Team Competition 1998
11:20

1. Each cycle of an air pump removes $\frac{1}{3}$ of the air remaining in a container. What fractional part of the air remains after 3 cycles?

   ANS: $\boxed{\frac{8}{27}}$

2. Adam can frame a house in 15 days, Burl can do it in 6 days, and Clark can do it in 10 days. Working together, how many days would it take for the three of them to frame a house?

   ANS: $\boxed{3}$ days

3. What is the perimeter of a regular hexagon inscribed in a circle of radius 10?

   ANS: $\boxed{60}$

4. Neglecting order of addition, in how many ways can 25 be written as a sum of 3 distinct primes?

   ANS: $\boxed{\text{2 ways}}$ $25 = 3 + 5 + 17 = 5 + 7 + 13$

5. Sarah averages 90% on multiple choice exams. What is the probability that she gets 100% on a 3-question quiz?

   ANS: $\boxed{.729}$ or $\boxed{\frac{729}{1000}}$

6. What is the greatest integer in $\frac{7}{27} + \frac{27}{7}$?

   ANS: $\frac{7}{27} + \frac{27}{7} = \frac{778}{189} = 4.11640\,21164\,0212\boxed{4}$

7. What is the statement of Fermat's Last Theorem?

   ANS: The equation $x^n + y^n = z^n$ has no positive integer solutions for any positive integer $n > 2$.

8. Pierre de Fermat was not a professional mathematician, although such famous results as Fermat's Last Theorem and Fermat's Little Theorem were named in his honor. What was his profession?

   ANS: Lawyer

9. An automobile is driven 17 000 miles with 5 tires rotated often for even wear. How many miles are there on each tire?

   ANS: 13 600

10. During a recent election, Alfie, Betty, and Gammer received votes for mayor. Alfie received $\frac{1}{4}$ as many votes as Betty and 3 times as many as Gammer. If the total number of votes was 17 600, how many did each person get?

    ANS: Alfie: 3300 Betty: 13200 Gammer: 1100

11. What is 9π radians equal to in degrees?

    ANS: 1620°

12. What are all the possible rational roots of $2x^3 - 5x^2 - x + 8 = 0$, as limited by the Rational Root Theorem?

    ANS: $\pm\frac{1}{2}, \pm 1, \pm 2, \pm 4, \pm 8$

13. A particle, initially at $(-2, -1)$ moves along a line of slope $\frac{1}{5}$ to a new position (x, y). Find y if $x = -27$.

    ANS: $y = -6$

14. Bo has \$4.40 in quarters and dimes. If Bo has 3 times as many dimes as quarters, how many of each type of coin does Bo have?

    ANS: 8 quarters and 24 dimes.

15. A pair of dice is rolled. What is the probability that the total is 2, 3 or 12?

    ANS: $\frac{1}{9}$ $(= \frac{4}{36})$

Team Competition 1998
11:40

1. What is the smallest number greater than 50 has the property that when divided by 15 the remainder is 1 and when divided by 18 the remainder is also 1?

   ANS: 271, since $271 = 15 \times 18 + 1$

2. What is the sum of the binomial coefficients (10 choose zero) plus (10 choose1) plus ■ plus (10 choose 10)?

   ANS: $1024 = 2^{10}$

3. The number 6 is perfect because $6 = 1 + 2 + 3$ is the sum of the proper divisors. Name the next perfect number.

   ANS: $\boxed{28} = 1 + 2 + 4 + 7 + 14$

4. An elevator moves at constant speed and it takes 21 seconds for an elevator to go from floor number 1 to floor number 4. How long should it take to go from floor number 1 to floor number 8?

   ANS: 49 seconds

5. For what choices of a does the polynomial $ax^2 - 12x + 1$ have two real roots?

   ANS: $a < 36$

6. A wheel of radius 1 foot rolls without slipping around the outside of a stationary wheel of radius 2 feet. Exactly how many rotations does the small wheel make?

   ANS: 3

7. Andrew Wiles has proven something that had remained an open problem for over 350 years. Either state the problem or name the person who claimed over 350 years ago to have a solution to the problem.

   ANS: $x^n + y^n = z^n$ has no positive integer solutions for $n > 2$ OR Fermat

8. Two cars start 2 miles apart and drive toward each other. One car goes 50 mph, the other 40 mph. After how many seconds do the two cars meet?

   ANS: 80 s

9. The sum of the squares of two positive integers is 164 and the difference of their squares is 36. What are the two integers?

   ANS: 8 and 10.

10. If a cube has a volume of $1000\,\text{cm}^3$, what is its surface area?

    ANS: $600\ \text{cm}^2$

11. Without using the word 'not', state the contrapositive of the statement, "If $x > 4$ then $x^2 > 16$".

    ANS: If $x^2 \leq 16$, then $x \leq 4$.

12. Bo is going to the store to buy candy that will cost somewhere between 5 cents and 26 cents. What is the fewest number of coins Bo can carry in order to be certain to have exact change to buy the candy?

    ANS: 1¢, 1¢, 1¢, 1¢, 5¢, 10¢, 10¢ $\boxed{\text{7 coins}}$

13. Brigitte plants a pumpkin seed. The area that is covered by the vine doubles every month. After 5 months the entire garden is covered. When was exactly half of the garden covered with the vine?

    ANS: $\boxed{\text{One month earlier}}$ or $\boxed{\text{after 4 months}}$

14. How many subsets does a set with 4 elements have?

    ANS: $16 = 2^4$

15. Assuming $0 < a < b$, express $\dfrac{a^b b^a}{a^a b^b}$ in terms of one quotient raised to a positive exponent.

    ANS: $\dfrac{a^b b^a}{a^a b^b} = \boxed{\left(\frac{a}{b}\right)^{b-a}}$

Team Competition 1998
12:00

1. If eggs weigh 1 oz each and a dozen eggs cost 90¢, what is the cost of a pound of eggs? (16oz = 1 pound)

   ANS: \$1.20

2. The sum of two integers is -3 and their product is -108. What are the two integers?

   ANS: 9 and -12.

3. What was the license plate number the Indian mathematician Ramanujan referred to when he said, "On the contrary, that's a very interesting number. It's the first number that can be written as the sum of 2 cubes in 2 different ways."

   ANS: 1729 $(= 12^3 + 1^3 = 10^3 + 9^3)$

4. A committee of 3 people is to be chosen from among 5 men and 8 women .How many ways can this be done if the committee must include at least one man and at least one woman?

   ANS: 220 $(= \binom{13}{3} - \binom{5}{3} - \binom{8}{3})$

5. What is the smallest 4-digit prime?

   ANS: 1009

6. The diagonal of a table with a square top is 6 ft. What is the area of the table top?

   ANS: 18

7. The equation

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right)$$

   provides an algorithm for approximating $\sqrt{2}$. Starting with $x_1 = 1$, what is x_3 (as a rational number)?

   ANS: $x_2 = \frac{1}{2}\left(1 + \frac{2}{1}\right) = \frac{3}{2}$, $x_3 = \frac{1}{2}\left(\frac{3}{2} + \frac{2}{3/2}\right) = \boxed{\frac{17}{12}} = \boxed{1\frac{5}{12}}$

8. According to Descartes' Rule of Signs, how many positive roots are there of the polynomial equation $2x^3 + 5x^2 - x - 8 = 0$?

   ANS: $\boxed{1}$ (One sign change in the sequence $++--$)

9. George is a 90% free-throw shooter. What is his expected score if he shoots two free throws?

   ANS: 1.8 or $\boxed{\frac{9}{5}}$

10. If the first term of a geometric sequence is $\frac{1}{3}$ and the second term is $\frac{4}{15}$, what is the fifth term?

    ANS: $\boxed{\frac{256}{1875}}$

11. The 4 faces of a regular tetrahedron are equilateral triangles. How many edges does a regular tetrahedron have?

    ANS: 6

12. What is the perimeter of an isosceles triangle with base 14 and area 42?

    ANS: $14 + 2\sqrt{85}$

13. How many positive divisors (including itself) does the number 81 have?

    ANS: 5 divisors are {1,3,9,27,81}

14. The probability of picking a dog to finish in the top 3 at the dog track is $\frac{1}{4}$. What is the probability of picking 2 straight losers?

    ANS: $\frac{9}{16}\left(= \left(\frac{3}{4}\right)^2\right)$

15. Two wheels are connected by a belt. One has a diameter of 30 centimeters and a speed of 100 rpm. The other has a speed of 300 rpm. What is its diameter?

    ANS: 10 centimeters

Team Competition 1998
12:20

1. Solve the equation $x\sqrt{0.09} = 9$

   ANS: $x = 30$

2. How far apart are the two points where the curves $y = x + 6$ and $y = x^2$ intersect?

   ANS: Points $(-2, 4)$,$(3, 9)$ are distance $\sqrt{(-2-3)^2 + (4-9)^2} = \boxed{5\sqrt{2}}$

3. If a regular polygon has 12 sides, what is the size, in degrees, of one of its interior angles?

   ANS: $150\,^\circ$

4. A rectangle's length is increased by 20% and its width is decreased by 20%. How much does its area change?

   ANS: $1.2 \times 0.8 = .96$ $\boxed{\text{decreases by } 4\%}$

5. What fraction is represented by the repeating decimal $0.9090909090\cdots$?

   ANS: $\boxed{\frac{10}{11}} = .909\,090\,909$

6. Henry explained his age by saying, "$\frac{2}{5}$ of my age less $\frac{1}{9}$ of my age next year is $\frac{1}{3}$ what my age was 5 years ago." What is his age now?

   ANS: $\frac{2}{5}a - \frac{1}{9}(a+1) = \frac{1}{3}(a-5)$, Solution is : $a = \boxed{35}$ years

7. Express the volume V of a cube as a function of the area A of one of its faces.

   ANS: $V = \boxed{A\sqrt{A}}$ or $V = A^{3/2}$

8. If 2 is the first prime, what is the eleventh prime?

   ANS: $2, 3, 5, 7, 11, 13, 17, 19, 23, 29, \boxed{31}$

9. Three positive integers have a sum of 10. What is the minimum possible value for the sum of their squares?

   ANS: $3^2 + 3^2 + 4^2 = \boxed{34}$

10. When this Hungarian mathematician died recently, the New York Times acknowledged his contributions to mathematics that included some 1500 published papers. Name this person.

    ANS: Paul Erdös

11. The equation $x_{n+1} = \frac{1}{2}\left(x_n + \frac{3}{x_n}\right)$ provides an algorithm for approximating $\sqrt{3}$. Starting with $x_1 = 1$, what is x_3 (as a rational number)?

    ANS: $x_2 = \frac{1}{2}\left(1 + \frac{3}{1}\right) = 2$, $x_3 = \frac{1}{2}\left(2 + \frac{3}{2}\right) = \boxed{\frac{7}{4}} = \boxed{1\frac{3}{4}}$

12. If $\sin x = \frac{1}{3}$ for some angle x between 0 and 90°, what is $\sin 2x$?

    ANS: $\sin 2x = 2 \sin x \cos x = 2\left(\frac{1}{3}\right)\sqrt{1 - \frac{1}{9}} = \boxed{\frac{4}{9}\sqrt{2}}$

13. What is the length of the arc on a circle of radius 10 subtended by a central angle of 54°

    ANS: $\boxed{3\pi}$

14. Three line segments joining the midpoints of the sides of a triangle determine a smaller triangle whose sides lie inside the larger triangle. What is the ratio of the area of the larger triangle to the area of the smaller triangle?

    ANS: $\boxed{4:1}$

15. A piece of string 64 inches long is cut into two pieces so that one piece is 6 inches shorter than the other. What are the lengths of the two pieces?

    ANS: $x + (x + 6) = 64$ implies $x = \boxed{29}$ in, $x + 6 = \boxed{35}$ in

Team Competition 1998
12:40

1. What number is halfway between $\frac{1}{4}$ and $\frac{1}{8}$?

   ANS: $\frac{3}{16}$ ($\frac{1}{2}\left(\frac{1}{4}+\frac{1}{8}\right) = \boxed{\frac{3}{16}}$

2. A solid statue is made by melting 10 cm^3 of metal and pouring it in to a mold. A larger model needs to be constructed by increasing each of its linear dimensions by a factor of 3. How much metal will the new statue require?

   ANS: $\boxed{270}$ cm^3

3. A cube has pyramids cut and discarded from each corner by passing planes through the midpoints of the edges adjacent to each of its vertices. How many edges does the new solid have?

   ANS: 8 vertices $\times 3$ edges/vertex $= \boxed{24}$ edges

4. A pair of dice is rolled. What is the probability that the total is 7 or 12?

   ANS: $\boxed{\frac{7}{36}}\left(=\frac{6}{36}+\frac{1}{36}\right)$

5. Sarah is an 80% free throw shooter. What is her expected score if she shoots a one and one (if she makes the first she gets a second chance)?

   ANS: $\boxed{\frac{36}{25}}$ or 1.44

6. Use the approximation $2^{10} \approx 10^3$ to estimate $\log_2\left(10^{42}\right)$.

   ANS: $\boxed{140}$

7. What is the distance between two opposite vertices of a cube of edge 10?

   ANS: $\boxed{10\sqrt{3}}$

8. What is the decimal (base 10) representation of the base 4 number $(121)_4$?

   ANS: $\boxed{25}$

9. In which quadrant is the center of the circle $x^2 + y^2 + 6x - 8y = 0$?

   ANS: $x^2 + 6x + y^2 - 8y + 25 = (x + 3)^2 + (y - 4)^2 = 25$, center $(-3, 4)$ $\boxed{\text{2nd}}$ quadrant

10. What is the largest integer that can be stored in a 6-bit computer word?

    ANS: $\boxed{63}$

11. A golf bag contains 13 balls, some yellow and the rest orange. Two balls are drawn at random from the bag and one is yellow, the other orange. What is the probability that exactly half of the balls in the bag are yellow?

    ANS: $\boxed{0}$

12. A wooden cube of edge 4 inches is painted red. The cube is then cut into 64 one-inch cubes by making 9 saw cuts. How many of the one-inch cubes have no red faces?

    ANS: The center of the large cube contains $2 \times 2 \times 2 = \boxed{8}$ one-inch cubes

13. If it takes 40 minutes to inflate a large spherical balloon to a radius of 2 meters, how long will it take to inflate a large spherical balloon to a radius of 6 meters?

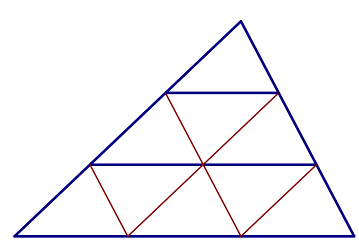
    ANS: $\boxed{1080}$ min

14. After what mathematician was the Cartesian coordinate system named?

    ANS: (René) Descartes

15. A triangular section of Old Town is divided into a smaller triangle and two trapezoids by two streets parallel to one of the boundary streets. The heights of the two trapezoids are equal to the height of the small triangle, and the area of the middle trapezoid is 12 acres. How many acres are there in the larger trapezoid?

    ANS: $\boxed{20}$ acres (Each small triangle has area 4 acres.)



Team Competition 1998
1:00

1. If the absolute value of x^2+4 is equal to the absolute value of x^2-12, what is x?

   ANS: $|x^2+4| = |x^2-12|$, Solution is: $x = \boxed{2, -2}$

2. What is the base 3 representation of the decimal number 35?

   ANS: $\boxed{(322)_3}$

3. Use the approximation $2^{10} \approx 10^3$ to approximate 2^{43} as a number in scientific notation $2^{43} \approx c \times 10^n$, where n is an integer and c is a number between 1 and 10.

   ANS: $2^{43} = 2 \times 2^{42} = 2 \times 2^2 \times (2^{10})^4 \approx 8 \times (10^3)^4 \approx \boxed{8 \times 10^{12}}$ ($2^{43} = 8796\,093\,022\,208$)

4. A bag of Halloween candy has 5pieces of chocolate and 4pieces of fruitbar. What is the probability that 4items selected at random are chocolate?

   ANS: $\boxed{\frac{5}{126}}(= \frac{\binom{5}{4}}{\binom{9}{4}})$

5. Sarah is a 60% free throw shooter. What is the probability that she misses 4 in a row?

   ANS: $\left(\frac{2}{5}\right)^4 = \boxed{\frac{16}{625}} = \boxed{.0\,256} = \boxed{2.56\%}$

6. The average of two numbers is 11.. What is the average of these two numbers together with 14?

   ANS: $\frac{11\times2+14}{3} = \boxed{12}$

7. A wooden cube of edge 4 inches is painted red. The cube is then cut into 64one-inch cubes by making 9 saw cuts. How many of the one-inch cubes have exactly 2 red faces?

   ANS: 12 edges $\times 2$ cubes per edge $= \boxed{24}$ cubes

8. If a single 60-Watt bulb provides sufficient light to read a newspaper 2 feet from the bulb, how many 60-Watt bulbs are required in a light fixture 4 feet from the newspaper in order to provide the same apparent level of brightness?

   ANS: $\boxed{4}$

9. What mathematician first resolved the Königsberg bridge problem?

   ANS: Euler

10. What is the greatest integer in the sum $\frac{19}{5} + \frac{5}{19}$?

    ANS: $\frac{19}{5} + \frac{5}{19} = 4.06315\,78947\,3684 = \boxed{4} + .06315789...$

11. A spider and a fly are at diagonally opposite corners of a closed cubical box of side length one. What is the shortest distance the spider can travel to reach the (stationary) fly by crawling along walls?

    ANS: $\sqrt{2^2 + 1^2} = \sqrt{5}$

12. Bo is going to the store to buy candy that will cost somewhere between 40 cents and 54 cents. What is the fewest number of coins Bo can carry in order to be certain to have exact change to buy the candy?

    ANS: 1¢, 1¢, 1¢, 1¢, 5¢, 10¢, 10¢,25 ¢ $\boxed{8}$ coins

13. John and Jill traded positions several times while rowing a canoe through the Boundary Waters of Minnesota. With Jill at the rear, the canoe went fast enough to complete the entire trip in 10 hours, and with John at the rear the trip would have taken 14 hours. The trip actually took 12 hours. For how many hours did Jill sit in the back of the canoe?

    ANS: $1 = \frac{1}{10}t + \frac{1}{14}(12 - t)$, Solution is: $t = \boxed{5 \text{ hours}}$

14. A pair of dice is rolled. What is the probability that the total is 10?

    ANS: $\frac{1}{12}(= \frac{3}{36})$

15. A city lot is twice as long as it is wide. By increasing its length 20 yards and its width 30 yards, the area will be increased by 2200 square yards. What are its dimensions?

    ANS: $(2x + 20)(x + 30) - 2x^2 = 2200$, $\boxed{\text{20 yards by 40 yards}}$

Tiebreak: The vertices of a quadrilateral are at the points $(0, 0)$, $(3, 1)$, $(4, 0)$,and $(2, -4)$. What is the area of the quadrilateral?

1. ANS: $Area = \frac{1}{2} \cdot 4 \cdot 1 + \frac{1}{2} \cdot 4 \cdot 4 = \boxed{10}$

Team Competition 1998
1:20

1. If 2 positive integers have a sum of 11, what is the maximum possible value for the sum of their squares?

   ANS: $\boxed{101} = 10^2 + 1^2$

2. In how many ways can ALLAN (spelled A-L-L-A-N) misspell his name, assuming he uses all the right letters (the right number of times)?

   ANS: $\frac{5!}{2!2!} - 1 = \boxed{29}$

3. How many numbers between 94 and 528 are divisible by 14?

   ANS: 31 ($7 \times 14 = 98, \ldots, 37 \times 14 = 518$, $37 - 7 + 1 = \boxed{31}$)

4. Write the next term of the sequence that begins: 4, 6, 11, 19, 30

   ANS: $\boxed{44}$

5. What former CSU Mathematics Professor was known as an "Euler Spoiler" for finding a counterexample to a famous conjecture of Euler and who helped develop the Bose-Chaudhuri-Hocquenghem error-correcting codes?

   ANS: R. C. Bose

6. What are the next two prime numbers greater than 50?

   ANS: $51 = 3 \times 17, \boxed{53}, 55 = 5 \times 11, 57 = 3 \times 19, \boxed{59}$

7. Farmer Jill raises goats and geese. If she counts 26 eyes and 42 feet, how many goats and how many geese does Jill have?

   ANS: $\boxed{\text{5 geese and 8 goats}}$.

8. If $f(x)$ is a linear function and $f(-5) = 3$ and $f(1) = -3$, what is $f(4)$?

   ANS: $\boxed{-6}$

9. How far apart are the two points of intersection of the two curves $y = x^2 + 1$ and $y = x + 7$?

   ANS: $\left\{ \begin{array}{c} y = x^2 + 1 \\ y = x + 7 \end{array} \right\}$, Sol ution is : $(-2, 5), (3, 10)$, $\sqrt{(3+2)^2 + (10-5)^2} = \boxed{5\sqrt{2}}$

10. A total of 6 liters of paint are required to paint the outside of a cubi cbox of volume $27\,\mathrm{m}^3$. How much paint is needed to paint theoutside o f a cubic box of volume $216\,\mathrm{m}^3$?

    ANS: $\boxed{24}$ liters

11. A computer sequentially computes integers by the following rule: If n is a square then multiply by 2; otherwise subtract 2. Starting at $n = 13$, what is the integer after 6 iterations?

    ANS: $13 \to 11 \to 9 \to 18 \to 16 \to 32 \to \boxed{30}$

12. What is the product of all the roots (real and complex) of the polynomial $x^3 - 82x^2 - 64x - 47$

    ANS: $\boxed{47}$ since the linear term in the product $(x-a)(x-b)(x-c)$ is $-abc$.

13. The 24 ways to write rearrangements of the letters MATH are listed in alphabetical order. Where in this list does MATH appear?

    ANS: $\boxed{\text{14th}}$ on the list right after MAHT and before MHAT (1–6 start with A, 7–12 with H, and MAHT is 13th.)

14. Where does the circle of radius 5 centered at the origin intersect the lin epassing through the origin with slope $-3/4$?

    ANS: $\left\{ \begin{array}{c} x^2 + y^2 = 25 \\ y = -\frac{3}{4}x \end{array} \right\}$, Solution is : $\boxed{\{y = 3, x = -4\}, \{y = -3, x = 4\}}$

15. A rectangle's length is increased by 30% and its width is decreased by 50%. How does its area change?

    ANS: decreases by $\boxed{35\%}$ ($1.3 \times .5 = .65$)

Team Competition 1998
1:35

1. Alpher, Beta, and Gamow are now 7, 11, and 13 years old, respectively. How many years will it be until they again have prime-numbered ages?

   ANS: Six years from now they will have ages 13, 17, and 19.

2. A ball thrown vertically into the air 100 feet, falls and rebounds to a height of 40 feet the first time, rebounds to 16 feet on the second bounce,and so forth. What is the entire distance the ball will have moved when it finally comes to rest?

   ANS: $2\sum_{i=0}^{\infty} 100 \cdot \left(\frac{2}{5}\right)^i = \frac{1000}{3} = \boxed{333\frac{1}{3}}$ ft

3. This research organization has employed applied mathematicians such as Ronald Graham, Richard Hamming, and Claude Shannon, each of whom have made major contributions to applied discrete mathematics. Name this organization.

   ANS: Bell Labs or AT&T Bell Laboratories

4. What is the greatest integer less than the sum $\frac{17}{3} + \frac{3}{17}$?

   ANS: $\frac{17}{3} + \frac{3}{17} = 5.843137255 = \boxed{5} + .843137255$

5. Kyle uses pure guessing on a TRUE/FALSE exam. Which of the following options give Kyle the best chance to score (at least) 50%? (A) A seven-question exam (guess correctly on 4, 5, 6, or 7 questions). (B) A five-question exam (guess correctly on 3, 4, or 5 questions). (C) The chances are equal.

   ANS: (C) Chances are equal (both probabilities are $\frac{1}{2}$)

6. What is the coefficient of y^4 in the expansion of $(x + y)^6$?

   ANS: $\boxed{15x^2}$ since $(x + y)^6 = x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + \boxed{15x^2}y^4 + 6xy^5 + y^6$

7. What is the radius of the sphere whose volume is 15 times its surface area?

   ANS: $\boxed{45}$

8. What is the sum of the roots of the polynomial $x^3 - x^2 - 100x + 100$? $(x-a)(x-b)(x-c) = x^3 - (a+b+c)x^2 + \cdots$

   ANS: $\boxed{1}$

9. Two sides of a nontrivial triangle have lengths 4 and 16.. What is the smallest integer length for the third side?

   ANS: $\boxed{13}$

10. The graph of a cubic polynomial crosses x-axis at $x = -3$, $x = -4$, and $x = 4$. In expanded form, what is one such polynomial?

    ANS: $\boxed{x^3 + 3x^2 - 16x - 48}$ or some nonzero multiple thereof

11. The compact disk UR2gly sells at outlet AC for \$12.95 less a discount of 15%, and at outlet DC for \$14.65 less a discount of 25%. Which outlet has the lower price?

    ANS: $AC = 12.95 \times .85 = 11.0075$, $\boxed{DC} = 14.65 \times .75 = 10.9875$

12. A total of 2000 raffle tickets are to be sold for \$1 each. The winner receives \$400. If you purchase 1 ticket, how much are your expected earnings?

    ANS: $\boxed{-\$0.80}$

13. What is the smallest positive integer whose square is larger than 194?

    ANS: $\boxed{14}$

14. A computer sequentially computes integers by the following rule: If $n$is asquare then multiply by 2; otherwise subtract 2. Starting at $n = 9$,what ist he integer after 6 iterations?

    ANS: $9, 18, 16, 32, 30, 28, \boxed{26}$

15. If $7^p = 3$ then what is 343^p?

    ANS: $\boxed{27}$

Team Competition 1998
1:50

1. A wooden cube of edge 4 inches is painted red. The cube is then cut into 64 one-inch cubes by making 9 saw cuts. How many of the one-inch cubes have exactly 3 red faces?

   ANS: 8 corners $\times$ 1 cube per corner $= \boxed{8}$ cubes

2. What is the area of the parallelogram with vertices $(0, 0)$, $(5, 10)$, $(7, 1)$, and $(12, 11)$?

   ANS: $\boxed{65}$

3. What is the area of a regular hexagon with sides of length 2?

   ANS: $\boxed{6\sqrt{3}}$

4. An orange has a diameter that is 95% fruit and 5% peel. To the nearest percent, what percentage of the total volume is the peel?

   ANS: $\boxed{14\%}$.

5. A basketball team has 12 players on the roster. How many different starting lineups are possible?

   ANS: $\binom{12}{5} = \boxed{792}$

6. What are the next 4 terms in the sequence that begins $1, 1, 2, 3, 5, 8$?

   ANS: $\boxed{13, 21, 34, 55}$ (Fibonacci sequence)

7. Sally invested \$6500, part at 5% and the rest at 15%. If the annual interest income from both investments was \$625.00, how much was invested at 5%?

   ANS: $0.05x + 0.15\,(6500 - x) = 625$, Solution is : $x = \boxed{\$3500.00}$

8. Divide 100 into 3 parts, such that $\frac{1}{7}$ of the first, $\frac{1}{5}$ of the second, and $\frac{1}{8}$ of the third are equal.

   ANS: $\boxed{35,\ 25,\ \text{and}\ 40}$

9. What is the area of the triangle bounded by the x-axis, the y-axis, and the line $y = 3x + 3$?

   ANS: $\boxed{\frac{3}{2}}$

10. At 6:00 a.m., Chad starts jogging at 4km/hr. At 9:00 a.m. Marzelle starts jogging from Chad's starting place at 5 km/hr. How far behind is Marzelle at 9:00 a.m.?

    ANS: $\boxed{12}$ km

11. A circle of diameter 4 contains a circle of diameter 1 in its interior. What is the area contained in the larger circle that is exterior to the small circle?

    ANS: $\pi 2^2 - \pi \left(\frac{1}{2}\right)^2 = \boxed{\frac{15\pi}{4}}$

12. What is the surface area of a sphere of volume $\frac{256\pi}{3}$?

    ANS: $\boxed{64\pi}$

13. How many minutes were there in October of this year?

    ANS: $31 \times 24 \times 60 = \boxed{44\,640}$ minutes

14. Sarah averages 50% on multiple choice exams. What is the probability that she gets at least 2 correct on a 3-question exam?

    ANS: $\left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^3 = \boxed{\frac{1}{2}}$

15. For whom was the computer language Ada named?

    ANS: Ada Lovelace

Team Competition 1998
2:05

1. Store A sells candy bars 3 for \$1.00. Store B sells candy bars individually for 40¢, but you get 5 for the price of 4. On Monday John bought some candy bars at store A. On Tuesday Jill bought some candy bars at store B.They compared notes and found that they had gotten the same number of candybars, and each had paid the same amount of money. What is the least amount of money that each of them could have spent?

   ANS: They each bought 6 candy bars and spent $\boxed{\$2.00}$

2. A mathematician named Wolfram started a company named Wolfram Research. What is its primary product?

   ANS: Mathematica

3. Find all the integer solutions of the equation $x^4 - 1 = 0$.

   ANS: $x = \boxed{1, -1}$ since $x^4 - 1 = (x-1)(x+1)(x^2+1)$

4. A bag contains 4 white balls, 5 red balls, and 4 green balls. If 3 balls are selected at random from the bag, what is the probability that they are all white?

   ANS: $\frac{2}{143}$ $\left(= \frac{\binom{4}{3}}{\binom{13}{3}} \right)$

5. The Center Ring Janitorial Supply owns a fleet of 3 vehicles: A car, which gets 35 miles per gallon, a van, which gets 25 miles per gallon, and atruck , which gets 15 miles per gallon. If in a typical week the car is driven 105 miles, the van is driven 150 miles and the truck is driven 240 miles, how many miles per gallon is Center Ring's fleet getting? (Give your answer to the nearest mile per gallon.)

   ANS: $\boxed{20}$ miles per gallon

6. A shirt has been marked down 20% and then 25% to \$30.00. What was the original price?

   ANS: $\boxed{\$50.00}$

7. The divergent series $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$ has a musical name. What is that name?

   ANS: $\boxed{\text{Harmonic Series}}$

8. What is the area of one face of a diamond-shaped kite whose two sticks are 60 and 48 inches long?

   ANS: $\boxed{1440}$

9. How many ways can the top 4 finishers be picked in an 8-person race?

   ANS: $\boxed{1680}$

10. The formula $e^{i\pi} + 1 = 0$ relates five of the most popular numbers in mathematics. What is π rounded to 10 significant digits?

    ANS: $\pi \approx 3.14159\,2654$

11. According to Descartes' Rule of Signs, how many positive real roots does the polynomial equation $3x^4 + 10x^2 + 5x + 4 = 0$ have?

    ANS: $\boxed{\text{None}}$ (since there are no sign changes)

12. Let $y = mx + b$ be the image when the line $-9x - 7y - 5 = 0$ is reflected across the x-axis. What is $m + b$?

    ANS: 2

13. If g is a function such that $g(1) = 2$, $g(2) = -1$, and

    $$g(n+1) = g(n) + 2g(n-1)$$

    for $n \geq 3$, what is $g(4)$?

    ANS: $g(3) = g(2) + 2g(1) = -1 + 4 = 3$, $g(4) = g(3) + 2g(2) = 3 - 2 = \boxed{1}$

14. List the following three numbers in increasing order: 2^{10}, $6!$, 10^3.

    ANS: $\boxed{6!} = 720 < \boxed{10^3} = 1000 < 2^{10} = \boxed{1024}$

15. What is the smallest value of the expression $x + \frac{1}{x}$ if x is a positive real number?

    ANS: $\left[x + \frac{1}{x}\right]_{x=1} = \boxed{2}$

Team Competition 1998
2:15

1. List the following three numbers in increasing order: 2^8, 3^5, 6^3.

   ANS: $\boxed{6^3} = 216 < \boxed{3^5} = 243 < \boxed{2^8} = 256$

2. In how many ways can 2 math books, 2 stat books, and 3 physics books be arranged on a shelf, assuming the books must be in groups by their category? (i.e.: math with math, stat with stat, etc.)

   ANS: $\boxed{144}$

3. In 1962 a mathematician named Edward O. Thorp wrote a book entitled *Beat the Dealer* that claimed to give a winning strategy, based upon large-scale computer simulations, for a certain card game. What is the name of that card game?

   ANS: Blackjack or 21

4. A biological brick grows 5% in length, 15% in width, and shrinks in height by 20%. Is the new volume larger, or smaller or the same as the original?

   ANS: $\boxed{\text{smaller}}$ $(1.05 \times 1.15 \times .8 = .966)$

5. The average of 10 numbers is 19.. The number 10 is discarded from the collection of 10 numbers. What is the average of the remaining 9 numbers?

   ANS: $\boxed{20}$

6. What is the least common multiple of 31, 40?

   ANS: $\boxed{1240}$

7. The 12 faces of a regular dodecahedron are pentagons. How many vertices does a regular dodecahedron have?

   ANS: $\frac{12\times5}{3} = \boxed{20}$

8. To the nearest minute, at what time between 9:30 AM and 10:00 AM are the minute hand and the hour hand at right angles?

   ANS: $45 + \frac{m}{12} = m + 15$, Solution is : $m = \frac{360}{11}$: 32.727272 727 $\boxed{9{:}33}$ AM

9. The 8 faces of a regular octahedron are equilateral triangles. How many vertices does a regular octahedron have?

   ANS: $\frac{8\times3}{4} = \boxed{6}$

10. If $m > 0$ and the points $(m, 9)$ and $(1, m)$ lie on a line with slope m, find m.

    ANS: $m = \boxed{3}$

11. A popular novel has 423 pages and 128592 words. What is the average number of words per page?

    ANS: $\frac{128592}{423} = \boxed{304}$

12. You currently earn \$6.50 per hour delivering pizza. You are due for a raise, and you figure the probability of a \$0.50 raise is 40% and the probability of a \$1.00 raise is 60%. What is your expected new salary?

    ANS: $\boxed{\$7.30}$

13. The polynomial $5x^7 + 21x^5 + 35x^3 + 35x + 18$ has one real root. How many imaginary roots does it have?

    ANS: $\boxed{6}$

14. How many integers between 101 and 999 are divisible by 3 or by 5?

    ANS: $\frac{999-102}{3} + 1 = 300$, $\frac{995-105}{5} + 1 = 179$, $\frac{990-105}{15} + 1 = 60$, $300 + 179 - 60 = \boxed{419}$

15. A triangle has 3 vertices and 3 edges. A tetrahedron has 4 vertices and 6 edges. How many vertices and edges does an n-dimensional tetrahedron have?

    ANS: $\boxed{n+1 \text{ vertices}}$ and $\boxed{n(n+1)/2 \text{ edges}}$

Team Competition 1998
Small School Final

1. In how many ways can one arrange the letters in OBOE?

   ANS: $\frac{4!}{2!} = \boxed{12}$

2. What famous mathematical object was named after the mathematician David Hilbert?

   ANS: Hilbert Space

3. If a sequence is defined by $x_1 = 1$, $x_2 = -4$, and $x_{n+1} = x_n - x_{n-1}$, what is x_5?

   ANS: $x_1 = 1$, $x_2 = -4$, $x_3 = (-4)-1 = -5$, $x_4 = -5-(-4) = -1$, $x_5 = -1-(-5) = \boxed{4}$

4. The 12 faces of a regular dodecahedron are pentagons. How many edges does a regular dodecahedron have?

   ANS: $\frac{12\times 5}{2} = \boxed{30}$

5. The sum of 4 consecutive integers is 106. What is the largest of the 4integers?

   ANS: $\boxed{28}$ ($25 + 26 + 27 + 28 = 106$)

6. An octahedron is a regular solid whose 8 faces are equilateral triangles.What is the distance between a pair of opposite vertices, assuming each edgeof the octahedron is of length $\sqrt{2}$?

   ANS: $\boxed{2}$

7. What is the smallest integer whose square is less than 182?

   ANS: $\boxed{-13}$

8. Find a point equidistant from the points $(-2,-2)$, $(2,2)$ and $(2,-2)$.

   ANS: $\boxed{(0,0)}$

9. The formula $e^{i\pi}+1=0$ relates five of the most popular numbers in mathematics. What is e rounded to 6 significant digits?

   ANS: $e \approx 2.718281828 \approx \boxed{2.71828}$

10. What is the 16th term of the arithmetic sequence that begins -7, -11, ...?

    ANS: $\boxed{-67}$ $(= -3 + 16\,(-4))$

11. Kyle uses pure guessing on a TRUE/FALSE exam. Which of the following options give Kyle the best chance to score (at least) 80%?

    (a) A ten-question exam (guess correctly on 8, 9, or 10 questions).

    (b) A five-question exam (guess correctly on 4 or 5 questions).

    (c) The chances are equal.

    ANS: (a) probability of 80% $\approx .054688$ $\boxed{\text{(b)}}$ probability of 80% $\approx .188$

12. How many committees of 4 men and 2 women can be formed from a group of 6 men and 6 women?

    ANS: $\boxed{225}$ $(= \binom{6}{4}\binom{6}{2}\ )$

13. Two baseball teams, the Green Socks and the Yellow Elbows, start a 7-game series.For each game, the odds are 10 to 9 in favor of the Green Socks.What is the probability that the Green Socks sweep the series in four games?

    ANS: $\boxed{\frac{10\,000}{130\,321}}$

14. A square is inscribed in a circle, which in turn is inscribed in a square. What percentage of the area of the large square is inside the small square?

    ANS: $\boxed{50\%}$

15. A spherical balloon's diameter decreases by 40%. By what percentage doe sthe surface area decrease?

    ANS: $\boxed{64\%}$

Team Competition 1998
Small School Final (Round 2)

1. What are the dimensions of a rectangle with area 154 and perimeter 50?

   ANS: $\boxed{\text{14 and 11}}$

2. What did the Norwegian mathematician Niels Henrik Abel prove about general fifth-degree polynomials?

   ANS: $\boxed{\text{Cannot be solved}}$ in terms of radicals involving the coefficients

3. How many different strings of length 7 can be formed using the letters in ELLIPSE?

   ANS: $\frac{7!}{2!2!} = \boxed{1260}$

4. Two baseball teams, the Padres and the Yankees, start a 7-game series. In how may different ways can the Yankees win the series four games to two?

   ANS: The Yankees win game 6 plus 3 of the first 5, so $\frac{5!}{3!2!} = \binom{5}{3} = \boxed{10}$

5. In how many ways can you have \$10 worth of dimes and quarters?

   ANS: $\boxed{21}$

6. Expand $(3x^3 + 2y^2)^3$

   ANS: $\boxed{27x^9 + 54x^6y^2 + 36x^3y^4 + 8y^6}$

7. Archie, Bob, and Claire are now 11, 17, and 19 years old, respectively. In how many years will they again have prime-numbered ages?

   ANS: $\boxed{\text{Twelve years from now}}$ they will have ages 23, 29, and 31.

8. What is the sum of the first 22 even integers: $2 + 4 + 6 + \cdots + 44$?

   ANS: $\boxed{506}$ ($= 2 \times \frac{22\times 23}{2}$)

9. If $f(x) = x^2 - 1$, what is $f(f(f(1)))$?

   ANS: $\boxed{0}$

10. How many edges does an n-dimensional cube have?

    ANS: $\boxed{n2^{n-1}}$

11. What is the area of an equilateral triangle inscribed in a circle of radius 6? $\sin\frac{2\pi}{3} = \frac{1}{2}\sqrt{3} = -\frac{1}{2}$

    ANS: $\boxed{27\sqrt{3}}$

12. If a mantel clock strikes the hours, how many times will it strike during a 24-hour period?

    ANS: $2\sum_{i=1}^{12} i = \boxed{156}$

13. A biological brick grows 5% in length, 25% in width, and shrinks in height by 23%. Is it larger or smaller than when it started out?

    ANS: $\boxed{\text{larger}}$ ($1.05 \times 1.25 \times .77 = 1.0106$)

14. If 2 is the first prime, what is the tenth prime?

    ANS: $2, 3, 5, 7, 11, 13, 17, 19, 23, \boxed{29}$

15. Let S be the set of the first 7 natural numbers. How many of the 128 subsets of S contain the number 2?

    ANS: $\boxed{64}$ or $\boxed{\text{Half}}$

Team Competition 1998
Large School Final

1. What is the sum of all the integers greater than 12 and less than 29?

   ANS: $\sum_{i=13}^{28} i = \boxed{328}$

2. This British mathematician lead a team of mathematicians and cryptologists during World War II that broke ciphertext generated by the German Enigma machine. Name this person.

   ANS: Alan Turing

3. Give an equation in the form $ax + by = c$, where a, b, and c are integers, for the line through $(2, 1)$ that is parallel to the line $7x + 3y = -9$.

   ANS: $\boxed{7x + 3y = 17}$

4. A deck of 52 cards is thoroughly shuffled and the cards are turned over two at a time. How many pairs (two cards of the form $\boxed{\text{A}}\boxed{\text{A}}$ or $\boxed{5}\boxed{5}$) do you expect to see?

   ANS: $26\left(\frac{3}{51}\right) = \boxed{\frac{26}{17}}$

5. Allison scored 65 on the first exam and 51 on the second exam. What must she average on the next two exams to bring her average for the four exams up to 75?

   ANS: $\boxed{92}$

6. Find all the roots of the equation $x^3 + 18x^2 + 92x + 120 = 0$.

   ANS: $\boxed{-2,\ -6,\ -10}$.

7. If a third degree polynomial has leading coefficient of -2 and roots 5, -3, and -5, what is its constant term? $-2(x-5)(x+3)(x+5) = -2x^3 - 6x^2 + 50x + 150$

   ANS: $\boxed{150}$ since $-2(x-5)(x+3)(x+5) = -2x^3 - 6x^2 + 50x + 150$

8. Cubic polynomials have three zeros, which in general are complex numbers.Knowing that 10 is a zero of the polynomial $x^3 - 16x^2 + 49x + 110$, what is the product of the two remaining real or complex zeros?

   ANS: $\boxed{-11}$ since $(x-10)(x-a)(x-b) = x^3 - (a+b+10)x^2 + (ab+10a+10b)x - 10ab = x^3 - 16x^2 + 49x + 110$

9. The set S has 12 elements. If a and b are (distinct) elements of S, how many subsets of S contain both a and b?

   ANS: $\boxed{1024} = 2^{10}$

10. A right triangle has legs of length 6 and 8. What is the radius of the circle that circumscribes the triangle?

    ANS: diameter $= 10$, radius $= \boxed{5}$

11. What is the sum of the first 20 positive integers?

    ANS: $\boxed{210}\left(= \frac{20\times 21}{2}\right)$

12. Marty bought a farm, a house, and a barn for \$342 000. If the house cost 2 times as much as the barn, and the farm 2 times as much as the house and barn together, how much did each cost?

    ANS: $\boxed{\text{Barn: } 38\,000 \qquad \text{House: } 76\,000 \qquad \text{Farm: } 228\,000}$

13. If g is a function such that $g(1) = 3$, $g(2) = -1$, and

    $$g(n+1) = g(n) + 2g(n-1)$$

    for $n \geq 3$, what is $g(4)$?

    ANS: $g(3) = g(2) + 2g(1) = -1 + 6 = 5$, $g(4) = g(3) + 2g(2) = 5 - 2 = \boxed{3}$

14. Name the five Platonic solids (regular polyhedra).

    ANS: $\boxed{\text{Tetrahedron, cube (or hexahedron), octahedron, dodecahedron,icosahedron}}$

15. Bonnie gets a salary of \$20,000 with a 10% yearly raise. What will her salary be after 4 years?

    ANS: $\boxed{\$29282.00}$

Team Competition 1998
Large School Final (Round 2)

1. If $f(x) = -3x - 2$, what is $f(f(f(3)))$?

   ANS: $\boxed{-95}$

2. A Stanford University mathematician/computer scientist who developed the $\TeX$ typesetting system and who wrote a three-volume series of books called *The Art of Computer Programming* stated, "Every bit of mathematics I have ever learned, I have found a use for someplace." What is this person's name?

   ANS: Donald Knuth

3. Which regular polyhedron has the same number of faces as vertices?

   ANS: $\boxed{\text{Tetrahedron}}$ 4 faces and 4 vertices

4. A computer sequentially computes integers n by the following rule: If n is odd then replace n by $3n + 1$; otherwise replace n by $\frac{n}{2}$. If n starts at 3, what is n after 5 iterations?

   ANS: $3 \to 10 \to 5 \to 16 \to 8 \to \boxed{4}$

5. What is the prime factorization of 127?

   ANS: $\boxed{127}$ (prime)

6. Grandma Josephine offers each of her 5 grandchildren the choice of 3 different kinds of cookies. If each grandchild only gets one cookie, in how many ways can the choices be made?

   ANS: $\boxed{243}$ $(= 3^5)$

7. Two circles are mutually tangent at one point, and the smaller circle passes through the center of the larger circle. What is the ratio between the circumferences of the two circles?

   ANS: $\boxed{2 : 1}$ or $\boxed{1 : 2}$

8. If a 400-Watt sound system can break a glass goblet placed 1 feet away from the speaker, how powerful a sound system would it take to break a similar glass goblet placed 2 feet from the speaker?

   ANS: $\boxed{1600}$ Watts

9. Find the area of the circle inscribed in a regular hexagon with sides of length 4.

   ANS: $\boxed{12\pi}$

10. A spherical balloon's diameter increases by 60%. By what percentage does the surface area change?

    ANS: $\boxed{256}$ %

11. At the local Dairy Queen, the "Monster Sundae" can be ordered with any of 6 flavors of ice cream plus none, any, some, or all of 6 toppings. If you order one such sundae every Saturday, how many weeks will it be before you must order the same sundae a second time?

    ANS: $\boxed{384} = 6 \times 2^6 = 384$

12. On January 1, 1998, an investment bond was purchased for \$1000 and earns 15% compounded annually. To the nearest dollar, what will the balance be on January 1, 2002?

    ANS: \$$\boxed{1749}$

13. If the line $y = mx$ touches the curve $y = x^2 + 1$ in exactly 1 point, what are the possibilities for m?

    ANS: $m = \boxed{\pm 2}$

14. If g is a function such that $g(1) = 1$,$g(2) = 1$,and

    $$g(n) = g(n-2) + 3g(n-1)$$

    for $n \geq 3$, what is $g(4)$?

    ANS: $g(3) = g(1) + 3g(2) = 4, g(4) = g(2) + 3g(3) = 1 + 3 \cdot 4 = \boxed{13}$

15. List the following three numbers in increasing order: $x = 1 + 2 + 3 + \cdots + 100$, $y = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7$,and $z = 4^6$.

    ANS: $\boxed{z} = 4^6 = 4096 < \boxed{y} = 7! = 5040 < \boxed{x} = \sum_{i=1}^{100} i = 5050$

Team Competition 1998
Emergency Exam

1. The graph of a cubic polynomial has x-intercepts 0 and 1 (only).What is a possible expression for the polynomial?

   ANS: $\boxed{x^2(x-1) = x^3 - x^2}$ or $\boxed{x(x-1)^2 = x^3 - 2x^2 + x}$ (or a nonzero multiple)

2. A baseball manager has selected 9 starters for a game. If the pitcher must bat last and the second baseman must bat first, how many different battingline -ups are possible?

   ANS: $7! = \boxed{5040}$

3. What positive number is 6 times as big as its reciprocal?

   ANS: $\boxed{\sqrt{6}}$ since $x = \frac{6}{x} \Longrightarrow x^2 = 6$

4. A circular pizza 12 inches in diameter is cut into 12 congruent slices .What is the perimeter of each slice?

   ANS: $\boxed{12 + \pi}$

5. Express the perimeter P of a square as a function of its area A.

   ANS: $A = \left(\frac{P}{4}\right)^2$, Solution is : $\boxed{P = 4\sqrt{A}}$

6. A shirt has been marked down 25% to \$37.50. What was the original price?

   ANS: \$$\boxed{50.00}$

7. It takes 1000 large square tiles to tile a room, or 1440 smaller square tiles whose edge is 1 inch less. How large is the room in square feet?

   ANS: $1000x^2 = 1440\left(x - \frac{1}{12}\right)^2$, Solution is : $x = \frac{1}{2}$, $1000\left(\frac{1}{2}\right)^2 = \boxed{250}$ ft^2

8. Given the circle $x^2 - 2x + y^2 = 0$ in the xy plane, what are the equations of the two vertical tangent lines to this circle?

   ANS: $\boxed{x = 0}$ and $\boxed{x = 2}$

9. This 20th century American mathematician introduced game theory as a mathematical discipline, conceived the idea of a self-stored computer program, and worked on the Manhattan project that developed the atomic bomb. Name this person.

   ANS: John von Neuman

10. John got a 10% raise last year and another 10% raise this year and his current salary is \$36 300.00. What was his salary before last year's raise?

    ANS: \$$\boxed{30\,000.00}$

11. An automobile travels at 90 kilometers per hour for 4 min, 30 km/h for 5 min, and then 15 km/h for 4 min. How far did it travel during the 13 minutes?

    ANS: $\boxed{\frac{19}{2}}=\boxed{9.5}$ km ($90 \times \frac{4}{60} + 30 \times \frac{5}{60} + 15 \times \frac{4}{60} = \frac{19}{2}$)

12. An octahedron is a regular solid whose 8 faces are equilateral triangles.What is the distance between a pair of opposite vertices, assuming each edgeof the octahedron is of length 1?

    ANS: $\sqrt{2}$

13. The number 12 is called abundant because the sum of the proper divisors $1 + 2 + 3 + 4 + 6 = 16$ is greater than 12. What is the next abundant number?

    ANS: $\boxed{18} < 1 + 2 + 3 + 6 + 9 = 21$

14. A pair of slacks priced at \$20 has been marked up 10% and then marked down 30%. What is the new price?

    ANS: \$$\boxed{15.40}$

15. What is the smallest positive integer n such that $n^2 - n + 41$ is not prime?

    ANS: $\boxed{41}$, since

n	1	2	3	4	5	$\cdots$	40	$\boxed{41}$
$n^2 - n + 41$	41	43	47	53	61	$\cdots$	1601	$1681 = 41^2$