CSU Math Day 1999
Team Competition
11:20 AM

1. Twenty students in an algebra class take a five-question true/false test. None of the students had studied for the test, so all of them used pure guessing. What is the probability that at least 10 students score 60% or higher on the test?

   ANS: $\boxed{50\%}$ or $\boxed{\frac{1}{2}}$

2. If eggs weigh 1.5 oz each and a dozen eggs cost 90¢, what is the cost of a pound of eggs? (16oz = 1 pound)

   ANS: $\boxed{80}$¢

3. What is the area of the region bounded by the x-axis, the vertical lines $x = 2$, $x = 4$, and the line $y = -x + 16$?

   ANS: $\boxed{26}$

4. What is the largest prime less than 500?

   ANS: $\boxed{499}$

5. In how many ways can 4 math book(s), 3 stat book(s), and 3 physics book(s) be arranged on a shelf, assuming the books must be in groups by their category? (ie: math with math, stat with stat, etc.)

   ANS: $3!4!3!3! = \boxed{5184}$

6. On January 1, 1998, an investment bond was purchased for $1000. To the nearest whole dollar, if it earns 15% compounded annually, what would the balance be on January 1, 2001?

   ANS: $1000 \times 1.15^3 = 1520.875 \approx \$\boxed{1521}$

7. How many different 6-letter words can be formed by rearranging the letters in SCHOOL?

   ANS: $\frac{6!}{2!} = \boxed{360}$

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8. What is the maximum number of pieces into which a circular pizza can be cut using 3 chops of a knife (with no intermediate rearrangements of the pieces)?

   ANS: $\boxed{7}$

9. Given a positive integer $x < 17$, what is the remainder of x^{17} when divided by 17?

   ANS: $\boxed{x}$ (Fermat's Little Theorem)

10. Completely factor the polynomial $3x^2 + 9x + 6$.

    ANS: $\boxed{3(x+1)(x+2)}$

11. In data processing terminology, what does FIFO mean?

    ANS: $\boxed{\text{First In First Out}}$

12. In how many ways can you have \$10 worth of dimes and quarters?

    ANS: $\boxed{21}$ (The number of quarters can be $0, 2, \ldots, 40$)

13. You currently earn \$5.50 per hour delivering pizza. You are due for a raise, and you figure the probability of a \$0.50 raise is 20% and the probability of a \$1.00 raise is 80%. What is your expected new salary?

    ANS: \$$\boxed{6.40}$

14. What is the area of the triangle bounded by the x-axis, the y-axis, and the line $y = x + 3$?

    ANS: $\boxed{\frac{9}{2}}$

15. List the following three numbers in increasing order: $x = 1 + 2 + 3 + \cdots + 100$, $y = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7$, and $z = 4^6$.

    ANS: $\boxed{z} = 4^6 = 4096 < \boxed{y} = 7! = 5040 < \boxed{x} = \sum_{i=1}^{100} i = 5050$

Tiebreaker Question: What is the prime power factorization of $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$?
ANS: $6! = \boxed{2^4 3^2 5}$

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1. What is the decimal representation of the base 9 number 234_9?

   ANS: $2 \times 9^2 + 3 \times 9 + 4 = \boxed{193}$

2. A regular tetrahedron has edges of length 11. What is the total surface area of the tetrahedron?

   ANS: $\boxed{121\sqrt{3}}$

3. An equilateral triangle has vertices at $(0, 0)$ and $(14, 0)$. Give one set of possible coordinates for the third vertex.

   ANS: $\boxed{(7, 7\sqrt{3})}$ or $\boxed{(7, -7\sqrt{3})}$

4. A circle of radius 1 is divided into 5 pieces. One of the pieces is 1/2 as large as each of the other four. What is the area of the smallest piece?

   ANS: $\boxed{\frac{\pi}{9}}$

5. A computer sequentially computes integers by the following rule: If n is odd then replace n by $3n + 1$; otherwise replace n by $n/2$. If n starts at 5, what is n after 5 iterations?

   ANS: $5 \to 16 \to 8 \to 4 \to 2 \to \boxed{1}$

6. If $f(x) = 3x - 5$, what is $f(f(\frac{1}{2}))$?

   ANS: $f\left(\frac{1}{2}\right) = -\frac{7}{2}$, $f\left(-\frac{7}{2}\right) = \boxed{-\frac{31}{2}}$

7. What are the next 4 terms in the sequence that begins $1, 1, 2, 3, 5, 8$?

   ANS: $\boxed{13, 21, 34, 55}$ (Fibonacci sequence)

8. A pair of dice is rolled 20 times. What is the expected number of times the total is 10?

   ANS: $\frac{3}{36} \times 20 = \boxed{\frac{5}{3}}$

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9. According to the Rational Root Theorem, what are all the possible rational roots of the polynomial $3x^3 - 4x^2 + 5x - 6$?

   ANS: $\boxed{\pm\frac{1}{3},\ \pm\frac{2}{3},\ \pm 1,\ \pm 2,\ \pm 3,\ \pm 6}$

10. A wooden cube of edge 4 inches is painted red. The cube is then cut into 64 one-inch cubes by making 9 saw cuts. How many of the one-inch cubes have no red faces?

    ANS: The center of the large cube contains $2 \times 2 \times 2 = \boxed{8}$ one-inch cubes

11. George is a 75% free-throw shooter. What is his expected score if he shoots two free throws?

    ANS: $\boxed{1.5}$ or $\boxed{\frac{3}{2}}$

12. An octahedron is a regular solid whose 8 faces are equilateral triangles. What is the distance between a pair of opposite vertices, assuming each edge of the octahedron is of length 1?

    ANS: $\boxed{\sqrt{2}}$

13. The equation $x_{n+1} = \frac{1}{2}\left(x_n + \frac{3}{x_n}\right)$ provides an algorithm for approximating $\sqrt{3}$. Starting with $x_1 = 1$, what is x_3 (as a rational number)?

    ANS: $x_2 = \frac{1}{2}(1+3) = 2$, $x_3 = \frac{1}{2}\left(2 + \frac{3}{2}\right) = \boxed{\frac{7}{4}} = \boxed{1\frac{3}{4}}$

14. Which regular polyhedron has the same number of faces as vertices?

    ANS: $\boxed{\text{Tetrahedron}}$ 4 faces and 4 vertices

15. Store A sells candy bars 3 for \$1.00. Store B sells candy bars individually for 40¢, but you get 5 for the price of 4. On Monday John bought some candy bars at store A. On Tuesday Jill bought some candy bars at store B. They compared notes and found that they had gotten the same number of candy bars, and each had paid the same amount of money. What is the least amount of money that each of them could have spent?

    ANS: They each bought 6 candy bars and spent $\boxed{\$2.00}$

Tiebreaker Question: What is the remainder when 22222 is divided by 3?
ANS: $22222 \bmod 3 = \boxed{1}$

CSU Math Day 1999
Team Competition
12:00 noon

1. List the following three numbers in increasing order: $x = 1 + 2 + 3 + \cdots + 40$, $y = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7$, and $z = 3^6$.

   ANS: $\boxed{y} = 6! = 720 < \boxed{z} = 3^6 = 729 < \boxed{x} = \sum_{i=1}^{40} i = 820$

2. What number is halfway between $\frac{1}{4}$ and $\frac{2}{9}$?

   ANS: $\frac{1}{2}\left(\frac{1}{4} + \frac{2}{9}\right) = \boxed{\frac{17}{72}}$

3. A wooden cube of edge 4 inches is painted red. The cube is then cut into 64 one-inch cubes by making 9 saw cuts. How many of the one-inch cubes have exactly 2 red faces?

   ANS: 12 edges $\times$ 2 cubes per edge $= \boxed{24}$ cubes

4. Two evenly-matched baseball teams, the Braves and the Yankees, start a 7-game series. In how may different ways can the Yankees win the series four games to two?

   ANS: The Yankees win game 6 plus 3 of the first 5, so $\frac{5!}{3!2!} = \binom{5}{3} = \boxed{10}$

5. The difference between two positive numbers is 6, and their product is twice the cube of the smaller number. What are the numbers?

$$\begin{aligned} x - y &= 6 \\ xy &= 2y^3 \end{aligned}$$

   Solution is : $\{y = 0, x = 6\}, \left\{y = -\frac{3}{2}, x = \frac{9}{2}\right\}, \{y = 2, x = 8\}$

   ANS: $\boxed{8, 2}$

6. In how many ways can 6 boys and 6 girls be teamed into pairs, if each pair must contain one girl?

   ANS: $6! = \boxed{720}$

7. What prime number is nearest to 1994?

   ANS: $\boxed{1993}$

8. Paul Erdös (pronounced "Air′-dish") wrote hundreds of papers in mathematics, many of which were coauthored by various mathematicians representing nearly every country on Earth. In which country was Paul Erdös born?

   ANS: $\boxed{\text{Hungary}}$

Team Competition

12:00 noon

1. In how many ways can the US Senate pick a committee of 4 from among its 100 members?

   ANS: $\binom{100}{4} = \boxed{3921\,225}$

2. What are the dimensions of a rectangle with area 104 and perimeter 42?

$$\begin{aligned} xy &= 104 \\ 2x + 2y &= 42 \end{aligned}$$

   ANS: Solution is : $\{y = 8, x = 13\}, \{x = 8, y = 13\}$ $\boxed{8 \text{ and } 13}$

3. A shelf will hold 20 calculus textbooks and 24 algebra textbooks, or 15 calculus textbooks and 36 algebra textbooks. How many calculus books alone will the shelf hold?

   ANS: $\left\{ \begin{array}{c} 20c + 24a = 1 \\ 15c + 36a = 1 \end{array} \right\}$, $a = \frac{1}{72}, c = \frac{1}{30}$. Shelf will hold $\boxed{30}$ calculus books

4. What is the area of a right triangle having a leg of length ℓ and hypotenuse of length 5?

   ANS: $A = \boxed{\frac{\ell\sqrt{25-\ell^2}}{2}}$

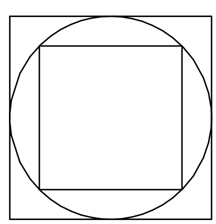
5. It takes 1000 square tiles to tile a room, or 1440 smaller tiles whose edge is 1 inch less. How large is the room in square feet?

   ANS: $1000x^2 = 1440\left(x - \frac{1}{12}\right)^2$, Solution is : $x = \frac{1}{2}$, $1000\left(\frac{1}{2}\right)^2 = \boxed{250}$ ft^2

6. A baseball manager has selected 9 starters for a game. If the pitcher must bat last and the second baseman must bat first, how many different batting line-ups are possible?

   ANS: $7! = \boxed{5040}$

7. A square is inscribed in a circle, which in turn is inscribed in a square. What percentage of the area of the large square is inside the small square?



   ANS: $\boxed{50\%}$

Tiebreaker Question: What is the remainder of 2^{10} is divided by 3?
ANS: $2^{10} \bmod 3 = \boxed{1}$

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1. The sum of 6 consecutive integers is 93. What is the largest of the 6 integers?

   ANS: $\boxed{18}$

2. How many different 6-place license plates are possible if the first two places are letters and the last 4 places are digits?

   ANS: $26^2 \times 10^4 = \boxed{6,760,000}$

3. State the contrapositive of the statement, "If $x^2 \le 9$, then $x \le 3$."

   ANS: $\boxed{\text{If } x > 3, \text{ then } x^2 > 9.}$

4. What is the area enclosed by the ellipse $\dfrac{x^2}{36} + \dfrac{y^2}{64} = 1$?

   ANS: $\boxed{48\pi}$

5. How many subsets does a set with 9 elements have?

   ANS: $\boxed{512} = 2^9$

6. An orange has a diameter that is 95% fruit and 5% peel. To the nearest percent, what percentage of the volume is the peel?

   ANS: $14.2625 \approx \boxed{14}$ %

7. An icosahedron has 20 faces and 12 vertices. A diagonal of such a solid is a line segment joining two vertices not lying in the same face. How many diagonals are there?

   ANS: $\frac{6\times 12}{2} = \boxed{36}$ Each vertex connected to 5 other vertices, so each vertex lies on 6 diagonals.

8. For whom was the computer language Ada named?

   ANS: Ada $\boxed{\text{Lovelace}}$

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9. The set S has 10 elements. If a and b are (distinct) elements of S, how many subsets of S contain both a and b?

   ANS: $\boxed{256} = 2^8$

10. The formula $e^{i\pi} + 1 = 0$ relates five of the most popular numbers in mathematics. What is π rounded to 10 significant digits?

    ANS: $\pi = 3.14159\,26535\,8979\ldots \approx \boxed{3.14159\,2654}$

11. If it takes 16 minutes to inflate a large spherical balloon to a radius of 2 meters, how long will it take to inflate a large spherical balloon to a radius of 5 meters?

    ANS: $\boxed{250}$ min

12. Which ancient civilization is responsible for dividing the circle into 360 equal parts (which we now call *degrees*)?

    ANS: $\boxed{\text{The Babylonians.}}$

13. Find all the roots of the equation $x^3 + 11x^2 + 38x + 40 = 0$?

    ANS: $\boxed{-2,\ -5,\ -4}$

14. How far is the point $(3, 0)$ from the line $3y = 4x$?

    ANS: $\frac{x}{3} = \frac{4}{5} \implies x = \boxed{\frac{12}{5}} = \boxed{2.4}$

15. Which is largest; π, 355/113, or 3.1416?

    ANS: $355/113 \approx 3.14159292$, $\pi \approx 3.141592654$, $\boxed{3.1416}$

Tiebreaker Question: Sometimes, always, or never: The product of two irrational numbers is irrational.
ANS: $\boxed{\text{sometimes}}$ $\sqrt{2} \times \sqrt{3} = \sqrt{6}$ is irrational, $\sqrt{2} \times \frac{1}{\sqrt{2}} = 1$ is rational

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1. A computer sequentially computes integers by the following rule: If n is odd then add 5 and divide the result by 2; otherwise add 3 . Starting at $n = 13$, what is the integer after 6 iterations?

   ANS: $13 \to \frac{13+5}{2} = 9 \to \frac{9+5}{2} = 7 \to \frac{7+5}{2} = 6 \to 9 \to 7 \to \boxed{6}$

2. The height of a rectangle is 25% less than its base. The perimeter of the rectangle is 14 inches. Find the area of the rectangle.

$$\begin{aligned} h &= \frac{3}{4}b \\ 2h + 2b &= 14 \end{aligned}$$

   ANS: Solution is : $\{b = 4, h = 3\}$ so area is $\boxed{12}$ in^2

3. How bits are there in a nibble?

   ANS: $\boxed{4}$ (a nibble is half a byte, which is 8 bits)

4. A bag contains 4 white balls, 5 red balls, and 3 green balls. If 3 balls are selected at random from the bag, what is the probability that they are white?

   ANS: $\frac{\binom{4}{3}}{\binom{12}{3}} = \boxed{\frac{1}{55}}$

5. In how many ways can one arrange the letters in OBOE?

   ANS: $\frac{4!}{2!} = \boxed{12}$

6. How many minutes were there in October of this year?

   ANS: $31 \times 24 \times 60 = \boxed{44\,640}$ minutes

7. The graph of the polynomial equation $y = x^4 - x^3 - 9x^2 + 7x + 6$ crosses the x-axis at $x = 3$. Factor the polynomial $x^4 - x^3 - 9x^2 + 7x + 6$.

   ANS: $(x - 3)(x^3 + 2x^2 - 3x - 2)$

8. One of the most influential mathematicians of all time was the ninth century Arab named Mohammed ibn-Musa al-Khwarizmi. His last name survives in mathematics today as the term “algorithm”. His most important work was *Al-jabr wa’l mugabālah.* What mathematical term was derived from this title?

   ANS: $\boxed{\text{Algebra}}$ (Source: Boyer and Merzbach, *A History of Mathematics*

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9. Two baseball teams, the Rockies and the Cardinals, begin a 7-game series. The odds makers give the Rockies 8 to 1 odds of winning the first game. Assuming the odds makers are good at their job, what is the probability that the Rockies will win the first game?

   ANS: $\frac{8}{9}$

10. A biological brick grows 25% in length, 25% in width, and shrinks in height by 35%. Is it larger or smaller than when it started out?

    ANS: $1.25 \times 1.25 \times .65 = 1.015\,625$ $\boxed{\text{larger}}$

11. Sarah is an 80% free throw shooter. What is her expected score if she shoots a one and one (if she makes the first she gets a second chance)?

    ANS: $\frac{4}{5} \times \frac{4}{5} \times 2 + \frac{4}{5} \times \frac{1}{5} \times 1 = \boxed{\frac{36}{25}}$ or $\boxed{1.44}$

12. How many strings of length 8 can be made with the letters in PARABOLA?

    ANS: $8!/3! = \boxed{6720}$

13. You are given seven points in the plane, no three of which lie on the same line. How many lines are there which pass through exactly two of these points?

    ANS: $\binom{7}{2} = \boxed{21}$

14. The sum of 4 consecutive integers is 58. What is the smallest of the 4 integers?

    ANS: $n + (n+1) + (n+2) + (n+3) = 58$, Solution is : $n = \boxed{13}$

15. If $m > 0$ and the points $(m, 9)$ and $(1, m)$ lie on a line with slope m, find m.

    ANS: $m = \frac{9-m}{m-1}$, Solution is : $\{m = 3\}, \{m = -3\}$ $\boxed{3}$

Tiebreaker Question: How many distinct real roots does the polynomial $x^5 - 1$ have?

ANS: $\boxed{\text{one}}$ $x^5 - 1$, roots: $\begin{cases} -.809\,016\,994\,4 - .587\,785\,252\,3i \\ -.809\,016\,994\,4 + .587\,785\,252\,3i \\ .309\,016\,994\,4 - .951\,056\,516\,3i \\ .309\,016\,994\,4 + .951\,056\,516\,3i \\ 1.0 \end{cases}$

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1. This British mathematician lead a team of mathematicians and cryptologists during World War II that broke ciphertext generated by the German Enigma machine. Name this person.

   ANS: Alan $\boxed{\text{Turing}}$

2. According to Descartes' Rule of Signs, how many positive real roots does the polynomial equation $3x^4 + 10x^2 + 5x + 4 = 0$ have?

   ANS: $\boxed{\text{None}}$ (since there are no sign changes)

3. Give an equation in the form $ax + by = c$, where a, b, and c are integers, for the line through $(4, -4)$ that is parallel to the line $y = \frac{1}{2}x - 9$.

   ANS: $\boxed{-x + 2y = -12}$ or $\boxed{x - 2y = 12}$

4. What is the sum of the first 88 positive integers?

   ANS: $\frac{88 \times 89}{2} = \boxed{3916}$

5. Let S be the set of the first 6 natural numbers. How many of the 64 subsets of S contain the number 2?

   ANS: $\boxed{32}$ or $\boxed{\text{Half}}$

6. In how many ways can ALLAN (spelled A-L-L-A-N) misspell his name, assuming he uses all the right letters (the right number of times)?

   ANS: $\frac{5!}{2!2!} - 1 = \boxed{29}$

7. Where does the circle of radius 5 centered at the origin intersect the line passing through the origin with slope $-3/4$?

   ANS: $\left\{ \begin{array}{c} x^2 + y^2 = 25 \\ y = -\frac{3}{4}x \end{array} \right\}$, Solution is : $\boxed{\{y = 3, x = -4\}, \{y = -3, x = 4\}}$

8. Who designed, but never build, an early "analytic engine" that led eventually to the Automatic Sequence Controlled Calculator developed jointly by IBM and Harvard University in 1944?

   ANS: Charles $\boxed{\text{Babbage}}$

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9. A 7-symbol license plate has 4 letters and 3 digits. What is the probability that at least one of the digits is a 0?

   ANS: $\boxed{\frac{271}{1000}}$

10. If g is a function such that $g(1) = 1$, $g(2) = -1$, and

$$g(n+1) = g(n) + 2g(n-1)$$

   for $n \geq 2$, what is $g(4)$?

   ANS: $g(3) = g(2) + 2g(1) = -1 + 2 = 1 \qquad g(4) = g(3) + 2g(2) = 1 + 2(-1) = \boxed{-1}$

11. A deck of 52 cards is thoroughly shuffled and the cards are turned over two at a time. How many pairs (two cards of the form $\boxed{\text{A}}\boxed{\text{A}}$ or $\boxed{5}\boxed{5}$) do you expect to see?

   ANS: $26\left(\frac{3}{51}\right) = \boxed{\frac{26}{17}}$

12. What is the sum of the roots of the polynomial $x^3 + 16x^2 + 73x + 90$?

   ANS: roots: $\{-5, -9, -2\}$ $\boxed{-16}$

13. A shirt has been marked down 35% and then 10% to \$23.40. What was the original price?

   ANS: \$$\boxed{40}$

14. A circular pizza is diameter 12 inches is cut into 12 congruent slices. What is the perimeter of each slice?

   ANS: $\boxed{12 + \pi}$

15. What mathematician first resolved the Königsberg bridge problem?

   ANS: $\boxed{\text{Euler}}$

Tiebreaker Question: Name the smallest integer whose square root is $> \pi$.
ANS: $\boxed{10}\sqrt{10} = 3.162\,277\,66 > \pi = 3.141\,592\,654 > \sqrt{9} = 3$

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1. List the following three numbers in increasing order: $\sqrt[6]{10}$, $\sqrt{2}$, $\sqrt[3]{3}$.

   ANS: $\boxed{\sqrt{2}} = 1.4142 < \boxed{\sqrt[3]{3}} = 1.4422 < \boxed{\sqrt[6]{10}} = 1.4678$

   note: $\left(\boxed{2^{1/2}}\right)^6 = 8 < \left(\boxed{3^{1/3}}\right)^6 = 9 < \left(\boxed{10^{1/6}}\right)^6 = 10$

2. What is the sum of the first seven odd positive integers?

   ANS: $1 + 3 + 5 + 7 + 9 + 11 + 13 = \boxed{49} = 7^2$

3. What Greek philosopher raised paradoxes that argued that motion is impossible?

   ANS: $\boxed{\text{Zeno}}$ of Elea

4. The distance between the points $(2, 4)$ and $(7, c)$ is 13. Find all the possible values for c.

   ANS: $(2 - 7)^2 + (4 - c)^2 = 13^2$, Solution is : $\boxed{\{c = 16\}, \{c = -8\}}$

5. 1800 cm^3 of paint are required to paint the outside of a cubic box of volume $1\,\text{m}^3$. How much paint is needed to paint the outside of a cubic box of volume $27\,\text{m}^3$ if the box has no lid?

   ANS: $\boxed{13\,500}$ cm^3

6. What is the largest integer that can be stored in a 6-bit computer word?

   ANS: $\boxed{63}$

7. The diameter of a square of edge a is $a\sqrt{2}$. The diameter of a cube of edge a is $a\sqrt{3}$. What is the diameter of a 4-dimensional cube of edge a?

   ANS: $\boxed{2a}$

8. What is the perimeter of an isosceles triangle with base 16 and area 48?

   ANS: $\boxed{36}$

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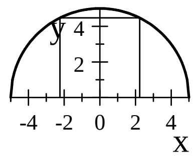
9. A square is inscribed in a semicircle of radius 9. Find the area of the square.



ANS: $\boxed{\frac{324}{5}} = \boxed{64\frac{4}{5}}$

10. Give the prime factorization of the smallest integer divisible by 10, 5, 1, and 2.

    ANS: $\boxed{2 \times 5}$

11. Who wrote the monumental *Principia mathematica*?

    ANS: Bertrand $\boxed{\text{Russell}}$ and Alfred North $\boxed{\text{Whitehead}}$

12. A set of points in the complex plane is determined by iteration of the function $z \to z^2 - \lambda$, where z and λ are complex numbers. What is the name of this set?

    ANS: $\boxed{\text{Mandelbrot}}$ set named after Benoit B. Mandelbrot

13. A pair of dice is rolled. What is the probability that the total is 5?

    ANS: $\boxed{\frac{1}{9}}$

14. How many distinct complex roots does the polynomial $x^5 - 1$ have?

    ANS: $\boxed{5}$

15. Partition 25 into two parts such that the difference of their square roots is 1.

    ANS: $\boxed{9,\ 16}$

Tiebreaker Question: Which has the larger surface area, a cube of volume 1 or a sphere of volume 1?
ANS: $\boxed{\text{cube}}$

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1. Art spent half his money plus 50¢, then half the remainder plus 50¢, then half of what he had left plus 50¢, when he found that he had \$2.00 remaining. How much did he start with?

   ANS: $\left(\left(\left(2+\frac{1}{2}\right)2+\frac{1}{2}\right)2+\frac{1}{2}\right)2 = \boxed{\$23.00}$

2. What is the equation of the line in the xy plane all of whose points are equidistant from the two points $(1,1)$ and $(-1,-1)$?

   ANS: $\boxed{y=-x}$

3. Two positive integers have a sum of 9. What is the smallest possible value for the sum of their cubes?

   ANS: $4^3+5^3=\boxed{189}$

4. Farmer Jill raises goats and geese. If she counts 32 eyes and 44 feet, how many goats and how many geese does Jill have?

   ANS: $\boxed{\text{10 geese and 6 goats}}$.

5. Five straight lines are drawn in the plane. What is the largest possible number of points of intersection?

   ANS: $\binom{5}{2}=\boxed{10}$

6. What is the volume of the tetrahedron whose vertices are $(0,0,0)$, $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$?

   ANS: $\boxed{1/6}$

7. If $4^p=5$ then what is 16^p?

   ANS: $\boxed{25}$

8. Which is larger, 1 cubic inch or 16 cubic centimeters? (use 1 in = 2.54 cm)

   ANS: $2.54^3 = 16.387064\text{ cm}^3 = \boxed{1\text{ in}^3}$

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9. Allison scored 76 on the first exam and 77 on the second exam. What must she average on the next two exams to bring her average for the four exams up to 80?

   ANS: $\boxed{\frac{167}{2}} = \boxed{83\frac{1}{2}}$

10. A pyramid is build out of cubical blocks by placing 64 blocks on the floor, 49 blocks on top of the bottom layer, and so forth. How many cubes are required to build the pyramid?

    ANS: $\sum_{i=1}^{8} i^2 = \boxed{204} = \frac{8\times9\times17}{6}$

11. George is a 90% free-throw shooter. What is his expected score if he shoots three free throws?

    ANS: $\boxed{\frac{27}{10}}$ or $\boxed{2.7}$

12. Two 3-digit integers consist of the same digits, but in the reverse order. What is the largest possible difference between the two numbers?

    ANS: $991 - 199 = 981 - 189 = \cdots = 901 - 109 = \boxed{792}$

13. Given the circle $x^2 - 2x + y^2 = 0$ in the xy plane, what are the equations of the two vertical tangent lines to this circle?

    ANS: $\boxed{x = 0}$ and $\boxed{x = 2}$

14. If a 500-Watt sound system can break a glass goblet placed 2 ft away from the speaker, how powerful a sound system would it take to break a similar glass goblet placed 12 ft from the speaker?

    ANS: $\boxed{18\,000}$-Watt

15. What is the area of the parallelogram with vertices $(0, 0)$, $(4, 6)$, $(7, 5)$, and $(11, 11)$?

    ANS: $\boxed{22}$

Tiebreaker Question: Name the largest integer whose 5th power is < 1000.
ANS: $\boxed{3}$ $3^5 = 243$, $4^5 = 1024$

CSU Math Day 1999
Team Competition
1:50 PM

1. What is the surface area of a spherical raindrop of diameter 0.5 millimeters?

   ANS: $4\pi\,(.5/2)^2 = \boxed{.25\pi}$ mm^2

2. A pyramid is built out of blocks by placing 100 blocks on the floor, placing 81 blocks on top of the bottom layer, and so forth. How many cubes are there in the pyramid?

   ANS: $\sum_{i=1}^{10} i^2 = \boxed{385} = \frac{10 \cdot 11 \cdot 21}{6}$

3. 4 positive integers have a sum of 14. What is the maximum possible value for the sum of their squares?

   ANS: $\boxed{124}$

4. What is the length of the arc on a circle of radius 9 subtended by a central angle of $120\,^\circ$?

   ANS: $\boxed{6\pi}$

5. What cubic curve was named after Maria Agnesi?

   ANS: $\boxed{\text{Witch of Agnesi}}$

6. For what choices of a does the polynomial $ax^2 - 6x + 2$ have no real roots?

   ANS: $\boxed{a > \frac{9}{2}}$

7. Ann keeps flies and spiders in a box in her dorm room during the Halloween season. There are a total of 19 creatures with 132 legs. How many flies and how many spiders does she have? (Flies have 6 legs and spiders have 8)

   ANS: $\boxed{\text{10 and 9}}$ since $10 \cdot 6 + 9 \cdot 8 = 132$

8. If the first term of a geometric sequence is $\frac{1}{3}$ and the second term is $\frac{4}{15}$, what is the fifth term?

   ANS: $\boxed{\frac{256}{1875}}$

Team Competition
1:50 PM

9. After what mathematician was the Cartesian coordinate system named?

   ANS: René $\boxed{\text{Descartes}}$

10. According to the Rational Root Theorem, what are all the possible rational roots of the polynomial $2x^3 - 5x^2 - 11x - 4$?

    ANS: $\boxed{\pm\frac{1}{2},\ \pm 1,\ \pm 2,\ \pm 4}$

11. A particle, initially at $(2,5)$, moves along a line of slope -2 to a new position (x,y). Find y if $x = -4$.

    ANS: $y = \boxed{17}$

12. What is the area of an equilateral triangle inscribed in a circle of radius 2?

    ANS: $\boxed{3\sqrt{3}}$

13. What is the area of the largest square that can be inscribed in the ellipse $\frac{x^2}{4} + \frac{y^2}{2} = 1$?

    ANS: $x = y \Longrightarrow \frac{3x^2}{4} = 1 \Longrightarrow 4x^2 = \boxed{\frac{16}{3}}$

14. This research organization has employed applied mathematicians such as Ronald Graham, Richard Hamming, and Claude Shannon, each of whom have made major contributions to applied discrete mathematics. Name this organization.

    ANS: $\boxed{\text{Bell Labs}}$ or $\boxed{\text{AT\&T Bell Laboratories}}$

15. List the following three numbers in increasing order: 2^{10}, $6!$, 10^3.

    ANS: $\boxed{6!} = 720 < \boxed{10^3} = 1000 < 2^{10} = \boxed{1024}$

Tiebreaker Question: A rectangle has perimeter 34 and area 52. What are the dimensions of the rectangle?
ANS: $\boxed{\text{4 by 13}}$

CSU Math Day 1999
Team Competition
2:05 PM

1. A triangle has sides of length 13, 10, and 13. Find the area of the triangle.

   ANS: Altitude is 12, so $Area = \frac{1}{2}(10)(12) = \boxed{60}$

2. What is the area of the smallest right triangle with all sides positive integers?

   ANS: $3-4-5$ right triangle has area $3 \cdot 2 = \boxed{6}$

3. 18 liters of paint are required to paint the outside of a cubic box of volume $64\,\mathrm{m}^3$. How much paint is needed to paint the outside of a cubic box of volume $125\,\mathrm{m}^3$?

   ANS: $\boxed{\frac{225}{8}}$

4. The age of a mathematician and her son added together is 49. In 3 years the mother will be 4 times as old as her son. How old is the mathematician?

   ANS: $\boxed{41}$

5. Sarah is a 60% free throw shooter. What is the probability that she misses 2 in a row?

   ANS: $\boxed{16.0\%}$ or $\boxed{.16}$ or $\boxed{\frac{4}{25}}$

6. A triangle has sides of length 8, 5, and 5. Find the area of the triangle.

   ANS: Altitude is 3, so $Area = \frac{1}{2}(8)(3) = \boxed{12}$

7. According to Descartes' Rule of Signs, how many negative real roots does the polynomial equation $x^4 - 8x^2 + 6x - 7 = 0$ have?

   ANS: $\boxed{\text{one}}$ (since there is one sign change if x is replaced by $-x$)

8. If a cube has a volume of $64\,\mathrm{cm}^3$, what is its surface area?

   ANS: $\boxed{96}\,\mathrm{cm}^2$

Team Competition
2:05 PM

9. If two cards are drawn from a standard deck of 52 cards, what is the probability that both are kings?

   ANS: $\frac{\binom{4}{2}}{\binom{52}{2}} = \frac{6}{1326} = \boxed{\frac{1}{221}}$

10. The quartic polynomial $6x^4 - 11x^3 - 66x^2 + 59x - 12$ has four real roots. What is the product of these four roots?

    ANS: $6\,(x+3)\,(x-4)\left(x-\frac{1}{2}\right)\left(x-\frac{1}{3}\right),\ -3 \cdot 4 \cdot \frac{1}{2} \cdot \frac{1}{3} = \boxed{-2}$

11. Find C so that the equation $x^2 - 20x + C = 0$ has exactly one real root.

    ANS: 100, $a \pm \sqrt{a^2 - C} = a$ when $\boxed{C = 100}$

12. The vertices of a quadrilateral are at the points $(0,0)$, $(3,1)$, $(4,0)$, and $(2,-4)$. What is the area of the quadrilateral?

    ANS: $Area = \frac{1}{2} \cdot 4 \cdot 1 + \frac{1}{2} \cdot 4 \cdot 4 = \boxed{10}$

13. What mathematician popularized the use of δ and ϵ in proofs involving limits?

    ANS: Augustin-Louis $\boxed{\text{Cauchy}}$

14. What are the next two prime numbers greater than 50?

    ANS: $51 = 3 \times 17, \boxed{53}, 55 = 5 \times 11, 57 = 3 \times 19, \boxed{59}$

15. A ball thrown vertically into the air 100 feet, falls and rebounds to a height of 40 feet the first time, rebounds to 16 feet on the second bounce, and so forth. What is the entire distance the ball will have moved when it finally comes to rest?

    ANS: $2\sum_{i=0}^{\infty} 100 \cdot \left(\frac{2}{5}\right)^i = \frac{1000}{3} = \boxed{333\frac{1}{3}}$ ft

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Tiebreaker Question: A rectangle has perimeter 34 and area 16. What are the dimensions of the rectangle?

ANS: $\boxed{\text{1 by 16}}$

CSU Math Day 1999
Team Competition
2:15 PM

1. The formula $e^{i\pi} + 1 = 0$ relates five of the most popular numbers in mathematics. What is $e^{2i\pi}$?

   ANS: $e^{2i\pi} = \boxed{1}$

2. Find x so that the average of the four numbers 35, 27, 47, and x is 32.

   ANS: $x = \boxed{19}$

3. A triangular section of Old Town is divided into a smaller triangle and two trapezoids by two streets parallel to one of the boundary streets. The heights of the two trapezoids are equal to the height of the small triangle, and the area of the middle trapezoid is 12 acres. How many acres are there in the larger trapezoid?

   ANS: $\boxed{20}$ acres

4. What is the smallest 3-digit prime?

   ANS: $\boxed{101}$

5. Who proved Fermat's Last Theorem

   ANS: Andrew $\boxed{\text{Wiles}}$

6. A wooden cube of edge 4 inches is painted red. The cube is then cut into 64 one-inch cubes by making 9 saw cuts. How many of the one-inch cubes have exactly one red face?

   ANS: 6 faces $\times$ 4 cubes per face = $\boxed{24}$ cubes

7. If a third degree polynomial has leading coefficient of -4 and roots -5, 1, and 2, what is its constant term?

   ANS: $\boxed{-40}$ since $-4\,(x+5)\,(x-1)\,(x-2) = -4x^3 - 8x^2 + 52x - 40$

8. What is the sum of the roots of the polynomial $x^2 - 31x + 68$?

   ANS: $\boxed{31}$, since $(x-a)(x-b) = x^2 - x\,(a+b) + ab \Rightarrow a+b = 31$ (roots are $\frac{31}{2} + \frac{1}{2}\sqrt{689}$, $\frac{31}{2} - \frac{1}{2}\sqrt{689}$)

Team Competition
2:15 PM

9. The polynomial $5x^7 + 21x^5 + 35x^3 + 35x + 18$ has one real root. How many imaginary roots does it have?

   ANS: $\boxed{6}$

10. List the following three numbers in increasing order: $\frac{3}{7}, \frac{4}{9}, \frac{7}{16}$

    ANS: $\boxed{\frac{3}{7}} = .428\,57 < \boxed{\frac{7}{16}} = .437\,5 < \boxed{\frac{4}{9}} = .444\,44$

    Note that $\frac{7}{16} = \frac{3+4}{7+9}$ is between $\frac{3}{7}$ and $\frac{4}{9}$

11. The sum of the squares of two positive integers is 394, and the difference of their squares is 56. What are the numbers?

    ANS: $\left\{ \begin{array}{c} x^2 + y^2 = 394 \\ x^2 - y^2 = 56 \end{array} \right\}$, Solution is : $\boxed{y = 13, x = 15}$

12. If g is a function such that $g(1) = -1$,$g(2) = 2$, and

$$g(n) = g(n-2) + 3g(n-1)$$

    for $n \geq 3$,what is $g(4)$?

    ANS: $g(3) = g(1) + 3g(2) = -1 + 6 = 5$,$g(4) = g(2) + 3g(3) = 2 + 15 = \boxed{17}$

13. What is the smallest positive integer whose square is larger than 688?

    ANS: $\boxed{27}$ since $26^2 = 676$ and $27^2 = 729$

14. What is the base 9 representation of the decimal number 618?

    ANS: $\boxed{756_9}$ since $7 \times 9^2 + 5 \times 9 + 6 = 618$

15. To the nearest minute, at what time between 10:30 AM and 11:00 AM are the minute hand and the hour hand at right angles?

    ANS: $50 + \frac{m}{12} = m + 15$, Solution is : $m = \frac{420}{11} = 38.18181818$ $\boxed{10{:}38}$ AM

Tiebreaker Question: How many vertices does a dodecahedron have?
ANS: $\frac{5\times12}{3} = \boxed{20}$

CSU Math Day 1999
Small School Team Final

1. A square has area α in^2 and has perimeter α in. What is α?

   ANS: $\alpha = x^2 = 4x$ so $x = 4$, $\alpha = \boxed{16}$

2. The compact disk UR2ugly sells at outlet AC for \$13.90 less a discount of 15%, and at outlet DC for \$15.70 less a discount of 25%. Which outlet has the lower price?

   ANS: $\boxed{\text{AC}}$ $13.90 \times .85 = \$11.815$, DC $15.90 \times .75 = \$11.925$

3. The number 12 is called abundant because the sum of the proper divisors $1+2+3+4+6 = 16$ is greater than 12. What is the next abundant number?

   ANS: $\boxed{18} < 1+2+3+6+9 = 21$

4. If $\binom{n}{4} + \binom{n}{0} = 6$, what is n?

   ANS: $\boxed{5}$

5. At the local Dairy Queen, the "Monster Sundae" can be ordered with any of 8 flavors of ice cream plus any or all of 4 toppings. If you order one such sundae every Saturday, how many weeks will it be before you must order the same sundae twice?

   ANS: $\boxed{128}$

6. Andrew Wiles has proven something that had remained an open problem for over 350 years. Either state the problem or name the person who claimed over 350 years ago to have a solution to the problem.

   ANS: $\boxed{x^n + y^n = z^n \text{ has no positive integer solutions for } n > 2}$ OR $\boxed{\text{Fermat}}$

7. How many committees of 3 men and 3 women can be formed from a group of 7 men and 5 women?

   ANS: $\binom{7}{3}\binom{5}{3} = \boxed{350}$

8. What is the area of a regular hexagon with sides of length 2?

   ANS: $\boxed{6\sqrt{3}}$

Small School Team Final
1:00 AM

9. Neglecting the order of addition, in how many ways can 30 be written as the sum of two primes?

   ANS: $\boxed{\text{3 ways}}$ $30 = 7 + 23 = 11 + 19 = 13 + 17$

10. A rectangle's length is increased by 40% and its width is decreased by 50%. How does its area change?

    ANS: $\boxed{\text{decreases by 30\%}}$

11. Name the Scottish mathematician who invented logarithms.

    ANS: $\boxed{\text{Napier}}$

12. What is the smallest positive integer n such that $n^2 - n + 41$ is not prime?

    ANS: 41, since

n	1	2	3	4	5	$\cdots$	40	$\boxed{41}$
$n^2 - n + 41$	41	43	47	53	61	$\cdots$	1601	$1681 = 41^2$

13. Give the points of intersection of the two curves $y = 20 - 6x$ and $y = 8 - 6x + 3x^2$.

    ANS: $\left\{ \begin{array}{c} y = 8 - 6x + 3x^2 \\ y = 20 - 6x \end{array} \right\}$, Solution is : $\boxed{(2,8),\ (-2,32)}$

14. A computer sequentially computes integers by the following rule: If n is odd then replace n by $3n + 1$; otherwise replace n by $n/2$. If n starts at 7, what is n after 5 iterations?

    ANS: $7 \to 22 \to 11 \to 34 \to 17 \to \boxed{52}$

15. A rectangle's length is increased by 20% and its width is decreased by 20%. How much does its area change?

    ANS: $1.2L \times .8W = \boxed{.96LW}$ or $\boxed{\text{decreases by 4\%}}$

Tiebreaker Question: How many real roots does the polynomial $x^2 + 4x + 5$ have?
ANS: $x^2 + 4x + 5$, roots: $-2 + i, -2 - i$ $\boxed{\text{None}}$ or $\boxed{\text{zero}}$

CSU Math Day 1999
Small Team Extra

1. At 6:00 a.m., Chad starts jogging at 5 km/hr. At 9:00 AM Marzelle starts jogging from Chads's starting place at 6 km/hr. How far behind is Marzelle at 11:00 AM?

   ANS: $\boxed{13}$ km

2. Kyle uses pure guessing on a TRUE/FALSE exam. Which of the following options give Kyle the best chance to score (at least) 80%? (A) A ten-question exam (guess correctly on 8, 9, or 10 questions). (B) A five-question exam (guess correctly on 4 or 5 questions). (C) The chances are equal.

   ANS: (A) probability of 80% $\approx .054688$ $\boxed{\text{(B)}}$ probability of 80% $\approx .188$

3. Name the smallest integer whose forth power is less than 1000.

   ANS: $(-5)^4 = 625$, $(-6)^4 = 1296$, etc. $\boxed{-5}$ is the smallest

4. What is the coefficient of xy^2 in the expansion of $(x+y)^3$?

   ANS: $\boxed{3}$

5. A wheel of radius 1 foot rolls without slipping around the outside of a stationary wheel of radius 2 feet. Exactly how many rotations does the small wheel make?

   ANS: $\boxed{3}$

6. During a recent election, Alfie, Betty, and Gammer received votes for mayor. Alfie received $\frac{1}{5}$ as many votes as Betty and 2 times as many as Gammer. If the total number of votes was 24 700, how many did each person get?

   ANS: $\boxed{\text{Alfie: 3800 Betty: 19 000 Gammer: 1900}}$

7. Bo is going to the store to buy candy that will cost somewhere between 5 cents and 26 cents. What is the fewest number of coins Bo can carry in order to be certain to have exact change to buy the candy?

   ANS: 1¢, 1¢, 1¢, 1¢, 5¢, 10¢, 10¢ $\boxed{7}$ coins

8. What is the diameter of a circle with area $4\pi\,\text{cm}^2$?

   ANS: $\boxed{4}$

9. Given 4 gallons of a 30% antifreeze/water mixture, how much pure antifreeze must be added to yield a 35% antifreeze/water mixture? (Answer to the nearest one-tenth of a gallon.)

   ANS: $\boxed{\frac{3}{10}}$ gal $\quad (\approx .307\,692)$

10. The sum of the squares of two positive integers is 116 and the difference of their squares is 84. What are the two integers?

$$\begin{aligned} x^2 + y^2 &= 116 \\ x^2 - y^2 &= 84 \end{aligned}$$

   ANS: Solution is : $\{y = 4, x = 10\}$ $\boxed{10 \text{ and } 4}$.

11. A pair of dice is rolled 30 times. What is the expected number of doubles?

   ANS: $30\left(\frac{1}{6}\right) = \boxed{5}$

12. How many rearrangements of the letters a, b, c, d, e, f have a listed before b and b listed before c?

   ANS: $\frac{6!}{3!} = \boxed{120}$

13. What is π radians equal to in degrees?

   ANS: 180°

14. A computer sequentially computes integers by the following rule: If n is a square then multiply by 2; otherwise subtract 1. Starting at $n = 9$, what is the integer after 6 iterations?

   ANS: $9 \to 18 \to 17 \to 16 \to 32 \to 31 \to \boxed{30}$

15. List the following three numbers in increasing order: e^π, π^e, 3^3.

   ANS: $\boxed{\pi^e} = 22.459 < \boxed{e^\pi} = 23.141 < \boxed{3^3} = 27$

Tiebreaker Question: What is the prime factorization of 143?

ANS: $143 = \boxed{11 \times 13}$

CSU Math Day 1999
Large School Team Final

1. Grandma Josephine offers each of her 6 grandchildren the choice of 2 different kinds of cookies. If each grandchild only gets one cookie, in how many ways can the choices be made?

   ANS: $2^6 = \boxed{64}$

2. A regular icosahedron has 20 faces, each of which is an equilateral triangle. If the midpoint of each face is connected with an edge to the midpoint of each adjacent face, what solid do these new edges determine?

   ANS: $\boxed{\text{Dodecahedron}}$ (12 faces, each face is a pentagon.)

3. A computer sequentialy computes integers by the following rule: If n is odd then replace n by $3n+1$; otherwise replace n by $n/2$. If n starts at 3, what is n after 5 iterations?

   ANS: $3 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow \boxed{4}$

4. 5 poker players are seated at a round table. How many rearrangements are possible, if the only considerations are who is seated at each person's left, and at each person's right? (Because of the betting order Arlene on your left is different from Arlene on your right.)

   ANS: $4! = \boxed{24}$

5. A 6-foot man casts a 5-foot shadow. A flag pole next to him casts a 22-foot shadow. How tall in the flag pole?

   ANS: $\boxed{\frac{132}{5}}$ ft or $\boxed{26.4}$ ft

6. A triangle has 3 vertices and 3 edges. A tetrahedron has 4 vertices and 6 edges. How many vertices and edges does an n-dimensional tetrahedron have?

   ANS: $\boxed{n+1 \text{ vertices}}$ and $\boxed{n(n+1)/2 \text{ edges}}$

7. A quadrilateral kite is made with a right angle at the top, angles of 2α left and right, and an angle of α at the bottom. What is α?

   ANS: $5\alpha = 360° - 90° \Longrightarrow \alpha = \boxed{54°}$ or $\boxed{\frac{3\pi}{10}}$

8. The number $\frac{2}{\frac{1}{a}+\frac{1}{b}}$ has a mean musical name. What is that name?

   ANS: $\boxed{\text{Harmonic}}$ Mean (of a and b)

9. What is the sum of all the integers greater than 18 and less than 38?

   ANS: $\sum_{i=19}^{37} i = \boxed{532}$

10. How many ways can the top 3 finishers be picked in a 9-person race?

    ANS: $9 \times 8 \times 7 = \boxed{504}$

11. What is the prime factorization of 127?

    ANS: $\boxed{127}$ (prime)

12. On a certain planet, a stone falls 64 feet in 4 seconds, how far will it fall in 7 seconds?

    ANS: $\boxed{196}$

13. Find an integer between 100 and 1000 that is both a perfect square and a perfect cube.

    ANS: $\boxed{729} = 9^3 = 27^2$

14. If $f(x) = x^{\frac{4}{5}}$, what is $f(32)$?

    ANS: $\boxed{16}$

15. John and Jill traded positions several times while rowing a canoe through the Boundary Waters of Minnesota. With Jill at the rear, the canoe went fast enough to complete the entire trip in 10 hours, and with John at the rear the trip would have taken 14 hours. The trip actually took 12 hours. For how many hours did Jill sit in the back of the canoe?

    ANS: $1 = \frac{1}{10}t + \frac{1}{14}(12 - t)$, Solution is: $t = \boxed{5 \text{ hours}}$

Tiebreaker Question: If n and $n+2$ and both primes, then the pair $n, n+2$ is called a pair of twin primes. Examples include $17, 19$ and $29, 31$. What is the smallest pair of three-digit twin primes?

ANS: $\boxed{101,\ 103}$

CSU Math Day 1999
Team Competition
Large Team Extra

1. What complex numbers are equal to their squares?

   ANS: $x = x^2$, Solution is: $x = \boxed{0}$, $x = \boxed{1}$

2. A piece of string 64 inches long is cut into two pieces so that one piece is 6 inches shorter than the other. What are the lengths of the two pieces?

   ANS: $x + (x + 6) = 64 \Longrightarrow$, Solution is : $x = \boxed{29}$ in, $x + 6 = \boxed{35}$ in

3. The 4 faces of a regular tetrahedron are equilateral triangles. How many edges does a regular tetrahedron have?

   ANS: $\boxed{6}$

4. The graph of a cubic polynomial has x-intercepts 0 and 1 (only). What is a possible expression for the polynomial?

   ANS: $\boxed{x^2(x-1)} = \boxed{x^3 - x^2}$ or $\boxed{x\,(x-1)^2} = \boxed{x^3 - 2x^2 + x}$ (or a nonzero multiple)

5. What is the least common denominator of $\frac{4}{85}$ and $\frac{1}{90}$?

   ANS: $\operatorname{lcm}(85, 90) = \boxed{1530}$

6. What is the sum of the first six odd positive integers?

   ANS: $1 + 3 + 5 + 7 + 9 + 11 = \boxed{36} = 6^2$

7. A biological brick grows 20% in length, 15% in width, and shrinks in height by 30%. Is it larger or smaller than when it started out?

   ANS: $\boxed{\text{smaller}}$ ($1.2 \times 1.15 \times (1 - .3) = .966$)

8. If a mantel clock strikes the hours, how many times will it strike during a 24-hour period?

   ANS: $2\sum_{i=1}^{12} i = \boxed{156}$

Team Competition

Large Team Extra

9. A bag of chicken feed will feed 12 chickens for 40 days. For how many days will it feed 30 chickens?

   ANS: $\boxed{16}$

10. Kyle uses pure guessing on a TRUE/FALSE exam. Which of the following options give Kyle the best chance to score (at least) 50%? (A) A seven-question exam (guess correctly on 4, 5, 6, or 7 questions). (B) A five-question exam (guess correctly on 3, 4, or 5 questions). (C) The chances are equal.

    ANS: $\boxed{\text{(C)}}$ Chances are equal (both probabilities are $\frac{1}{2}$)

11. What did the Norwegian mathematician Niels Henrik Abel prove about general fifth-degree polynomials?

    ANS: $\boxed{\text{Cannot be solved}}$ in terms of radicals involving the coefficients

12. 3000 raffle tickets are to be sold for \$1 each. The winner receives \$600. If you purchase 1 ticket, how much are your expected earnings?

    ANS: $-1 + 600 \times \frac{1}{3000} = \boxed{-\ \$0.80}$

13. What is the perimeter of a right triangle with legs 8 and 15?

    ANS: $s^2 = 8^2 + 15^2$, $s = 17$, $P = 8 + 15 + 17 = \boxed{40}$

14. The average of your first 6 exams is 82. What is your new average if your lowest exam score of 57 is dropped?

    ANS: $\boxed{87} = \frac{6\times 82-57}{5}$

15. A basketball team has 12 players on the roster. How many different starting lineups are possible?

    ANS: $\binom{12}{5} = \boxed{792}$

Tiebreaker Question: How many distinct complex roots does the polynomial $x^6 - 1$ have?

ANS: $\boxed{\text{six}}$ $x^6 - 1$, roots:
$$\begin{array}{c} -1.0 \\ -.5 - .866\,025\,4i \\ -.5 + .866\,025\,4i \\ .5 - .866\,025\,4i \\ .5 + .866\,025\,4i \\ 1.0 \end{array}$$