

Part I: Short Answer Give a short answer to the following questions in the space provided. You do not need to show work.

1. How many ways can $n = 11 \cdot 6 \cdot 2008$ be factored as a product of two natural numbers exactly one of which is a power of a prime number? (251 is prime).

ANSWER: _____

2. List all real solutions x of the equation $|x + 2| = |3x - 8|$.

ANSWER: _____

3. You have only a 3 gallon and a 5 gallon bucket and are required to to fetch exactly 4 gallons of water from the river. It not possible to mark the buckets in any way. Write a sequence of pairs of numbers (a_i, b_i) where $i = 1, 2, \dots$, where a_i records the water amount in the 5 gallon bucket and b_i records the water amount in the 3 gallon bucket that starts with $(0, 0)$ and ends with $(4, 0)$ in which successive terms could actually be used to finish your peculiar job.

ANSWER: _____

4. A bag contains 2 red balls, 3 green balls and 3 white balls. Two balls are drawn randomly from the bag. What is the probability that the drawn balls are the same color?

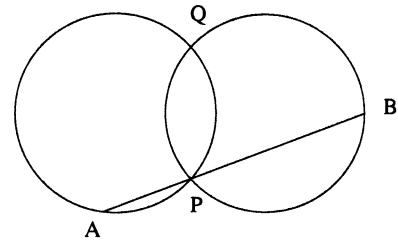
ANSWER: _____

5. A school bus travels 72 mph downhill, 63 mph on the level and 56 mph uphill. It takes one hour for the bus to make it from Pleasantville to Fort Fun and 70 min to return. How many miles is the (one way) trip?

ANSWER: _____

Part II: Long Answer Answer the following questions as completely as possible. Show all work for partial credit.

6. Two circles of the same radii intersect at points P and Q . A line through P intersects the circles at points A and B as in the figure. Prove that the perpendicular bisector of AB passes through Q .



Part I: Short Answer Give a short answer to the following questions in the space provided. You do not need to show work.

1. How many ways can $n = 11 \cdot 6 \cdot 2008$ be factored as a product of two natural numbers greater than 1, exactly one of which is even? (251 is prime)

ANSWER: _____

2. List all real solutions x of the equation $|x + 3| = |4x - 9|$.

ANSWER: _____

3. You have only a 3 gallon and a 8 gallon bucket and are required to to fetch exactly 7 gallons of water from the river. It not possible to mark the buckets in any way. Write a sequence of pairs of numbers (a_i, b_i) where $i = 1, 2, \dots$, where a_i records the water amount in the 8 gallon bucket and b_i records the water amount in the 3 gallon bucket that starts with $(0, 0)$ and ends with $(7, 0)$ in which successive terms could actually be used to finish your peculiar job.

ANSWER: _____

4. A bag contains 4 red balls, 2 green balls and 2 white balls. Two balls are drawn randomly from the bag. What is the probability that the drawn balls are the same color?

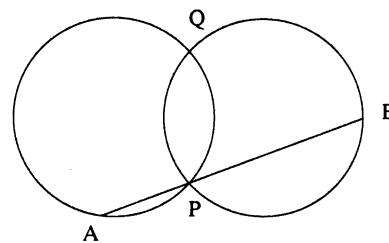
ANSWER: _____

5. A school bus travels 72 mph downhill, 63 mph on the level and 56 mph uphill. It takes one hour for the bus to make it from Pleasantville to Fort Fun and 70 min to return. How many miles is the (one way) trip?

ANSWER: _____

Part II: Long Answer Answer the following questions as completely as possible. Show all work for partial credit.

6. Two circles of the same radii intersect at points P and Q . A line through P intersects the circles at points A and B as in the figure. Prove that the perpendicular bisector of AB passes through Q .



Part I: Short Answer Give a short answer to the following questions in the space provided. You do not need to show work.

1. How many ways can $n = 11 \cdot 6 \cdot 2008$ be factored as a product of two natural numbers both of which are even? (251 is prime).

ANSWER: _____

2. List all real solutions x of the equation $|x + 2| = |4x - 4|$.

ANSWER: _____

3. You have only a 4 gallon and a 7 gallon bucket and are required to to fetch exactly 6 gallons of water from the river. It not possible to mark the buckets in any way. Write a sequence of pairs of numbers (a_i, b_i) where $i = 1, 2, \dots$, where a_i records the water amount in the 7 gallon bucket and b_i records the water amount in the 4 gallon bucket that starts with $(0, 0)$ and ends with $(6, 0)$ in which successive terms could actually be used to finish your peculiar job.

ANSWER: _____

4. A bag contains 5 red balls, 2 green balls and 1 white ball. Two balls are drawn randomly from the bag. What is the probability that the drawn balls are the same color?

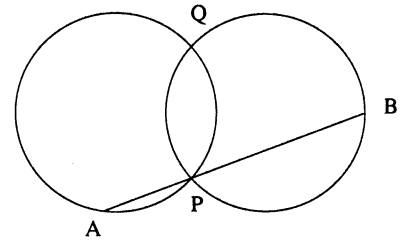
ANSWER: _____

5. A school bus travels 72 mph downhill, 63 mph on the level and 56 mph uphill. It takes one hour for the bus to make it from Pleasantville to Fort Fun and 70 min to return. How many miles is the (one way) trip?

ANSWER: _____

Part II: Long Answer Answer the following questions as completely as possible. Show all work for partial credit.

6. Two circles of the same radii intersect at points P and Q . A line through P intersects the circles at points A and B as in the figure. Prove that the perpendicular bisector of AB passes through Q .



Part I: Short Answer Give a short answer to the following questions in the space provided. You do not need to show work.

1. How many ways can $n = 11 \cdot 6 \cdot 2008$ be factored as a product of two natural numbers exactly one of which is even?. (251 is prime)

ANSWER: _____

2. List all real solutions x of the equation $|x - 1| = |2x + 5|$.

ANSWER: _____

3. You have only a 4 gallon and a 9 gallon bucket and are required to to fetch exactly 6 gallons of water from the river. It not possible to mark the buckets in any way. Write a sequence of pairs of numbers (a_i, b_i) where $i = 1, 2, \dots$, where a_i records the water amount in the 9 gallon bucket and b_i records the water amount in the 4 gallon bucket that starts with $(0, 0)$ and ends with $(6, 0)$ in which successive terms could actually be used to finish your peculiar job.

ANSWER: _____

4. A bag contains 3 red balls, 3 green balls and 2 white balls. Two balls are drawn randomly from the bag. What is the probability that the drawn balls are the same color.

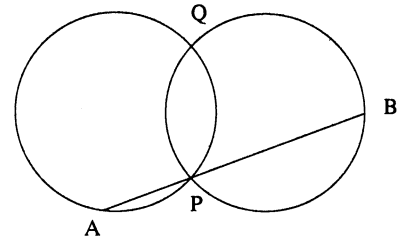
ANSWER: _____

5. A school bus travels 72 mph downhill, 63 mph on the level and 56 mph uphill. It takes one hour for the buss to make from Pleasantville to Fort Fun and 70 min to return. How many miles is the (one way) trip?

ANSWER: _____

Part II: Long Answer Answer the following questions as completely as possible. Show all work for partial credit.

6. Two circles of the same radii intersect at points P and Q . A line through P intersects the circles at points A and B as in the figure. Prove that the perpendicular bisector of AB passes through Q .



7. Suppose a, b are positive real numbers and $a^2 + b^2 = 4$.

Show that

$$\frac{ab}{a+b+2} \leq \sqrt{2} - 1.$$

8. **Dominos** are 1 by 2 rectangles. A **rectangle of dominos** is a rectangle with sides of length a and b made with $ab/2$ dominos. A rectangle of dominos can be **factored** if it can be broken into two smaller rectangles of dominos (without “breaking” any dominos). A rectangle of dominos is **prime** if it contains more than one domino and cannot be factored.

Note that every rectangle of dominos made of 3 dominos (and having area 6) can be factored and therefore is not prime.

- (a) Show that no 6 by 6 rectangle of dominos is prime.
- (b) Find a smaller prime rectangle of dominos.

9. Let n be a non-square positive integer, and let m be the largest integer less than the square root of n . Find all values of m and n such that m^3 divides n^2 .

10. f is a function from the positive integers to the positive integers satisfying the following:

a. $f(n + 1) > f(n)$ for all n

b. $f(f(f(n))) = 4n$

Compute $f(2008)$

11. A group of 21 boys and 21 girls are taking a math test. The bad news is that no contestant solved more than six problems. The good news is that for each girl and each boy, at least one problem was solved by both of them. Prove that there was a problem that was solved by at least three girls and at least 3 boys.