CSU Math Day 2000
Team Competition
11:20

1. What is the prime factorization of $4! = 24$?

   ANS: $\boxed{2^3 3}$

2. Write the next term of the sequence that begins: 4, 10, 18, 28, 40, ...

   ANS: $\boxed{54}$

3. What is the length of the arc on a circle of radius 10 subtended by a central angle of $180\,^\circ$?

   ANS: $\boxed{10\pi}$

4. How far apart are the two points where the curves $y = x + 6$ and $y = x^2$ intersect?

   ANS: $\left\{ \begin{array}{c} y = x + 6 \\ y = x^2 \end{array} \right\}$, Solution is : $\{(-2, 4), (3, 9)\}$, distance $\boxed{5\sqrt{2}}$

5. An equilateral triangle has vertices at $(0, 0)$ and $(12, 0)$. Give one set of possible coordinates for the third vertex.

   ANS: $\boxed{(6, 6\sqrt{3})}$ or $\boxed{(6, -6\sqrt{3})}$

6. Ann keeps flies and spiders in a box in her dorm room during the Halloween season. There are a total of 17 creatures with 124 legs. How many flies and how many spiders does she have? (Flies have 6 legs and spiders have 8)

   ANS: $\boxed{6}$ flies and $\boxed{11}$ spiders.

7. What is the contrapositive of the statement, "If $x > 4$ then $x^2 > 16$"?

   ANS: $\boxed{\text{If } x^2 \leq 16, \text{ then } x \leq 4.}$

8. It takes 1000 square tiles to tile a room, or 1440 smaller tiles whose edge is 1 inch less. How large is the room in square feet?

   ANS: $1000x^2 = 1440\left(x - \frac{1}{12}\right)^2$, Solution is : $x = \frac{1}{2}$, $1000\left(\frac{1}{2}\right)^2 = \boxed{250}$ ft^2

Team Competition

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9. What is the area of an equilateral triangle inscribed in a circle of radius 6?

   ANS: $\boxed{27\sqrt{3}}$

10. What is the equation of the line in the xy plane all of whose points are equidistant from the two points $(1, 1)$ and $(-1, -1)$?

    ANS: $\boxed{y = -x}$ or $\boxed{x + y = 0}$

11. What is the greatest common divisor of 228, 452?

    ANS: $\boxed{4}$

12. Expand $(3x^3 + 3y^3)^3$

    ANS: $\boxed{27x^9 + 81x^6y^3 + 81x^3y^6 + 27y^9}$

13. Use the approximation $2^{10} \approx 10^3$ to estimate $\log_2(10^{42})$.

    ANS: $\log_2(10^{42}) = \log_2\left((10^3)^{14}\right) \approx \log_2\left((2^{10})^{14}\right) = \log_2(2^{140}) = \boxed{140}$

14. Two circles are mutually tangent at one point, and the smaller circle passes through the center of the larger circle. What is the ratio between the circumferences of the two circles?

    ANS: $\boxed{2:1}$ or $\boxed{1:2}$

15. If 2 is the first prime, what is the seventh prime?

    ANS: $2, 3, 5, 7, 11, 13, \boxed{17}, 19, 23, 29$

Tiebreaker Question:

Name the largest integer whose 5th power is < 1000.
ANS: $\boxed{3}$ $3^5 = 243$, $4^5 = 1024$

CSU Math Day 2000
Team Competition
11:40

1. In the early 1900s a self-taught Indian mathematician sent some of his unusual mathematical formulas to the English mathematician G. H. Hardy, who recognized the depth of the formulas and invited this mathematician to England. This exceptional Indian mathematician was the subject of a public television documentary. A biography, *The man Who Knew Infinity*, was published in 1991. Name this mathematician.

   ANS: Srinivasa $\boxed{\text{Ramanujan}}$ (RA·MA·NU′JUN)

2. What is the area of a regular octagon of edge 1?

   ANS: Large $\square-$ four corners $= \left(1+2\left(\frac{1}{\sqrt{2}}\right)\right)^2 - 4\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)^2 = \boxed{2+2\sqrt{2}}$

3. George Boole (BOOL) developed a mathematical system called Boolean algebra. In this algebra, OR is denoted by "plus". What is the value of $1+1$ under this system?

   ANS: $1+1=\boxed{1}$

4. What is the perimeter of an isosceles triangle with base 16 and area 48?

   ANS: 36

5. What is the product of the next two primes numbers greater than 25?

   ANS: $29 \times 31 = \boxed{899} = 30^2 - 1^2$

6. Allison scored 78 on the first exam and 74 on the second exam. What must she average on the next two exams to bring her average for the four exams up to 85?

   ANS: 94

7. What is the volume of a sphere of surface area 400π?

   ANS: $\frac{4000}{3}\pi$

8. Find C so that the equation $x^2 - 20x + C = 0$, has exactly one real root.

   ANS: $x^2 - 20x + C = 0$, Solutions $\left\{x = 10 + \sqrt{100 - C}\right\}$ and $\left\{x = 10 - \sqrt{100 - C}\right\}$ are the same when $C = \boxed{100}$

Team Competition

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9. What is the least common multiple of 79, 100?

   ANS: 7900

10. A spherical balloon's diameter decreases by 50%. By what percentage does the surface area decrease?

    ANS: 75%

11. The sum of the squares of two positive integers is 394, and the difference of their squares is 56. What are the numbers?

    ANS: $\left\{ \begin{array}{c} x^2+y^2=394 \\ x^2-y^2=56 \end{array} \right\}$, Solution is : $\boxed{y=13, x=15}$

12. What is the maximum number of pieces into which a circular pizza can be cut using 3 chops of a knife (with no intermediate rearrangements of the pieces)?

    ANS: 7

13. An orange has a diameter that is 75% fruit and 25% peel. To the nearest percent, what percentage of the volume is the peel?

    ANS: 58% $\quad (\approx 57.812\,5\%)$

14. In which quadrant do the two lines $y = 26 + 2x$ and $y = 6 - 3x$ intersect?

    ANS: $\left\{ \begin{array}{c} y=26+2x \\ y=6-3x \end{array} \right\}$, Solution is : $(-4, 18)$ $\boxed{\text{second quadrant}}$

15. What is the greatest integer less than the sum $\frac{17}{3} + \frac{3}{17}$?

    ANS: $\frac{17}{3} + \frac{3}{17} = 5.843137255 = \boxed{5} + .843137255$

Tiebreaker Question:

Arrange the following three numbers from smallest to largest: $\frac{1}{8}$, $\frac{1}{9}$, or $\frac{2}{17}$.

ANS: $\boxed{\frac{1}{9}} = 0.111\,111\,111\,1 < \boxed{\frac{2}{17}} = 0.117\,647\,058\,8 < \boxed{\frac{1}{8}} = 0.125$

CSU Math Day 2000
Team Competition
12:00

1. What is the radius of the sphere whose volume is 12 times its surface area?

   ANS: $\boxed{36}$

2. From what language was the term 'algebra' derived?

   ANS: $\boxed{\text{Arabic}}$

3. This cipher scrambled messages by assigning numbers to letters and creating ciphertext by adding 3 and reducing the result modulo 26. Name this cipher.

   ANS: $\boxed{\text{Caesar cipher}}$ (used by Julius Caesar)

4. Art, Betty and Claude are now 5, 7, and 11 years old. How many years will it be until they again have prime-numbered ages?

   ANS: $\boxed{\text{Six years from now}}$ they will have ages 11, 13, and 17.

5. A computer sequentially computes integers by the following rule: If n is a square then multiply by 2; otherwise subtract 2. Starting at $n = 13$, what is the integer after 6 iterations?

   ANS: $13 \to 11 \to 9 \to 18 \to 16 \to 32 \to \boxed{30}$

6. A wooden cube of edge 4 inches is painted red. The cube is then cut into 64 one-inch cubes by making 9 saw cuts. How many of the one-inch cubes have exactly one red face?

   ANS: 6 faces $\times$ 4 cubes per face = $\boxed{24}$ cubes

7. What is $80\,^\circ$ equal to in radians?

   ANS: $\boxed{\frac{4}{9}\pi}$

8. Solve the system of equations

$$\begin{aligned} -x + y + z &= 1 \\ x - y + z &= 1 \\ x + y - z &= 1 \end{aligned}$$

   ANS: By symmetry, $\boxed{x = y = z = 1}$

Team Competition
12:00

9. List the following three numbers in increasing order: π, 22/7, and 3.14?

   ANS: $\boxed{3.14} < \boxed{\pi} \approx 3.141592654 < \boxed{22/7} \approx 3.1429$

10. In how many ways can 6 boys and 6 girls be teamed into pairs, if each pair must contain one girl?

    ANS: 6! $=\boxed{720}$

11. A ladder leans against a vertical wall. The bottom of the ladder is 7 feet from the wall and the top of the ladder is 12 feet above the floor. How long is the ladder?

    ANS: $\boxed{\sqrt{193}}$ feet

12. The formula $e^{i\pi} + 1 = 0$ relates five of the most popular numbers in mathematics. What is e rounded to 6 significant digits?

    ANS: $e \approx 2.718281828 \approx \boxed{2.71828}$

13. In which quadrant is the center of the circle $x^2 + y^2 + 6x - 8y = 0$?

    ANS: $x^2 + 6x + y^2 - 8y + 25 = (x+3)^2 + (y-4)^2 = 25$, center $(-3, 4)$ $\boxed{\text{2nd}}$ quadrant

14. Every mathematician has an Erdös (AIR′·DISH) number. How is this number defined?

    ANS: Paul Erdös has an Erdös number of 0. Those who published a joint paper with Erdös have an Erdös number of 1.Those who have a joint paper with someone with Erdös number n has an Erdös number $\leq n+1$. The Erdös number is the length of the shortest such path.

15. If $f(x) = 4x + 5$, what is $f(f(\frac{3}{2}))$?

    ANS: 49

Tiebreaker Question:

What is the largest number that correctly completes the statement, "Every cubic polynomial with real coefficients has at least _____ real root(s)."
ANS: $\boxed{\text{one}}$

CSU Math Day 2000
Team Competition
12:20

1. What is the area of the largest regular octagon that can be inscribed in a square of side 1?

   ANS:

$$\begin{aligned} x + 2\left(\frac{x}{\sqrt{2}}\right) &= 1, \text{ Solution is: } x = \frac{1}{\sqrt{2}+1} \\ area &= 1 - x^2 = 1 - \frac{1}{\left(\sqrt{2}+1\right)^2} \\ &= \boxed{\frac{2}{1+\sqrt{2}}} = \boxed{2\left(\sqrt{2}-1\right)} = \boxed{2\sqrt{2}-2} \end{aligned}$$

2. What is the smallest positive integer n such that $n^2 - n + 11$ is not prime?

   ANS: 11, since

n	1	2	3	4	5	6	7	8	9	10	$\boxed{11}$
$n^2 - n + 11$	11	13	17	23	31	41	53	67	83	101	$121 = 11^2$

3. A square is inscribed in a circle, which in turn is inscribed in a square. What percentage of the area of the large square is inside the small square?

   ANS: 50%

4. Which real numbers are equal to their cubes?

   ANS: $x = x^3$, Solution is : $x = \boxed{0}, x = \boxed{1}, x = \boxed{-1}$

5. Solve the equation $x\sqrt{0.64} = 4$

   ANS: $x = 5$

6. According to Descartes' Rule of Signs, how many negative real roots does the polynomial equation $x^4 - 8x^2 + 6x - 7 = 0$ have?

   ANS: $\boxed{\text{one}}$ (since there is one sign change if x is replaced by $-x$)

7. If g is a function such that $g(1) = 2$, $g(2) = -1$, and

$$g(n+1) = g(n) + 2g(n-1)$$

   for $n \geq 3$, what is $g(4)$?

   ANS: $g(3) = g(2) + 2g(1) = -1 + 4 = 3$, $g(4) = g(3) + 2g(2) = 3 - 2 = \boxed{1}$

8. Factor completely the integer $2^5 + 3^5$.

   ANS: $2^5 + 3^5 = 275 = \boxed{5^2 11}$

Team Competition
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9. What is the sum of the first 16 even integers: 2, 4, 6, ... 32?

   ANS: 272

10. What is the smallest integer whose square is less than 706?

    ANS: -26

11. Completely factor the polynomial $x^3 + 5x^2 - 16x - 80$

    ANS: $(x+4)(x-4)(x+5)$

12. A pair of slacks priced at \$20 has been marked up 20% and then marked down 30%. What is the new price?

    ANS: \$16. 80

13. What is the cube root of 1 million?

    ANS: 100

14. What former University of Colorado student went on to help found Apple Computer?

    ANS: Steve Wozniak

15. What is the largest integer that can be stored in a 6-bit computer word?

    ANS: 63

Tiebreaker Question:

What is the greatest common divisor of 203 and 217?
ANS: $\gcd(203, 217) = \boxed{7}$

CSU Math Day 2000
Team Competition
12:40

1. If $\sin x = \frac{1}{3}$ and $0 < x < \pi/2$, what is $\sin 2x$?

   ANS: $\sin 2x = 2 \sin x \cos x = 2 \cdot \frac{1}{3}\sqrt{1 - \frac{1}{9}} = \boxed{\frac{4}{9}\sqrt{2}}$

2. What is the smallest value of the expression $2x + \frac{1}{2x}$ if x is a positive real number?

   ANS: $\left[2x + \frac{1}{2x}\right]_{x=1/2} = \boxed{2}$

3. The 12 faces of a regular dodecahedron are pentagons. How many edges does a regular dodecahedron have?

   ANS: $\frac{12 \times 5}{2} = \boxed{30}$

4. Name the five Platonic solids (regular polyhedra).

   ANS: Tetrahedron, cube (or hexahedron), octahedron, dodecahedron, icosahedron

5. A baseball manager has selected 9 starters for a game. If the pitcher must bat last and the second baseman must bat first, how many different batting line-ups are possible?

   ANS: $7! = \boxed{5040}$

6. A Stanford University mathematician/computer scientist who developed the TEX typesetting system and who wrote a three-volume series of books called *The Art of Computer Programming* stated, "Every bit of mathematics I have ever learned, I have found a use for someplace." What is this person's name?

   ANS: Donald Knuth

7. The difference between two positive numbers is 6, and their product is twice the cube of the smaller number. What are the numbers?

   ANS: $8, 2$

8. What is the greatest integer in the sum $\frac{19}{5} + \frac{5}{19}$?

   ANS: $\frac{19}{5} + \frac{5}{19} = 4.\,06315\,78947\,3684 = \boxed{4} + .\,06315\,78947\,3684$

Team Competition
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9. If the surface area of a cube is equal to 6, what is its volume?

   ANS: 1

10. An icosahedron has 20 faces and 12 vertices. A diagonal of such a solid is a line segment joining two vertices not lying in the same face. How many diagonals are there?

    ANS: $\frac{6\times 12}{2} = \boxed{36}$ Each vertex connected to 5 other vertices, so each vertex lies on 6 diagonals.

11. Two wheels are connected by a belt. One has a diameter of 50 centimeters and a speed of 200 rmp. The other has a speed of 400 rpm. What is its diameter?

    ANS: 25

12. The plane can be completely tiled using equilateral triangles, or using squares, or using regular hexagons. These give the three regular tessellations of the plane. A semi-regular tessellation is a tiling that uses at least two different regular polygons, and where every vertex is congruent to every other vertex. How many semi-regular tessellations are there?

    ANS: 8

13. What is the sum of the first five odd positive integers?

    ANS: $1 + 3 + 5 + 7 + 9 = \boxed{25} = 5^2$

14. The distance between the points $(2, 4)$ and $(7, c)$ is 13. Find all the possible values for c.

    ANS: $(2-7)^2 + (4-c)^2 = 13^2$, Solution is : $\boxed{\{c = 16\}, \{c = -8\}}$

15. Two 3-digit integers consist of the same digits, but in the reverse order. What is the largest possible difference between the two numbers?

    ANS: $991 - 199 = 981 - 189 = \cdots = 901 - 109 = \boxed{792}$

Tiebreaker Question:

What is the remainder when 22222 is divided by 3?
ANS: $22222 \bmod 3 = \boxed{1}$

CSU Math Day 2000
Team Competition
1:00

1. What former CSU Mathematics Professor was known as an "Euler Spoiler" for finding a counterexample to a famous conjecture of Euler and who helped develop the Bose (BOZE)-Chaudhuri (CHAW·DRE)-Hocquenghem (HOAK·ENG·EM) error-correcting codes?

   ANS: R. C. $\boxed{\text{Bose}}$

2. What is the perimeter of a right triangle with legs 8 and 15?

   ANS: $s^2 = 8^2 + 15^2$, $s = 17$, $P = 8 + 15 + 17 = \boxed{40}$

3. What is the surface area of a sphere of volume $\frac{256}{3}\pi$?

   ANS: $\boxed{64\pi}$

4. How many edges does an n-dimensional cube have?

   ANS: $\boxed{n2^{n-1}}$

5. Let $y = mx + b$ be the image when the line $-7x + 8y + 1 = 0$ is reflected across the x-axis. What is $m + b$?

   ANS: $\boxed{-\frac{3}{4}}$

6. A pair of dice is rolled. What is the probability that the total is 5?

   ANS: $\boxed{\frac{1}{9}}$

7. What is the product of all the roots (real and complex) of the polynomial $x^3 + 30x^2 - 15x - 46$?

   ANS: $\boxed{46}$ since the linear term in the product $(x - a)(x - b)(x - c)$ is $-abc$.

8. What is the 13th term of the arithmetic sequence that begins 5, 10, 15. . . ?

   ANS: $\boxed{65}$

Team Competition
1:00

9. What is the coefficient of x^6 in the expansion of $(2+x^2)^4$?
   ANS: $(2+x^2)^4 = 16 + 32x^2 + 24x^4 + \boxed{8}x^6 + x^8$

10. A pair of dice is rolled. What is the probability that the total is 2 or 9 or 11?
    ANS: $\frac{1}{36} + \frac{4}{36} + \frac{2}{36} = \boxed{\frac{7}{36}}$

11. Bo is going to the store to buy candy that will cost somewhere between 5 cents and 26 cents. What is the fewest number of coins Bo can carry in order to be certain to have exact change to buy the candy?
    ANS: 1¢, 1¢, 1¢, 1¢, 5¢, 10¢, 10¢ $\boxed{7}$ coins

12. If the interior angle of a regular polygon is 108°, how many sides does it have?
    ANS: $\boxed{5}$ sides

13. Pierre de Fermat was not a professional mathematician, although such famous results as Fermat's Last Theorem and Fermat's Little Theorem were named in his honor. What was his profession?
    ANS: $\boxed{\text{Lawyer}}$

14. What is the smallest 3-digit prime?
    ANS: $\boxed{101}$

15. A right triangle has legs of length 6 and 8. What is the radius of the circle that circumscribes the triangle?
    ANS: diameter = 10, radius = $\boxed{5}$

Tiebreaker Question:

Which has the larger surface area, a cube of volume 1 or a sphere of volume 1?
ANS: $\boxed{\text{cube}}$

CSU Math Day 2000
Team Competition
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1. Sarah averages 50% on multiple choice exams. What is the probability that she gets at least 2 correct on a 3-question exam?

   ANS: $\left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) = \boxed{\frac{1}{2}}$

2. What is the area of the parallelogram with vertices $(0,0)$, $(1,9)$, $(7,5)$, and $(8,14)$?

   ANS: $\boxed{58}$

3. Find all positive roots of the equation $x^4 - 5x^2 + 4 = 0$.

   ANS: $x^4 - 5x^2 + 4 = 0$, Solution is : $x = \boxed{1}, x = \boxed{2}, x = -2, x = -1$

4. What is the midpoint of the line segment joining the two points $(-4,-5)$ and $(-8,-3)$?

   ANS: $\boxed{(-6,-4)}$

5. Who wrote the monumental Principia Mathematica?

   ANS: Bertrand $\boxed{\text{Russell}}$ and Alfred North $\boxed{\text{Whitehead}}$

6. A pyramid is built out of blocks by placing 100 blocks on the floor, placing 81 blocks on top of the bottom layer, and so forth. How many cubes are there in the pyramid?

   ANS: $\sum_{i=1}^{10} i^2 = \boxed{385} = \frac{10 \cdot 11 \cdot 21}{6}$

7. If 2 is the first prime, what is the fifteenth prime?

   ANS: $2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, \boxed{47}$

8. What is the prime factorization of 77?

   ANS: 7×11

Team Competition

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9. A city lot is twice as long as it is wide. By increasing its length 20 yards and its width 30 yards, the area will be increased by 2200 square yards. What are its dimensions?

   ANS: $(2x + 20)(x + 30) - 2x^2 = 2200$, $\boxed{\text{20 yards by 40 yards}}$

10. Which of the following best describes how many 6-symbol license plates are possible if the symbols come from the letters A-Z together with the digits 0-9? (a) 2–3 million (b) 20–30 million (c) 200–300 million (d) 2–3 billion

    ANS: $36^6 = 21767\,82336 \approx \boxed{\text{2–3 billion}}$ or $\boxed{\text{(d)}}$

11. Which mathematician amazed his grade school teacher by quickly summing the integers from 1 to 100?

    ANS: $\boxed{\text{Gauss}}$

12. A computer sequentially computes integers by the following rule: If n is a square then multiply by 2; otherwise subtract 1. Starting at $n = 9$, what is the integer after 6 iterations?

    ANS: $9 \to 18 \to 17 \to 16 \to 32 \to 31 \to \boxed{30}$

13. What can you say about a parallelogram whose diagonals have equal length?

    ANS: $\boxed{\text{It is a rectangle.}}$

14. Sarah averages 90% on multiple choice exams. What is the probability that she gets exactly 2 correct on a 3-question exam?

    ANS: $\boxed{.243}$ or $\boxed{\frac{243}{1000}}$

15. What is the highest power of 10 dividing 20!?

    ANS: $20! = \underline{20} \cdot 19 \cdots \underline{15} \cdots \underline{10} \cdots \underline{5} \cdots 2 \cdot 1 = 2432\,90200\,81766\,40000$ so $\boxed{4}$ or $\boxed{10^4}$

Tiebreaker Question:

How many edges does an octahedron have?
ANS: $\frac{3\times 8}{2} = \boxed{12}$

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1. How many distinct complex roots does the polynomial $x^6 + 1$ have?

   ANS: 6

2. Which regular polyhedron has the same number of faces as vertices?

   ANS: $\boxed{\text{Tetrahedron}}$ 4 faces and 4 vertices

3. In an algebra class of 26 students, each student shakes hands with each of the other students exactly once. How many handshakes are there?

   ANS: 325

4. If a sequence is defined by $x_1 = 1$, $x_2 = 3$, and $x_{n+1} = x_n \cdot x_{n-1}$, what is x_4?

   ANS: $x_3 = 3$, $x_4 = \boxed{9}$

5. The graph of a cubic polynomial crosses the x-axis at $x = -3$, $x = 2$, and $x = 4$. In expanded form, what is one such polynomial?

   ANS: $x^3 - 3x^2 - 10x + 24$ or some nonzero multiple thereof

6. Kyle uses pure guessing on a TRUE/FALSE exam. Which of the following options give Kyle the best chance to score (at least) 50%? (A) A seven-question exam (guess correctly on 4, 5, 6, or 7 questions). (B) A five-question exam (guess correctly on 3, 4, or 5 questions). (C) The chances are equal.

   ANS: $\boxed{\text{(C)}}$ Chances are equal (both probabilities are $\frac{1}{2}$)

7. 600 cm^3 of paint are required to paint the outside of a cube of volume $1\,\text{m}^3$. How much paint is needed to paint the outside of a cubic box of volume $8\,\text{m}^3$ if the box has no lid?

   ANS: 2000 cm^3

8. What is the smallest perfect cube larger than 51?

   ANS: $4^3 = 64$

Team Competition

1:35

9. What did the Norwegian mathematician Niels Henrik Abel prove about general fifth-degree polynomials?

   ANS: $\boxed{\text{Cannot be solved}}$ in terms of radicals involving the coefficients

10. You currently earn \$5.00 per hour delivering pizza. You are due for a raise, and you figure the probability of a \$0.50 raise is 20% and the probability of a \$1.00 raise is 80%. What is your expected new salary?

    ANS: $\$5.90$

11. A computer sequentially computes integers by the following rule: If n is a square then multiply by 2; otherwise subtract 2. Starting at $n = 9$, what is the integer after 6 iterations?

    ANS: $9, 18, 16, 32, 30, 28, \boxed{26}$

12. How many rearrangements of the letters a, b, c, d, e, f have a listed before b and b listed before c?

    ANS: $\frac{6!}{3!} = \boxed{120}$

13. What are the next two prime numbers greater than 90?

    ANS: 91=7×13, 93=3×31, $\boxed{97, 101}$

14. In how many ways can one rearrange the letters in OBOE?

    ANS: $\frac{4!}{2!} = \boxed{12}$

15. Art spent half his money plus 50¢, then half the remainder plus 50¢, then half of what he had left plus 50¢, when he found that he had \$2.00 remaining. How much did he start with?

    ANS: $\left(\left(\left(2 + \frac{1}{2}\right) 2 + \frac{1}{2}\right) 2 + \frac{1}{2}\right) 2 = \boxed{\$23.00}$

Tiebreaker Question:

What integer is closest to 12.8×23.5?
ANS: $12.8 \times 23.5 = 300.8 = \boxed{301} - .2$

CSU Math Day 2000
Team Competition
1:50

1. What is the volume of a spherical raindrop of diameter 2 millimeters?

   ANS: $\frac{4\pi}{3}(1)^3 = \boxed{\frac{4\pi}{3}}$ mm^2

2. If the absolute value of $x^2 + 4$ is equal to the absolute value of $x^2 - 12$, what is x?

   ANS: $|x^2 + 4| = |x^2 - 12|$, Solution is: $\boxed{x = 2, x = -2}$

3. Factor the polynomial $x^3 + 4x^2 + 6x + 4$ as a product of a linear and a quadratic, using the fact that -2 is a root.

   ANS: $\boxed{(x + 2)(x^2 + 2x + 2)}$

4. What is the smallest positive integer with exactly 5 positive integer divisors?

   ANS: $\boxed{16}$ (divisors are $\{1, 2, 4, 8, 16\}$)

5. The number of times a pendulum oscillates in a given time varies inversely as the square root of its length. If a 40 inch penculum oscillates once per second, what is the length of a pendulum that oscillates twice each second?

   ANS: $\boxed{10}$ in

6. A wooden cube of edge 4 inches is painted red. The cube is then cut into 64 one-inch cubes by making 9 saw cuts. How many of the one-inch cubes have exactly 3 red faces?

   ANS: 8 corners $\times$ 1 cube per corner = $\boxed{8}$ cubes

7. A wooden cube of edge 4 inches is painted red. The cube is then cut into 64 one-inch cubes by making 9 saw cuts. How many of the one-inch cubes have no red faces?

   ANS: The center of the large cube contains $2 \times 2 \times 2 =$ $\boxed{8}$ one-inch cubes

8. The compact disk UR2gly was originally marked at \$12.00, was marked up 50%, then marked down 50%. What was the new selling price?

   ANS: $\$12.00 * 1.5 * .5 = \$9 = \$\boxed{9.00}$

Team Competition

1:50

9. What is the sum of the first six odd positive integers?

   ANS: $1+3+5+7+9+11 = \boxed{36} = 6^2$

10. According to Descartes' Rule of Signs, how many positive real roots does the polynomial equation $3x^4 + 10x^2 + 5x - 4 = 0$ have?

    ANS: $\boxed{\text{one}}$ (since there is one sign change)

11. What is the greatest integer in the sum $\frac{11}{3} + \frac{3}{11}$?

    ANS: $\frac{11}{3} + \frac{3}{11} = \frac{130}{33} = 3.939393939$, so greatest integer is $\boxed{3}$

12. What is the sum of all the integers greater than 5 and less than 25?

    ANS: 285

13. This research organization has employed applied mathematicians such as Ronald Graham, Richard Hamming, and Claude Shannon, each of whom have made major contributions to applied discrete mathematics. Name this organization.

    ANS: Bell Labs or AT&T Bell Laboratories

14. The 4 faces of a regular tetrahedron are equilateral triangles. How many edges does a regular tetrahedron have?

    ANS: 6

15. A particle, initially at $(-3, 5)$, moves along a line of slope 1 to a new position (x, y). Find y if $x = -12$.

    ANS: $y = -4$

Tiebreaker Question:

What is the remainder if 123456789 is divided by 3?
ANS: $123456789 \bmod 3 = \boxed{0}$

CSU Math Day 2000
Team Competition
2:05

1. Express the perimeter P of a square as a function of its area A.

   ANS: $A = \left(\frac{P}{4}\right)^2$, Solution is : $\boxed{P = 4\sqrt{A}}$

2. What is the area of a right triangle having a leg of length ℓ and hypotenuse of length 25?

   ANS: $A = \boxed{\frac{\ell\sqrt{25-\ell^2}}{2}}$

3. What is the decimal representation of the base 5 number 123_5?

   ANS: $\boxed{38}$

4. A mathematician named Wolfram started a company named Wolfram Research. What is its primary product?

   ANS: $\boxed{\text{Mathematica}}$

5. A shelf will hold 20 calculus textbooks and 24 algebra textbooks, or 15 calculus textbooks and 36 algebra textbooks. How many calculus books alone will the shelf hold?

   ANS: $\left\{ \begin{array}{l} 20c + 24a = 1 \\ 15c + 36a = 1 \end{array} \right\}$, $a = \frac{1}{72}, c = \frac{1}{30}$ $\boxed{\text{Shelf will hold 30 calculus books}}$

6. A solid statue is made by melting $9\,\text{cm}^3$ of metal and pouring it into a mold. A larger model needs to be constructed by increasing each of its linear dimensions by a factor of 5. How much metal will the new statue require?

   ANS: $\boxed{1125}\,\text{cm}^3$

7. Use the approximation $2^{10} \approx 10^3$ to approximate 2^{43} as a number in scientific notation $2^{43} \approx c \times 10^n$, where n is an integer and c is a number between 1 and 10.

   ANS: $2^{43} = 8 \times (2^{10})^4 \approx 8 \times (10^3)^4 = \boxed{8 \times 10^{12}} = 8000000000000$

8. A computer sequentialy computes integers by the following rule: If n is odd then replace n by $3n + 1$; otherwise replace n by $n/2$. If n starts at 3, what is n after 5 iterations?

   ANS: $3 \to 10 \to 5 \to 16 \to 8 \to \boxed{4}$

Team Competition

2:05

9. A golf bag contains 13 balls, some yellow and the rest orange. Two balls are drawn at random from the bag and one is yellow, the other orange. What is the probability that exactly half of the balls in the bag are yellow?

   ANS: 0

10. A basketball team has 12 players on the roster. How many different starting lineups are possible?

    ANS: $\binom{12}{5} = \boxed{792}$

11. A baseball manager has selected 9 starters for a game. If the pitcher must bat last, how many different batting line-ups are possible?

    ANS: $8! = \boxed{40\,320}$

12. A wheel of radius 1 foot rolls without slipping around the outside of a stationary wheel of radius 2 feet. Exactly how many rotations does the small wheel make?

    ANS: $\boxed{3}$

13. What is the area of the finite region bounded by the x-axis, the y-axis, and the line $y = -4x + 16$?

    ANS: $\frac{1}{2} \cdot 16 \cdot 4 = \boxed{32}$

14. A multiple-choice exam has 15 questions with 3 choices per question. If you answer the questions randomly, what is the expected number of correct responses?

    ANS: $\boxed{5}$

15. How many vertices does an n-dimensional cube have?

    ANS: $\boxed{2^n}$

Tiebreaker Question:

What is the smallest perfect square > 1000?
ANS: $31^2 = 961$, $32^2 = \boxed{1024}$

CSU Math Day 2000
Team Competition
2:15

1. Find all the roots of the equation $x^3 - x^2 - 72x - 180 = 0$?

   ANS: 10, -3, -6.

2. If two cards are drawn from a standard deck of 52 cards, what is the probability that both are kings?

   ANS: $\frac{\binom{4}{2}}{\binom{52}{2}} = \frac{6}{1326} = \boxed{\frac{1}{221}}$

3. In data processing terminology, what does FIFO mean?

   ANS: First In First Out

4. If it takes 54 minutes to inflate a large spherical balloon to a radius of 3 meters, how long will it take to inflate a large spherical balloon to a radius of 5 meters?

   ANS: 250 min

5. How many different strings of length 7 can be formed using the letters in ELLIPSE?

   ANS: $\frac{7!}{2!2!} = \boxed{1260}$

6. Bonnie gets a salary of \$50,000 with a 15% yearly raise. What will her salary be after 2 years?

   ANS: \$66125.00

7. Divide 90 into 3 parts, such that $\frac{1}{9}$ of the first, $\frac{1}{3}$ of the second, and $\frac{1}{6}$ of the third are equal.

   ANS: 45, 15, and 30

8. 5 positive integers have a sum of 11. What is the maximum possible value for the sum of their squares?

   ANS: 53

Team Competition

2:15

9. If a third degree polynomial has leading coefficient of -2 and roots -3, -4, and -5, what is its constant term?

   ANS: 120

10. How many strings of length 6 can be made from the letters in CONOCO?

    ANS: $\frac{6!}{3!2!} = \boxed{60}$

11. What is the volume of the tetrahedron whose vertices are $(0,0,0)$, $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$?

    ANS: $1/6$

12. On a four-question true/false exam, correct answers are worth 3 points, wrong answers 0, and blanks count 1 point. How many different responses will result in a total score of 4?

    ANS: Form 1111 or 3100, so $1 + \frac{4!}{2!} = \boxed{13}$

13. To the nearest one-hundredth of an inch, what is the smallest diameter log needed for cutting a square post of dimensions 4 in by 4 in?

    ANS: $4\sqrt{2} = 5.\,65685\,42494\,9238 \approx \boxed{5.66}$ in

14. The equation $x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right)$ provides an algorithm for approximating $\sqrt{2}$. Starting with $x_1 = 1$, what is x_3 (as a rational number)?

    ANS: $x_2 = \frac{1}{2}(1+2) = \frac{3}{2}$, $x_3 = \frac{1}{2}\left(\frac{3}{2} + \frac{2}{3/2}\right) = \boxed{\frac{17}{12}} = \boxed{1\frac{5}{12}}$

15. The vertices of a quadrilateral are at the points $(0,0)$, $(3,1)$, $(4,0)$, and $(2,-4)$. What is the area of the quadrilateral?

    ANS: $Area = \frac{1}{2}\cdot 4\cdot 1 + \frac{1}{2}\cdot 4\cdot 4 = \boxed{10}$

Tiebreaker Question:

Factor 1998 as a product of primes.

ANS: $1998 = \boxed{2\times 3^3\times 37}$ or $\boxed{2\times 3\times 3\times 3\times 37}$

CSU Math Day 2000
Small School Final

1. The diagonal of a table with a square top is 20 feet. What is the area of the table top?

   ANS: $\frac{20}{\sqrt{2}}\,\text{ft} \times \frac{20}{\sqrt{2}}\,\text{ft} = \boxed{200}\,\text{ft}^2$

2. A rectangle's length is increased by 30% and its width is decreased by 50%. How does its area change?

   ANS: $\boxed{\text{decreases by } 35\%}$

3. A piece of string 64 inches long is cut into two pieces so that one piece is 6 inches shorter than the other. What are the lengths of the two pieces?

   ANS: $x + (x+6) = 64 \Longrightarrow$, Solution is : $x = \boxed{29}$ in, $x + 6 = \boxed{35}$ in

4. In how many ways can 3 math book(s), 3 stat book(s), and 3 physics book(s) be arranged on a shelf, assuming the books must be in groups by their category? (ie: math with math, stat with stat, etc.)

   ANS: $3!\,(3!)^3 = \boxed{1296}$

5. The midpoints of two edges of a triangle are connected with a line to subdivide the original triangle into a trapezoid and a small triangle. If the area of the small triangle is 5, what is the area of the trapezoid?

   ANS: $\boxed{15}$

6. The sum of two integers is 6 and their product is -27. What are the two integers?

   ANS: $\boxed{9}$ and $\boxed{-3}$.

7. Sarah is an 80% free throw shooter. What is the probability that she misses 4 in a row?

   ANS: . $\boxed{16\%}$ or $\boxed{.00\,16}$ or $\boxed{\frac{1}{625}}$

8. How many integers between 101 and 999 are divisible by 3 or by 5?

   ANS: $\frac{999-102}{3} + 1 = 300$, $\frac{995-105}{5} + 1 = 179$, $\frac{990-105}{15} + 1 = 60$, $300 + 179 - 60 = \boxed{419}$

9. If $f(x) = -3x + 2$, what is $f(f(f(0)))$?
   ANS: $\boxed{14}$

10. How far apart are the two points of intersection of the two curves $y = x^2 + 1$ and $y = x + 7$?
    ANS: $\left\{ \begin{array}{c} y = x^2 + 1 \\ y = x + 7 \end{array} \right\}$, Solution is : $(-2, 5), (3, 10)$, $\sqrt{(3+2)^2 + (10-5)^2} = \boxed{5\sqrt{2}}$

11. If g is a function such that $g(1) = -1$, $g(2) = 2$, and
$$g(n) = g(n-2) + 3g(n-1)$$
for $n \geq 3$, what is $g(4)$?
    ANS: $g(3) = g(1) + 3g(2) = -1 + 6 = 5$, $g(4) = g(2) + 3g(3) = 2 + 15 = \boxed{17}$

12. Find a point equidistant from the points $(-1, -1)$, $(1, 1)$, and $(1, -1)$.
    ANS: $\boxed{(0, 0)}$

13. To the nearest minute, at what time between 9:30 a.m. and 10:00 a.m. are the minute hand and the hour hand at right angles?
    ANS: $45 + \frac{m}{12} = m + 15$, Solution is : $m = \frac{360}{11}$: $32.\,72727\,27272\,727$ $\boxed{9{:}33}$ a.m.

14. What is the area of the triangle bounded by the x-axis, the y-axis, and the line $y = x + 3$?
    ANS: $\boxed{\frac{9}{2}}$

15. The height of a rectangle is 50% less than its base. The perimeter of the rectangle is 36 inches. Find the area of the rectangle.
    ANS: $\left\{ \begin{array}{c} h = \frac{b}{2} \\ 2h + 2b = 36 \end{array} \right\}$, Solution is: $\{b = 12, h = 6\}$, $6 \times 12 = \boxed{72}$

Tiebreaker Question:

What is the prime factorization of 105?
ANS: $105 = \boxed{3 \times 5 \times 7}$

CSU Math Day 2000
Small School Final (Round 2)

1. Farmer Jill raises goats and geese. If she counts 26 eyes and 38 feet, how many goats and how many geese does Jill have?

   ANS: $\boxed{7}$ geese and $\boxed{6}$ goats.

2. A Social Security number has nine digits. Assuming the digits are random, what is the expected number of fives in a social security number?

   ANS: $9 \times \frac{1}{10} = \boxed{\frac{9}{10}}$

3. What Greek philosopher raised paradoxes that argued that motion is impossible?

   ANS: $\boxed{\text{Zeno}}$ of Elea

4. A 6-foot man casts a 12-foot shadow. A flag pole next to him casts a 54-foot shadow. How tall in the flag pole?

   ANS: $\boxed{27}$ ft

5. A shirt has been marked down 15% and then 30% to \$23. 80. What was the original price?

   ANS: \$$\boxed{40}$

6. At the local Dairy Queen, the "Monster Sundae" can be ordered with any of 8 flavors of ice cream plus any or all of 5 toppings. If you order one such sundae every Saturday, how many weeks will it be before you must order the same sundae twice?

   ANS: $\boxed{256}$

7. What is the statement of Fermat's Last Theorem?

   ANS: $\boxed{\text{The equation } x^n + y^n = z^n \text{ has no positive integer solutions for integers } n > 2.}$

8. What is the sum of the roots of the polynomial $x^2 - 32x + 70$?

   ANS: $\boxed{32}$, since $(x-a)(x-b) = x^2 - x(a+b) + ab \Rightarrow a + b = 32$

   $x^2 - 32x + 70 = 0$ has solutions $\left\{x = 16 + \sqrt{186}\right\}, \left\{x = 16 - \sqrt{186}\right\}$ and $16 + \sqrt{186} + 16 - \sqrt{186} = 32$

9. How many primes are there between 100 and 110?
   ANS: $\boxed{\text{Four}}$ (101, 103, 107, 109)

10. What are the next two prime numbers greater than 50?
    ANS: $51 = 3 \times 17, \boxed{53}, 55 = 5 \times 11, 57 = 3 \times 19, \boxed{59}$

11. Give the points of intersection of the two curves $y = 20 - 6x$ and $y = 8 - 6x + 3x^2$.
    ANS: $\left\{ \begin{array}{c} y = 8 - 6x + 3x^2 \\ y = 20 - 6x \end{array} \right\}$, Solution is : $\boxed{(2, 8),\ (-2, 32)}$

12. What is the area of the smallest right triangle with all sides positive integers?
    ANS: $3 - 4 - 5$ right triangle has area $3 \cdot 2 = \boxed{6}$

13. What is the perimeter of a regular hexagon inscribed in a circle of radius 8?
    ANS: $\boxed{48}$

14. Three line segments joining the midpoints of the sides of a triangle determine a smaller triangle whose sides lie inside the larger triangle. What is the ratio of the area of the larger triangle to the area of the smaller triangle?
    ANS: $\boxed{4:1}$

15. A regular icosahedron has 20 faces, each of which is an equilateral triangle. If the midpoint of each face is connected with an edge to the midpoint of each adjacent face, what solid do these new edges determine?
    ANS: $\boxed{\text{Dodecahedron}}$ (12 faces, each face is a pentagon.)

Tiebreaker Question:

How many distinct complex roods does $x^3 - x^2 + x - 1$ have?
ANS: $\boxed{\text{three}}$ $x^3 - x^2 + x - 1$, roots: i, $-i$, 1

CSU Math Day 2000
Large School Final

1. The compact disk UR2ugly sells at outlet AC for \$15.49 less a discount of 20%, and at outlet DC for \$16.95 less a discount of 30%. Which outlet has the lower price?

   ANS: AC, $15.49 \times .8 = \$12.39$; $\boxed{\text{DC}}$, $16.95 \times .7 = \$11.86$

2. A deck of 52 cards is thoroughly shuffled and the cards are turned over two at a time. What is the expected number of pairs (two cards of the form $\boxed{\text{A}}\boxed{\text{A}}$ or $\boxed{5}\boxed{5}$) ?

   ANS: $26\left(\frac{3}{51}\right) = \boxed{\frac{26}{17}}$

3. George is a 75% free-throw shooter. What is his expected score if he shoots two free throws?

   ANS: $\boxed{1.5}$ or $\boxed{\frac{3}{2}}$

4. What is the volume of the largest cube that can fit inside a sphere of radius $\sqrt{3}$?

   ANS: $Volume = 2^3 = \boxed{8}$

5. During the 1970's Steve Jobs and Steve Wozniak starting selling electronic equipment out of a garage. What company did they start?

   ANS: $\boxed{\text{Apple Computer}}$

6. According to the Rational Root Theorem, what are all the possible rational roots of the polynomial $3x^3 - 4x^2 + 5x - 6$?

   ANS: $\boxed{\pm\frac{1}{3}, \pm\frac{2}{3}, \pm1, \pm2, \pm3, \pm6}$

7. How many different 6-letter words can be formed by rearranging the letters in SCHOOL?

   ANS: $\frac{6!}{2!} = \boxed{360}$

8. What is the name of the surface formed by rotating a circle about a line which does not touch the circle?

   ANS: A $\boxed{\text{torus}}$

9. If 2 cards are drawn from a standard deck of 52 cards, what is the probability that one is a heart and the other is a diamond?

   ANS: $\frac{\binom{13}{1}\binom{13}{1}}{\binom{52}{2}} = \boxed{\frac{13}{102}}$

10. What is the coefficient of a^6 in the expansion of $(3+2a^3)^3$?

    ANS: $(3+2a^3)^3 = 27 + 54a^3 + \boxed{36}a^6 + 8a^9$

11. What number is halfway between $\frac{1}{8}$ and $\frac{6}{7}$?

    ANS: $\boxed{\frac{55}{112}}$

12. The 24 ways to write rearrangements of the letters MATH are listed in alphabetical order. Where in this list does MATH appear?

    ANS: $\boxed{\text{14th}}$ on the list right after MAHT and before MHAT (1-6 start with A, 7-12 with H, and MAHT is 13th.)

13. The polynomial $5x^7 + 21x^5 + 35x^3 + 35x + 18$ has one real root. How many imaginary roots does it have?

    ANS: $\boxed{6}$

14. In 1962 a mathematician named Edward O. Thorp wrote a book entitled *Beat the Dealer* that claimed to give a winning strategy, based upon large-scale computer simulations, for a certain card game. What is the name of that card game?

    ANS: $\boxed{\text{Blackjack}}$ or $\boxed{21}$

15. Give the slope-intercept equation for a line through the origin that contains no other points with integer coordinates.

    ANS: $\boxed{y = \alpha x, \alpha \text{ any irrational number}}$

Tiebreaker Question:

What is the greatest common divisor of 115 and 145?
ANS: $\gcd(115, 145) = \boxed{5}$

CSU Math Day 2000
Large School Final (Round 2)

1. A circular pizza with diameter 12 inches is cut into 12 congruent slices. What is the perimeter of each slice?

   ANS: $\boxed{12+\pi}$

2. The equation $x_{n+1} = \frac{1}{2}\left(x_n + \frac{3}{x_n}\right)$ provides an algorithm for approximating $\sqrt{3}$. Starting with $x_1 = 1$, what is x_3 (as a rational number)?

   ANS: $x_2 = \frac{1}{2}(1+3) = 2$, $x_3 = \frac{1}{2}\left(2+\frac{3}{2}\right) = \boxed{\frac{7}{4}} = \boxed{1\frac{3}{4}}$

3. What is the coefficient of x^3y^4 in the expansion of $(x+y)^7$?

   ANS: $\boxed{35}$

4. An octahedron is a regular solid whose 8 faces are equilateral triangles. What is the distance between a pair of opposite vertices, assuming each edge of the octahedron is of length $\sqrt{2}$?

   ANS: $\boxed{2}$

5. A regular tetrahedron has edges of length 5. What is the total surface area of the tetrahedron?

   ANS: $\boxed{25\sqrt{3}}$

6. Assuming $0 < a < b$, express $\dfrac{a^b b^a}{a^a b^b}$ in terms of one quotient raised to a positive exponent.

   ANS: $\dfrac{a^b b^a}{a^a b^b} = \left(\frac{a}{b}\right)^b \left(\frac{b}{a}\right)^a = \boxed{\left(\frac{a}{b}\right)^{b-a}}$

7. Given the circle $x^2 - 2x + y^2 = 0$ in the xy plane, what are the equations of the two vertical tangent lines to this circle?

   ANS: $\boxed{x=0}$ and $\boxed{x=2}$

8. What is the area of the largest square that can be inscribed in the ellipse $\frac{x^2}{4} + \frac{y^2}{2} = 1$?

   ANS: $x = y \Longrightarrow \frac{3x^2}{4} = 1 \Longrightarrow 4x^2 = \boxed{\frac{16}{3}}$

9. Maple is a widely-used computer algebra system. In what country was it developed?
   ANS: $\boxed{\text{Canada}}$

10. What is the converse of the statement, "If roses are red, then violets are blue"?
    ANS: $\boxed{\text{If violets are blue, then roses are red.}}$

11. Which is larger, 1 cubic inch or 16 cubic centimeters? (use $1\,\text{in} = 2.54\,\text{cm}$)
    ANS: $(2.54\,\text{cm})^3 = 16.387064\,\text{cm}^3 = \boxed{1\,\text{in}^3} > 16\,\text{cm}^3$

12. In which quadrant do the two lines $y = 4 - 2x$ and $y = 18 + 5x$ intersect?
    ANS: Point $(-2, 8)$ is in the $\boxed{\text{second quadrant}}$

13. The sum of 4 consecutive integers is 62. What is the smallest of the 4 integers?
    ANS: $\boxed{14}$

14. If the absolute value of $x + 1$ is equal to the absolute value of $x - 1$, what is x?
    ANS: $x = \boxed{0}$

15. If $f(x)$ is a linear function and $f(-2) = 9$ and $f(-4) = 19$, what is $f(3)$?
    ANS: $\boxed{-16}$

Tiebreaker Question:

Which of the following three numbers is smallest: e^π, π^e, or 3^3?
ANS: $e^\pi = 23.\,140\,692\,64$, $\boxed{\pi^e} = 22.\,459\,157\,7$, $3^3 = 27$