomial  $ax^2 - 8x + 2$  have two real roots?

ANS:  $a < 8$

6. If 2 is the first prime, what is the seventh prime?

ANS: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

7. George is a 90% free-throw shooter. What is his expected score if he shoots three free throws?

ANS:  $\frac{27}{10}$  or 2.7

8. In an algebra class of 26 students, each student shakes hands with each of the other students exactly once. How many handshakes are there?

ANS: 325

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9. A ladder leans against a vertical wall. The bottom of the ladder is 8 feet from the wall and the top of the ladder is 10 feet above the floor. How long is the ladder?

ANS:  $2\sqrt{41}$  feet

10. What is the distance between two opposite vertices of a cube of edge  $2\sqrt{3}$ ?

ANS: 6

11. After what mathematician was the Cartesian coordinate system named?

ANS: (René) Descartes

12. Write the expression  $\frac{98x+84}{(7x+6)^2}$  in partial fraction form. (ie:  $\frac{A}{Bx+C} + \frac{D}{Ex+F}$ , where  $A, B, C, D, E,$  and  $F$  are integers)

ANS:  $\frac{14}{7x+6}$

13. Marty bought a farm, a house, and a barn for \$459 000. If the house cost 2 times as much as the barn, and the farm 2 times as much as the house and barn together, how much did each cost?

ANS: Barn: 51 000 House: 102 000 Farm: 306 000

14. What is the prime factorization of 127?

ANS: 127 (prime)

15. What is the smallest 4-digit prime?

ANS: 1009

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## Tiebreaker Question

What is the greatest common divisor of 105 and 143?

ANS:  $gcd(105, 143) = \boxed{1}$

CSU Math Day 2001  
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12:00

1. What is the perimeter of a right triangle with legs 8 and 15?

ANS:  $s^2 = 8^2 + 15^2$ ,  $s = 17$ ,  $P = 8 + 15 + 17 = \boxed{40}$

2. John and Jill traded positions several times while rowing a canoe through the Boundary Waters of Minnesota. With Jill at the rear, the canoe went fast enough to complete the entire trip in 10 hours, and with John at the rear the trip would have taken 14 hours. The trip actually took 12 hours. For how many hours did John sit in the back of the canoe?

ANS: 7 hours

3. What fraction is represented by the repeating decimal  $.666\ 666\ 6\ \dots$ ?

ANS:  $\frac{2}{3}$

4. In which quadrant do the two lines  $y = 26 + 2x$  and  $y = 6 - 3x$  intersect?

ANS:  $\left\{ \begin{array}{l} y = 26 + 2x \\ y = 6 - 3x \end{array} \right\}$ , Solution is :  $(-4, 18)$  second quadrant

5. A wooden cube of edge 4 inches is painted red. The cube is then cut into 64 one-inch cubes by making 9 saw cuts. How many of the one-inch cubes have exactly one red face?

ANS:  $6 \text{ faces} \times 4 \text{ cubes per face} = \boxed{24}$  cubes

6. Brigitte plants a pumpkin seed. The area that is covered by the vine doubles every month. After 5 months the entire garden is covered. When was exactly half of the garden covered with the vine?

ANS: One month earlier or after 4 months

7. Two wheels are connected by a belt. One has a diameter of 30 centimeters and a speed of 400 rpm. The other has a speed of 300 rpm. What is its diameter?

ANS: 40

8. How far apart are the two points of intersection of the two curves  $y = x^2 + 1$  and  $y = x + 7$ ?

ANS:  $\left\{ \begin{array}{l} y = x^2 + 1 \\ y = x + 7 \end{array} \right\}$ , Solution is :  $(-2, 5), (3, 10), \sqrt{(3+2)^2 + (10-5)^2} = \boxed{5\sqrt{2}}$

# Team Competition

12:00

9. A circle of diameter 4 contains a circle of diameter 1 in its interior. What is the area contained in the larger circle that is exterior to the smaller circle?

ANS:  $\pi 2^2 - \pi \left(\frac{1}{2}\right)^2 = \boxed{\frac{15\pi}{4}}$

10. What are the dimensions of a rectangle with area 104 and perimeter 42?

ANS: 8 and 13

11. List the following solids in order increasing volume: A sphere of radius 3, a cube of edge 5, and a pyramid whose height is 6 and whose base is a square of edge 8.

ANS:  $\boxed{\text{Sphere}} < \boxed{\text{Cube}} < \boxed{\text{Pyramid}}$  since  $\frac{4}{3}\pi 3^3 = 113.1 < 5^3 = 125 < \frac{1}{3}8^2 \cdot 6 = 128$

12. If  $f(x) = x^2 - 1$ , what is  $f(f(f(-1)))$ ?

ANS: 0

13. Three positive integers have a sum of 10. What is the minimum possible value for the sum of their squares?

ANS:  $3^2 + 3^2 + 4^2 = \boxed{34}$

14. Name a mathematician who made early contributions to differential calculus.

ANS: Pierre  $\boxed{\text{Fermat}}$  or John  $\boxed{\text{Wallis}}$  or Isaac  $\boxed{\text{Newton}}$  or Gottfried  $\boxed{\text{Leibniz}}$  or Isaac  $\boxed{\text{Barrow}}$

15. What is the least common multiple of 79, 100?

ANS: 7900

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## Tiebreaker Question

Which of the following three numbers is largest:  $a = \frac{234}{468}$ ,  $b = \frac{69}{161}$ , or  $c = \frac{175}{315}$ ?

ANS:  $a = \frac{234}{468} = \frac{1}{2}$ ,  $b = \frac{69}{161} = \frac{3}{7}$ ,  $\boxed{c = \frac{175}{315} = \frac{5}{9}}$

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1. Which mathematician amazed his grade school teacher by quickly summing the integers from 1 to 100?

ANS: Gauss

2. Write the next term of the sequence that begins: 4, 10, 18, 28, 40■

ANS: 54

3. What is the cube root of 1 million?

ANS: 100

4. What is the reduced form of the fraction  $\frac{148}{258}$ ?

ANS:  $\frac{74}{129}$

5. How many integers between 101 and 999 are divisible by 3 or by 5?

ANS:  $\frac{999-102}{3} + 1 = 300$ ,  $\frac{995-105}{5} + 1 = 179$ ,  $\frac{990-105}{15} + 1 = 60$ ,  $300 + 179 - 60 = \boxed{419}$

6. Allison scored 66 on the first exam and 58 on the second exam. What must she average on the next two exams to bring her average for the four exams up to 75?

ANS: 88

7. What is the greatest integer in  $\frac{7}{27} + \frac{27}{7}$ ?

ANS:  $\frac{7}{27} + \frac{27}{7} = \frac{778}{189} = 4.11640211640212 \boxed{4}$

8. Express the volume  $V$  of a cube as a function of the area  $A$  of one of its faces.

ANS:  $V = x^3 = (x^2)^{3/2} = A^{3/2}$ , so  $\boxed{V = A^{3/2}}$  or  $\boxed{V = A\sqrt{A}}$

# Team Competition

## 12:20

9. Let  $y = mx + b$  be the image when the line  $3x + 8y + 9 = 0$  is reflected across the  $x$ -axis. What is  $m + b$ ?

ANS:  $\frac{3}{2}$

10. Two 3-digit integers consist of the same digits, but in the reverse order. What is the largest possible difference between the two numbers?

ANS:  $991 - 199 = 981 - 189 = \dots = 901 - 109 = \boxed{792}$

11. Grandma Josephine offers each of her 6 grandchildren the choice of 4 different kinds of cookies. If each grandchild only gets one cookie, in how many ways can the choices be made?

ANS: 4096

12. The graph of the polynomial equation  $y = x^4 - x^3 - 9x^2 + 7x + 6$  crosses the  $x$ -axis at  $x = 3$ . Factor the polynomial  $x^4 - x^3 - 9x^2 + 7x + 6$ .

ANS:  $(x - 3)(x^3 + 2x^2 - 3x - 2)$

13. CAS describes a type of computer software. What do the letters stand for?

ANS: Computer Algebra Systems

14. The equation  $x_{n+1} = \frac{1}{2} \left( x_n + \frac{3}{x_n} \right)$  provides an algorithm for approximating  $\sqrt{3}$ . Starting with  $x_1 = 1$ , what is  $x_3$  (as a rational number)?

ANS:  $x_2 = \frac{1}{2}(1 + 3) = 2$ ,  $x_3 = \frac{1}{2} \left( 2 + \frac{3}{2} \right) = \boxed{\frac{7}{4}} = \boxed{1\frac{3}{4}}$

15. List the following three numbers in increasing order:  $2^8$ ,  $3^5$ ,  $6^3$ .

ANS:  $\boxed{6^3} = 216 < \boxed{3^5} = 243 < \boxed{2^8} = 256$

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### Tiebreaker Question

What is the remainder when 1000 is divided by 11?

ANS:  $1000 \bmod 11 = \boxed{10}$

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1. What is the name of the surface formed by rotating a circle about a line which does not touch the circle?

ANS: A torus

2. An elevator moves at constant speed and it takes 24 seconds for an elevator to go from floor number 1 to floor number 5. How long should it take to go from floor number 1 to floor number 7?

ANS: 36 sec

3. Alpher, Beta, and Gamov are now 7, 11, and 13 years old, respectively. How many years will it be until they again have prime-numbered ages?

ANS: Six years from now they will have ages 13, 17, and 19.

4. According to Descartes' Rule of Signs, how many negative real roots does the polynomial equation  $x^4 - 8x^2 + 6x - 7 = 0$  have?

ANS: one (since there is one sign change if  $x$  is replaced by  $-x$ )

5. A biological brick grows 25% in length, 5% in width, and shrinks in height by 24%. Is it larger or smaller than when it started out?

ANS: smaller

6. In data processing terminology, what does FIFO mean?

ANS: First In First Out

7. What is the sum of the first seven odd positive integers?

ANS:  $1 + 3 + 5 + 7 + 9 + 11 + 13 = \text{49} = 7^2$

8. Factor completely the integer  $2^5 + 3^5$ .

ANS:  $2^5 + 3^5 = 275 = \text{5^2 11}$

# Team Competition

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9. How many committees of 4 men and 2 women can be formed from a group of 5 men and 7 women?

ANS: 105

10. An orange has a diameter that is 75% fruit and 25% peel. To the nearest percent, what percentage of the volume is the peel?

ANS: 58% ( $\approx 57.8125\%$ )

11. A triangle has 3 vertices and 3 edges. A tetrahedron has 4 vertices and 6 edges. How many vertices and edges does an  $n$ -dimensional tetrahedron have?

ANS:  $n + 1$  vertices and  $n(n + 1)/2$  edges

12. How many 2-person groups can be made out of a group of 2 people?

ANS: 1

13. A wooden cube of edge 4 inches is painted red. The cube is then cut into 64 one-inch cubes by making 9 saw cuts. How many of the one-inch cubes have exactly 2 red faces?

ANS: 12 edges  $\times$  2 cubes per edge = 24 cubes

14. What is the vertex of the parabola with equation  $y = x^2 - 8x + 11$ ?

ANS:  $(4, -5)$  since  $y = x^2 - 8x + 11 = (x - 4)^2 - 5$

15. A multiple-choice exam has 15 questions with 3 choices per question. If you answer the questions randomly, what is the expected number of correct responses?

ANS: 5

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Tiebreaker Question

Factor 1001 as a product of primes.

ANS:  $1001 = 7 \times 11 \times 13$

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1. What is the area of a regular octagon of edge 1?

ANS: Large  $\square$  - four corners =  $\left(1 + 2\left(\frac{1}{\sqrt{2}}\right)\right)^2 - 4\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)^2 = \boxed{2 + 2\sqrt{2}}$

2. How far apart are the two points at which the curves  $x^2 + y^2 = 169$  and  $x = 5$  intersect?

ANS:  $\left\{ \begin{array}{l} x = 5 \\ x^2 + y^2 = 169 \end{array} \right\}$ , Points (5, 12) and (5, -12), distance  $\boxed{24}$

3. The set S has 7 elements. If  $a$  and  $b$  are (distinct) elements of S, how many subsets of S contain both  $a$  and  $b$ ?

ANS:  $32 = 2^5$

4. In a river that flows at  $\frac{8}{3}$  miles per hour, it takes a certain swimmer 5 times as long to swim a certain distance up stream as it does to swim the same distance down stream. What is her speed in still water?

ANS: 4

5. The Center Ring Janitorial Supply owns a fleet of 3 vehicles: A car, which gets 40 miles per gallon, a van, which gets 25 miles per gallon, and a truck, which gets 10 miles per gallon. If in a typical week the car is driven 160 miles, the van is driven 275 miles and the truck is driven 100 miles, how many miles per gallon is Center Ring's fleet getting?

ANS:  $\frac{107}{5} = 21\frac{2}{5}$  miles per gallon

6. Andrew Wiles has proven something that had remained an open problem for over 350 years. Either state the problem or name the person who claimed over 350 years ago to have a solution to the problem.

ANS:  $x^n + y^n = z^n$  has no positive integer solutions for  $n > 2$  OR Fermat

7. Bo is going to the store to buy candy that will cost somewhere between 5 cents and 26 cents. What is the fewest number of coins Bo can carry in order to be certain to have exact change to buy the candy?

ANS: 1¢, 1¢, 1¢, 1¢, 5¢, 10¢, 10¢  $\boxed{7}$  coins

8. How many primes are there between 90 and 100?

ANS: One (it is 97)

# Team Competition

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9. Pierre de Fermat was not a professional mathematician, although such famous results as Fermat's Last Theorem and Fermat's Little Theorem were named in his honor. What was his profession?

ANS: Lawyer

10. What is the sum of the binomial coefficients (5 choose 0) plus (5 choose 1) plus ■ plus (5 choose 5)?

ANS:  $32 = 2^5$

11. What is the statement of Fermat's Last Theorem?

ANS: The equation  $x^n + y^n = z^n$  has no positive integer solutions for  $n > 2$ .

12. What is the smallest value of the expression  $2x + \frac{1}{2x}$  if  $x$  is a positive real number?

ANS:  $\left[2x + \frac{1}{2x}\right]_{x=1/2} = \boxed{2}$

13. A Spanish port was protected by large cannons, and each cannon had a pile of cannon balls nearby stacked neatly in the shape of a tetrahedron. If each bottom layer contained a total of 21 cannon balls, how many cannon balls were in each pile?

ANS:  $21 + 15 + 10 + 6 + 3 + 1 = \boxed{56}$

14. A punch bowl at a Halloween party is the shape of a truncated cone of height 8 inches, base diameter 12 inches, and top diameter 12 inches. The orange plastic punch cups are also truncated cones of height 2 inches, base diameter 3 inches, and top diameter 3 inches. How many cups of punch will the punch bowl hold?

ANS: 64

15. A 6-foot man casts a 12-foot shadow. A flag pole next to him casts a 59-foot shadow. How tall is the flag pole?

ANS:  $\frac{59}{2}$  ft

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## Tiebreaker Question

What is the remainder when 1 000 000 is divided by 3?

ANS:  $1\,000\,000 \bmod 3 = \boxed{1}$

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1. What is the radius of the sphere whose volume is 19 times its surface area?

ANS: 57

2. If 84 centimeters of wire are used to build the skeleton of edges of a cube, what is the volume of the cube?

ANS: 343

3. What is the volume of the tetrahedron whose vertices are  $(0, 0, 0)$ ,  $(1, 0, 0)$ ,  $(0, 1, 0)$ , and  $(0, 0, 1)$ ?

ANS:  $1/6$

4. Give the prime factorization of the smallest integer divisible by 9, 4, 2, and 1.

ANS:  $2^23^2$

5. In 1962 a mathematician named Edward O. Thorp wrote a book entitled *Beat the Dealer* that claimed to give a winning strategy, based upon large-scale computer simulations, for a certain card game. What is the name of that card game?

ANS: Blackjack or 21

6. A computer sequentially computes integers by the following rule: If  $n$  is odd then replace  $n$  by  $3n + 1$ ; otherwise replace  $n$  by  $n/2$ . If  $n$  starts at 7, what is  $n$  after 5 iterations?

ANS:  $7 \rightarrow 22 \rightarrow 11 \rightarrow 34 \rightarrow 17 \rightarrow \boxed{52}$

7. How many rearrangements of the letters  $a, b, c, d, e, f$  have  $a$  listed before  $b$  and  $b$  listed before  $c$ ?

ANS:  $\frac{6!}{3!} = \boxed{120}$

8. List the following three numbers in increasing order:  $e^\pi$ ,  $\pi^e$ ,  $3^3$ .

ANS:  $\boxed{\pi^e} = 22.459 < \boxed{e^\pi} = 23.141 < \boxed{3^3} = 27$

# Team Competition

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9. A city lot is twice as long as it is wide. By increasing its length 20 yards and its width 30 yards, the area will be increased by 2200 square yards. What are its dimensions?

ANS:  $(2x + 20)(x + 30) - 2x^2 = 2200$ , 20 yards by 40 yards

10. A triangle has sides of length 8, 5, and 5. Find the area of the triangle.

ANS: Altitude is 3, so  $Area = \frac{1}{2} (8) (3) =$ 12

11. What are the next two prime numbers greater than 50?

ANS:  $51 = 3 \times 17$ , 53,  $55 = 5 \times 11$ ,  $57 = 3 \times 19$ , 59

12. A shelf will hold 20 calculus textbooks and 24 algebra textbooks, or 15 calculus textbooks and 36 algebra textbooks. How many calculus books alone will the shelf hold?

ANS:  $\left\{ \begin{array}{l} 20c + 24a = 1 \\ 15c + 36a = 1 \end{array} \right\}$ ,  $a = \frac{1}{72}$ ,  $c = \frac{1}{30}$  Shelf will hold 30 calculus books

13. If  $g$  is a function such that  $g(1) = 2$ ,  $g(2) = 1$ , and

$$g(n + 1) = g(n) - 2g(n - 1)$$

for  $n \geq 3$ , what is  $g(4)$ ?

ANS:  $g(3) = g(2) - 2g(1) = 1 - 4 = -3$ ,  $g(4) = g(3) - 2g(2) = -3 - 2 =$ -5

14. In how many ways can ALLAN (spelled A-L-L-A-N) misspell his name, assuming he uses all the right letters (the right number of times)?

ANS:  $\frac{5!}{2!2!} - 1 =$ 29

15. Two baseball teams, the Rockies and the Cardinals, begin a 7-game series. The odds makers give the green Rockies 8 to 1 odds of winning the first game. Assuming the odds makers are good at their job, what is the probability that the Rockies will win the first game?

ANS:  $\frac{8}{9}$

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## Tiebreaker Question

What is the least common multiple of 21 and 37?

ANS:  $\text{lcm}(21, 37) =$ 777

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1. How far is the point  $(3,0)$  from the line  $3y = 4x$ ?

ANS:  $\frac{x}{3} = \frac{4}{5} \implies x = \boxed{\frac{12}{5}} = \boxed{2.4}$

2. The sum of two integers is 20 and their product is 99. What are the two integers?

ANS: 11 and 9.

3. Find  $x$  so that the average of the four numbers 46, 31, 37, and  $x$  is 37.

ANS:  $x = 34$

4. In how many ways can you have \$10 worth of dimes and quarters?

ANS: 21

5. A triangular section of Old Town is divided into a smaller triangle and two trapezoids by two streets parallel to one of the boundary streets. The heights of the two trapezoids are equal to the height of the small triangle, and the area of the middle trapezoid is 12 acres. How many acres are there in the larger trapezoid?

ANS:  $\boxed{20}$  acres

6. What is the area of a right triangle having a leg of length  $\ell$  and hypotenuse of length 25?

ANS:  $A = \frac{\ell\sqrt{25-\ell^2}}{2}$

7. You are given seven points in the plane, no three of which lie on the same line. How many lines are there which pass through exactly two of these points?

ANS:  $\binom{7}{2} = \boxed{21}$

8. A bag of Halloween candy has 5 pieces of chocolate and 5 pieces of fruit bar. What is the probability that 3 items selected at random are chocolate?

ANS:  $\frac{1}{12}$

# Team Competition

## 1:35

9. What number is halfway between  $\frac{1}{4}$  and  $\frac{2}{9}$ ?

ANS:  $\frac{17}{72}$

10. What is the smallest positive integer with exactly 5 positive integer divisors?

ANS: 16 (divisors are  $\{1, 2, 4, 8, 16\}$ )

11. If 2 cards are drawn from a standard deck of 52 cards, what is the probability that one is a heart and the other is a diamond?

ANS:  $\frac{\binom{13}{1}\binom{13}{1}}{\binom{52}{2}} = \frac{13}{102}$

12. If eggs weigh 1.5 oz each and a dozen eggs cost 90¢, what is the cost of a pound of eggs? (16oz = 1 pound)

ANS: 80¢

13. Five straight lines are drawn in the plane. What is the largest possible number of points of intersection?

ANS:  $\binom{5}{2} = 10$

14. What is the diameter of a circle with area  $289\pi$  cm<sup>2</sup>?

ANS: 34

15. Sarah averages 50% on multiple choice exams. What is the probability that she gets at least 2 correct on a 3-question exam?

ANS:  $\left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) = \frac{1}{2}$

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### Tiebreaker Question

What is the greatest common divisor of 165 and 147?

ANS:  $gcd(165, 147) = 3$

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1. How many distinct complex roots does the polynomial  $x^5 - 1$  have?

ANS: 5

2. If two cards are drawn from a standard deck of 52 cards, what is the probability that both are kings?

ANS:  $\frac{\binom{4}{2}}{\binom{52}{2}} = \frac{6}{1326} = \boxed{\frac{1}{221}}$

3. The 4 faces of a regular tetrahedron are equilateral triangles. How many edges does a regular tetrahedron have?

ANS: 6

4. What did the Norwegian mathematician Niels Henrik Abel prove about general fifth-degree polynomials?

ANS: Cannot be solved in terms of radicals involving the coefficients

5. At 6:00 AM, Chad starts jogging at 5 km/h. At 6:00 AM Marzelle starts jogging from Chad's starting place at 7 km/h. How far behind is Marzelle at 11:00 AM?

ANS: 11 km

6. Who wrote the monumental *Principia mathematica*?

ANS: Bertrand Russell and Alfred North Whitehead

7. The 12 faces of a regular dodecahedron are pentagons. How many vertices does a regular dodecahedron have?

ANS:  $\frac{12 \times 5}{3} = \boxed{20}$

8. How many primes are there between 100 and 110?

ANS: Four (101, 103, 107, 109)

# Team Competition

1:50

9. The age of a mathematician and her son added together is 62. In 3 years the mother will be 3 times as old as her son. How old is the mathematician?

ANS: 48

10. A diagonal of a rectangle is  $\sqrt{149}$  and the area of the rectangle is 70. What are the dimensions of the rectangle?

ANS: 7 by 10

11. CAD describes a type of computer software. What do the letters stand for?

ANS: Computer-Aided Design

12. What is the area of one face of a diamond-shaped kite whose two sticks are 20 and 50 inches long?

ANS: 500

13. A triangle has sides of length 13, 10, and 13. Find the area of the triangle.

ANS: Altitude is 12, so  $Area = \frac{1}{2} (10) (12) = \boxed{60}$

14. If  $m > 0$  and the points  $(m, 9)$  and  $(1, m)$  lie on a line with slope  $m$ , find  $m$ .

ANS:  $m = 3$

15. What is the converse of the statement, "If roses are red, then violets are blue"?

ANS: If violets are blue, then roses are red.

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## Tiebreaker Question

What is the slope of the line perpendicular to the line  $7x - 4y + 3 = 0$ ?

ANS:  $7x - 4y + 3 = 0$ , Solution is :  $y = \frac{7}{4}x + \frac{3}{4}$ ,  $\perp$  slope is  $\boxed{-\frac{4}{7}}$

CSU Math Day 2001  
Team Competition  
2:05

1. If a mantel clock strikes the hours, how many times will it strike during a 24-hour period?

ANS:  $2 \sum_{i=1}^{12} i = \boxed{156}$

2.  $\frac{1}{3}$  of the air in a container is removed by each cycle of an air pump. What fractional part of the air remains after 4 cycles?

ANS:  $\frac{16}{81}$

3. What is the perimeter of an isosceles triangle with base 16 and area 48?

ANS: 36

4. What can you say about a parallelogram whose diagonals have equal length?

ANS: It is a rectangle.

5. How many vertices does an  $n$ -dimensional cube have?

ANS:  $2^n$

6. A square is inscribed in a circle, which in turn is inscribed in a square. What percentage of the area of the large square is inside the small square?

ANS: 50%

7. A shirt has been marked down 5% to \$42.75. What was the original price?

ANS: \$45

8. What complex numbers are equal to their squares?

ANS:  $x = x^2$ , Solution is:  $x = \boxed{0}$ ,  $x = \boxed{1}$

# Team Competition

2:05

9. What is the smallest positive integer with exactly 6 positive divisors?

ANS: 12 (divisors are  $\{1, 2, 3, 4, 6, 12\}$ )

10. The number of times a pendulum oscillates in a given time varies inversely as the square root of its length. If a 40 inch pendulum oscillates once per second, what is the length of a pendulum that oscillates twice each second?

ANS:  $\boxed{10}$  in

11. An icosahedron has 20 faces and 12 vertices. A diagonal of such a solid is a line segment joining two vertices not lying in the same face. How many diagonals are there?

ANS:  $\frac{6 \times 12}{2} = \boxed{36}$  Each vertex connected to 5 other vertices, so each vertex lies on 6 diagonals.

12. A piece of string 64 inches long is cut into two pieces so that one piece is 6 inches shorter than the other. What are the lengths of the two pieces?

ANS:  $x + (x + 6) = 64 \implies$ , Solution is :  $x = \boxed{29}$  in,  $x + 6 = \boxed{35}$  in

13. A golf bag contains 13 balls, some yellow and the rest orange. Two balls are drawn at random from the bag and one is yellow, the other orange. What is the probability that exactly half of the balls in the bag are yellow?

ANS: 0

14. What was the license plate number the Indian mathematician Ramanujan referred to when he said, "On the contrary, that's a very interesting number. It's the first number that can be written as the sum of 2 cubes in 2 different ways."

ANS: 1729 ( $= 12^3 + 1^3 = 10^3 + 9^3$ )

15. If a right triangle has hypotenuse  $c$  and legs  $a$  and  $b$ , then  $c^2 = a^2 + b^2$ . After whom is this result named?

ANS: Pythagoras

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## Tiebreaker Question

Factor 2222 as a product of primes.

ANS:  $2222 = \boxed{2 \times 11 \times 101}$

CSU Math Day 2001  
Team Competition  
2:15

1. When this Hungarian mathematician died recently, the New York Times acknowledged his contributions to mathematics that included some 1500 published papers. Name this person.

ANS: Paul Erdős

2. If a stone falls 64 feet in 2 seconds, how far will it fall in 5 seconds?

ANS:  $16 \cdot 5^2 = \boxed{400}$  ft

3. What cubic curve was named after Maria Agnesi?

ANS: Witch of Agnesi

4. Find a point equidistant from the points  $(-2, -2)$ ,  $(2, 2)$ , and  $(2, -2)$ .

ANS:  $(0, 0)$

5. A computer sequentially computes integers by the following rule: If  $n$  is odd then replace  $n$  by  $3n + 1$ ; otherwise replace  $n$  by  $n/2$ . If  $n$  starts at 3, what is  $n$  after 5 iterations?

ANS:  $3 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow \boxed{4}$

6. Bo is going to the store to buy candy that will cost somewhere between 40 cents and 54 cents. What is the fewest number of coins Bo can carry in order to be certain to have exact change to buy the candy?

ANS:  $1\text{¢}, 1\text{¢}, 1\text{¢}, 1\text{¢}, 5\text{¢}, 10\text{¢}, 10\text{¢}, 25\text{¢}$   $\boxed{8}$  coins

7. How many minutes were there in October of this year?

ANS:  $31 \times 24 \times 60 = \boxed{44640}$  minutes

8. The quartic polynomial  $6x^4 - 11x^3 - 66x^2 + 59x - 12$  has four real roots. What is the product of these four roots?

ANS:  $6(x + 3)(x - 4)(x - \frac{1}{2})(x - \frac{1}{3}), -3 \cdot 4 \cdot \frac{1}{2} \cdot \frac{1}{3} = \boxed{-2}$

# Team Competition

## 2:15

9. What is the greatest integer in the sum  $\frac{19}{5} + \frac{5}{19}$ ?

ANS:  $\frac{19}{5} + \frac{5}{19} = 4.06315789473684 = \boxed{4} + .06315789473684$

10. What mathematician first resolved the Königsberg bridge problem?

ANS: Euler

11. The compact disk UR2ugly sells at outlet AC for \$15.49 less a discount of 20%, and at outlet DC for \$16.95 less a discount of 30%. Which outlet has the lower price?

ANS: AC,  $15.49 \times .8 = \$12.39$ ;  $\boxed{\text{DC}}$ ,  $16.95 \times .7 = \$11.86$

12. Sarah averages 50% on multiple choice exams. What is the probability that she gets at least 3 correct on a 5-question exam?

ANS:  $(\frac{1}{2})^5 + 5(\frac{1}{2})^4(\frac{1}{2}) + 10(\frac{1}{2})^3(\frac{1}{2})^2 = \boxed{\frac{1}{2}}$

13. A pair of dice is rolled 50 times. What is the expected number of times the total is 9?

ANS:  $\frac{50}{9}$

14. What is the smallest positive integer  $n$  such that  $n^2 - n + 41$  is not prime?

ANS: 41, since

$n$	1	2	3	4	5	...	40	$\boxed{41}$
$n^2 - n + 41$	41	43	47	53	61	...	1601	$1681 = 41^2$

15. What is the greatest common divisor of 130, 126?

ANS: 2

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### Tiebreaker Question

How many faces does an octahedron have?

ANS:  $\boxed{8}$

CSU Math Day 2001  
Team Competition  
Small School Final

1. The compact disk UR2gly sells at outlet AC for \$12.95 less a discount of 15%, and at outlet DC for \$14.65 less a discount of 25%. Which outlet has the lower price?

ANS:  $AC = 12.95 \times .85 = 11.0075$ ,  $\boxed{DC} = 14.65 \times .75 = 10.9875$

2. If the line  $y = mx$  touches the curve  $y = x^2 + 1$  in exactly 1 point, what are the possibilities for  $m$ ?

ANS:  $m = \boxed{\pm 2}$

3. Use the approximation  $2^{10} \approx 10^3$  to approximate  $2^{43}$  as a number in scientific notation  $2^{43} \approx c \times 10^n$ , where  $n$  is an integer and  $c$  is a number between 1 and 10.

ANS:  $2^{43} = 8 \times (2^{10})^4 \approx 8 \times (10^3)^4 = \boxed{8 \times 10^{12}} = 8000000000000$

4. A basketball team has 12 players on the roster. How many different starting lineups are possible?

ANS:  $\binom{12}{5} = \boxed{792}$

5. Three line segments joining the midpoints of the sides of a triangle determine a smaller triangle whose sides lie inside the larger triangle. What is the ratio of the area of the larger triangle to the area of the smaller triangle?

ANS: 4 : 1

6. Which is largest;  $\pi$ ,  $355/113$ , or 3.1416?

ANS:  $355/113 \approx 3.14159292$ ,  $\pi \approx 3.141592654$ ,  $\boxed{3.1416}$

7. Completely factor the polynomial  $2x^3 - x^2 - 26x + 40$ .

ANS:  $(2x - 5)(x - 2)(x + 4)$

8. Find all the roots of the equation  $x^3 - x^2 - 72x - 180 = 0$  ?

ANS: 10, -3, -6.

# Team Competition

## Small School Final

9. The average of your first 6 exams is 82. What is your new average if your lowest exam score of 57 is dropped?

ANS: 87

10. Express the perimeter  $P$  of a square as a function of its area  $A$ .

ANS:  $A = \left(\frac{P}{4}\right)^2$ , Solution is :  $P = 4\sqrt{A}$

11. Who designed, but never build, an early “analytic engine” that led eventually to the Automatic Sequence Controlled Calculator developed jointly by IBM and Harvard University in 1944?

ANS: Charles Babbage

12. A computer sequentially computes integers by the following rule: If  $n$  is a square then multiply by 2; otherwise subtract 3. Starting at  $n = 12$ , what is the integer after 6 iterations?

ANS:  $12 \rightarrow 9 \rightarrow 18 \rightarrow 15 \rightarrow 12 \rightarrow 9 \rightarrow \boxed{18}$

13. A contemporary, prolific, Hungarian mathematician is reputed to have said, “A mathematician is a machine for converting coffee into theorems.” Name this mathematician.

ANS: Paul Erdős

14. Find all positive roots of the equation  $x^4 - 5x^2 + 4 = 0$ .

ANS:  $x^4 - 5x^2 + 4 = 0$ , Solution is :  $x = \boxed{1}, x = \boxed{2}, x = -2, x = -1$

15. To the nearest minute, at what time between 9:30 a.m. and 10:00 a.m. are the minute hand and the hour hand at right angles?

ANS:  $45 + \frac{m}{12} = m + 15$ , Solution is :  $m = \frac{360}{11} : 32.7272727272727 \boxed{9:33}$  a.m.

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### Tiebreaker Question

Which of the following three numbers is smallest:  $e^\pi$ ,  $\pi^e$ , or  $3^3$ ?

ANS:  $e^\pi = 23.14069264$ ,  $\boxed{\pi^e = 22.45915771}$ ,  $3^3 = 27$

CSU Math Day 2001  
Team Competition  
Small School Final (Round 2)

1. The formula  $e^{i\pi} + 1 = 0$  relates five of the most popular numbers in mathematics. What is  $e$  rounded to 6 significant digits?

ANS:  $e \approx 2.718281828 \approx \boxed{2.71828}$

2. How many ways can the top 3 finishers be picked in a(n) 9-person race?

ANS: 504

3. A spherical balloon's diameter increases by 30%. By what percentage does the surface area change?

ANS: 69%

4. What is the coefficient of  $x^7$  in the expansion of  $(x - 1)^8$ ?

ANS:  $-8$

5. Give the points of intersection of the two curves  $y = 20 - 6x$  and  $y = 8 - 6x + 3x^2$ .

ANS:  $\left\{ \begin{array}{l} y = 8 - 6x + 3x^2 \\ y = 20 - 6x \end{array} \right\}$ , Solution is :  $\boxed{(2, 8), (-2, 32)}$

6. Find all the integer solutions of the equation  $x^4 + 2x^2 - 3 = 0$ .

ANS:  $x = 1, -1$  since  $x^4 + 2x^2 - 3 = (x - 1)(x + 1)(x^2 + 3)$

7. A baseball manager has selected 9 starters for a game. If the pitcher must bat last and the second baseman must bat first, how many different batting line-ups are possible?

ANS:  $7! = \boxed{5040}$

8. Two circles are mutually tangent at one point, and the smaller circle passes through the center of the larger circle. What is the ratio between the circumferences of the two circles?

ANS:  $\boxed{2 : 1}$  or  $\boxed{1 : 2}$

# Team Competition

## Small School Final (Round 2)

9. Where does the circle of radius 5 centered at the origin intersect the line passing through the origin with slope  $-3/4$ ?

ANS:  $\left\{ \begin{array}{l} x^2 + y^2 = 25 \\ y = -\frac{3}{4}x \end{array} \right\}$ , Solution is :  $\boxed{\{y = 3, x = -4\}, \{y = -3, x = 4\}}$

10. If  $f(x) = 3x - 5$ , what is  $f(f(\frac{1}{2}))$ ?

ANS:  $-20$

11. A regular icosahedron has 20 faces, each of which is an equilateral triangle. If the midpoint of each face is connected with an edge to the midpoint of each adjacent face, what solid do these new edges determine?

ANS: Dodecahedron (12 faces, each face is a pentagon.)

12. A pair of dice is rolled. What is the probability that the total is 8 or 6?

ANS:  $\frac{5}{18}$

13. According to the Rational Root Theorem, what are all the possible rational roots of the polynomial  $2x^3 - 5x^2 - 11x - 4$ ?

ANS:  $\pm\frac{1}{2}, \pm 1, \pm 2, \pm 4$

14. If the surface area of a cube is equal to 150, what is its volume?

ANS: 125

15. What is the highest power of 10 dividing  $20!$ ?

ANS:  $20! = \underline{20} \cdot 19 \cdots \underline{15} \cdots \underline{10} \cdots \underline{5} \cdots 2 \cdot 1 = 2432\ 90200\ 81766\ 40000$  so  $\boxed{4}$  or  $\boxed{10^4}$

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### Tiebreaker Question

Of all the rational numbers strictly between  $\frac{2}{3}$  and  $\frac{3}{4}$ , which rational number has the smallest denominator?

$$\frac{2}{3} < \frac{2+3}{3+4} = \boxed{\frac{5}{7}} < \frac{3}{4}$$

CSU Math Day 2001  
Team Competition  
Large School Final

1. On a certain planet, a stone falls 45 feet in 3 seconds, how far will it fall in 8 seconds?

ANS: 320

2. Two positive integers have a sum of 9. What is the smallest possible value for the sum of their cubes?

ANS:  $4^3 + 5^3 = \boxed{189}$

3. John and Jill traded positions several times while rowing a canoe through the Boundary Waters of Minnesota. With Jill at the rear, the canoe went fast enough to complete the entire trip in 10 hours, and with John at the rear the trip would have taken 14 hours. The trip actually took 12 hours. For how many hours did Jill sit in the back of the canoe?

ANS:  $1 = \frac{1}{10}t + \frac{1}{14}(12 - t)$ , Solution is:  $t = \boxed{5 \text{ hours}}$

4. How far apart are the two points with the curves  $y = x + 6$  and  $y = x^2$  intersect?

ANS:  $\left\{ \begin{array}{l} y = x + 6 \\ y = x^2 \end{array} \right\}$ , Solution is :  $\{(-2, 4), (3, 9)\}$ , distance  $\boxed{5\sqrt{2}}$

5. A pair of dice is rolled. What is the probability that the total is 5?

ANS:  $\frac{1}{9}$

6. What society first developed the concept of zero?

ANS: The Hindus

7. If the first term of a geometric sequence is  $\frac{1}{4}$  and the second term is  $\frac{3}{20}$ , what is the fifth term?

ANS:  $\frac{81}{2500}$

8. From what language was the term 'algebra' derived?

ANS: Arabic

# Team Competition

## Large School Final

9. How many different 6-place license plates are possible if the first two places are letters and the last 4 places are digits?

ANS:  $26^2 \times 10^4 = \boxed{6,760,000}$

10. The sum of 5 consecutive integers is 60. What is the largest of the 5 integers?

ANS: 14

11. What is the midpoint of the line segment joint  $(12, -6)$  and  $(2, -4)$ ?

ANS:  $(7, -5)$

12. How many different 6-letter words can be formed by rearranging the letters in SCHOOL?

ANS:  $\frac{6!}{2!} = \boxed{360}$

13. Two baseball teams, the Green Socks and the Yellow Elbows, start a 7-game series. For each game, the odds are 9 to 4 in favor of the Green Socks. What is the probability that the Green Socks sweep the series in four games?

ANS:  $\frac{6561}{28561}$  or .22971885

14. If  $g$  is a function such that  $g(1) = 3$ ,  $g(2) = -1$ , and

$$g(n+1) = g(n) + 2g(n-1)$$

for  $n \geq 3$ , what is  $g(4)$ ?

ANS:  $g(3) = g(2) + 2g(1) = -1 + 6 = 5$ ,  $g(4) = g(3) + 2g(2) = 5 - 2 = \boxed{3}$

15. If a single 60-Watt bulb provides sufficient light to read a newspaper 2 feet from the bulb, how many 60-Watt bulbs are required in a light fixture 10 feet from the newspaper in order to provide the same apparent level of brightness?

ANS: 25

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### Tiebreaker Question

Factor 123 as a product of primes.

ANS:  $123 = \boxed{3 \times 41}$

CSU Math Day 2001  
Team Competition  
Large School Final (Round 2)

1. If the absolute value of  $x^2 + 4$  is equal to the absolute value of  $x^2 - 6$ , what is  $x$ ?

ANS:  $|x^2 + 4| = |x^2 - 6|$ , Solution is :  $\boxed{x = 1, x = -1}$

2. Two sides of a nontrivial triangle have lengths 4 and 14. What is the smallest integer length for the third side?

ANS: 11

3. Let S be the set of the first 6 natural numbers. How many of the 64 subsets of S contain the number 2?

ANS: 32 or Half

4. What is the volume of a sphere of surface area  $400\pi$ ?

ANS:  $\frac{4000}{3}\pi$

5. A ball thrown vertically into the air 100 feet, falls and rebounds to a height of 40 feet the first time, rebounds to 16 feet on the second bounce, and so forth. What is the entire distance the ball will have moved when it finally comes to rest?

ANS:  $2 \sum_{i=0}^{\infty} 100 \cdot \left(\frac{2}{5}\right)^i = \frac{1000}{3} = \boxed{333\frac{1}{3}}$  ft

6. If a 100-Watt sound system can break a glass goblet placed 2 feet away from the speaker, how powerful a sound system would it take to break a similar glass goblet placed 4 feet from the speaker?

ANS: 400

7. If  $f(x)$  is a linear function and  $f(-2) = 9$  and  $f(-4) = 19$ , what is  $f(3)$ ?

ANS:  $-16$

8. What is the volume of the largest cube that can fit inside a sphere of radius  $\sqrt{3}$ ?

ANS:  $Volume = 2^3 = \boxed{8}$

# Team Competition

## Large School Final (Round 2)

9. The 12 faces of a regular dodecahedron are pentagons. How many edges does a regular dodecahedron have?

ANS:  $\frac{12 \times 5}{2} = \boxed{30}$

10. Given 6 gallons of a 20% antifreeze/water mixture, how much pure antifreeze must be added to yield a 70% antifreeze/water mixture?

ANS: 10 gallons

11. If 2 is the first prime, what is the fifteenth prime?

ANS: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43,  $\boxed{47}$

12. What is the 15th term of the arithmetic sequence that begins  $-17, -27, -37$ ?

ANS:  $-157$

13. The diagonal of a table with a square top is 20 feet. What is the area of the table top?

ANS:  $200 \text{ ft}^2$

14. If a cube has a volume of  $64 \text{ cm}^3$ , what is its surface area?

ANS:  $96 \text{ cm}^2$

15. An ant and a drop of honey are at diagonally opposite corners of a rectangular box of dimensions 1 by 2 by 3. What is the shortest distance the ant can travel to reach the (stationary) drop of honey by crawling along walls?

ANS:  $\sqrt{3^2 + 3^2} = \sqrt{18} = \boxed{3\sqrt{2}}$

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### Tiebreaker Question

Which of the following three numbers is the largest:  $8^{-10}$ ,  $9^{-9}$ ,  $10^{-8}$ ?

ANS:  $8^{-10} = \frac{1}{1073741824} < 9^{-9} = \frac{1}{387420489} < \boxed{10^{-8} = \frac{1}{100000000}}$