Part I: Short answer

1. Suppose an (analog) clock shows 1:05h. Then the two arrows are meeting in the same position. How often will such a meeting happen in the 14 hour period starting at 1:30h?

   ANSWER:_____

2. How many numbers from 1 to 1000 are divisible by neither 3 nor 5?

   ANSWER:_____

3. If $f(z) = z^2 - z$, what is $f(f(f(i)))$? (where $i = \sqrt{-1}$.)

   ANSWER:_____

4. What is the area of a triangle with side lengths 10, 10 and 16?

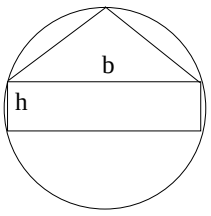
   ANSWER:_____

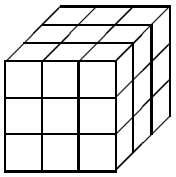
5. A builders store sells fence in units of 10ft and 3ft. Notice that it is impossible to build a 4 foot long fence from these pieces without cutting a piece. What is the greatest such length of fence that cannot be built from these pieces without cutting at least one piece?

   ANSWER:_____

Part II: Show all work

6. Inscribe a rectangle of base b and height h and an isosceles triangle of base b in a circle of radius one as shown. For what values of h have the triangle and the rectangle the same area?



7. Consider a wooden cube with a side length of 3 inch. We want to cut this cube into $3^3 = 27$ small cubes of side length 1 inch each as shown in the picture.

The saw used can only cut straight (planar) cuts, but we are permitted to rearrange pieces in between cuts to cut two pieces simultaneously with one cut.
What is the minimum number of cuts we will need and why is this the minimum?

8. Let A be any set of 20 distinct integers chosen from the arithmetic progression 1,4,7,10,...,100. Prove that there must be two distinct integers in A whose sum is 104.

9. Let S be a set of real numbers that is closed under multiplication. ("Closed under multiplication" means: Whenever $a, b \in S$, then $a \cdot b \in S$.)
Let T and U be disjoint subsets of S whose union is S. Given that the product of any three (not necessarily distinct) elements of T is in T and that the product of any three elements of U is in U, show that at least one of the two subsets T and U is closed under multiplication.

10. Consider the triangle in $\mathbb{R}^2$, defined by the three points $(0, 0)$, $(30, 0)$ and $(3, 9)$. How many points with integral coordinates (e.g: $(5, 1)$) are contained in it or bordered by it?

11. In a set of sixteen people, four are English, four are Scots, four are Irish, and four Welsh. There are four each of ages 35, 45, 55, and 65. Four are lawyers, four are doctors, four are soldiers and four clergymen. Four are single, four married, four widowed, and four divorced. Finally four play piano, four play violin, four play trumpet and four just listen to the radio.
No two persons who are the same in one category are the same in another category. I.e. if person A and person B are both English, they must have different ages. If person C and D are both doctors, they must have different nationality and so on.
Three of the piano players are: a single English lawyer of 65, a married Scottish soldier of 55, and a widowed Irish doctor of 45. Furthermore, the Irish trumpet player is 35, the Scottish violinist is 45, and the English clergymen is 55. What can you say about the Welsh lawyer?