

**Part I: Short Answer** Give a short answer to the following questions in the space provided. You do not need to show work.

1. Craig can shovel a certain driveway in 15 minutes. Denzel can shovel the same driveway in 12 minutes. How long, to the nearest minute, will it take them to shovel the driveway if they work together?

ANSWER: \_\_\_\_\_

2. A sporting goods store ordered an equal number of orange and yellow tennis balls, but the tennis ball company delivered 20 extra yellow balls, making the ratio of yellow balls to orange balls in the shipment 7 : 5. How many tennis balls did the store originally order?

ANSWER: \_\_\_\_\_

3. If  $1/5$  of the air is let out of a spherical balloon, by how much does its surface area decrease?

ANSWER: \_\_\_\_\_

4. We say that a prime number  $p$  is *the sum of two squares* if  $p = x^2 + y^2$  for some integers  $x$  and  $y$ . How many primes  $p$  with  $7 \leq p \leq 31$  are *not* the sum of two squares?

ANSWER: \_\_\_\_\_

5. Define a function  $f$  with complex values by  $f(z) = (z - i)^2$ . What is  $f(f(f(1)))$ ?

ANSWER: \_\_\_\_\_

**Part I: Short Answer** Give a short answer to the following questions in the space provided. You do not need to show work.

1. Craig can shovel a certain driveway in 60 minutes. Denzel can shovel the same driveway in 40 minutes. How long, to the nearest minute, will it take them to shovel the driveway if they work together?

ANSWER: \_\_\_\_\_

2. A sporting goods store ordered an equal number of orange and yellow tennis balls, but the tennis ball company delivered 30 extra yellow balls, making the ratio of yellow balls to orange balls in the shipment 6 : 5. How many tennis balls did the store originally order?

ANSWER: \_\_\_\_\_

3. If  $1/3$  of the air is let out of a spherical balloon, by how much does its surface area decrease?

ANSWER: \_\_\_\_\_

4. We say that a prime number  $p$  is *the sum of two squares* if  $p = x^2 + y^2$  for some integers  $x$  and  $y$ . How many primes  $p$  with  $5 \leq p \leq 31$  are *not* the sum of two squares?

ANSWER: \_\_\_\_\_

5. Define a function  $f$  with complex values by  $f(z) = (z + i)^2$ . What is  $f(f(f(-1)))$ ?

ANSWER: \_\_\_\_\_

**Part I: Short Answer** Give a short answer to the following questions in the space provided. You do not need to show work.

1. Craig can shovel a certain driveway in 20 minutes. Denzel can shovel the same driveway in 30 minutes. How long, to the nearest minute, will it take them to shovel the driveway if they work together?

ANSWER: \_\_\_\_\_

2. A sporting goods store ordered an equal number of orange and yellow tennis balls, but the tennis ball company delivered 30 extra yellow balls, making the ratio of yellow balls to orange balls in the shipment 7 : 5. How many tennis balls did the store originally order?

ANSWER: \_\_\_\_\_

3. If  $1/2$  of the air is let out of a spherical balloon, by how much does its surface area decrease?

ANSWER: \_\_\_\_\_

4. We say that a prime number  $p$  is *the sum of two squares* if  $p = x^2 + y^2$  for some integers  $x$  and  $y$ . How many primes  $p$  with  $2 \leq p \leq 31$  are *not* the sum of two squares?

ANSWER: \_\_\_\_\_

5. Define a function  $f$  with complex values by  $f(z) = (z + i)^2$ . What is  $f(f(f(1)))$ ?

ANSWER: \_\_\_\_\_

**Part I: Short Answer** Give a short answer to the following questions in the space provided. You do not need to show work.

1. Craig can shovel a certain driveway in 10 minutes. Denzel can shovel the same driveway in 15 minutes. How long, to the nearest minute, will it take them to shovel the driveway if they work together?

ANSWER: \_\_\_\_\_

2. A sporting goods store ordered an equal number of orange and yellow tennis balls, but the tennis ball company delivered 20 extra yellow balls, making the ratio of yellow balls to orange balls in the shipment 8 : 7. How many tennis balls did the store originally order?

ANSWER: \_\_\_\_\_

3. If  $1/4$  of the air is let out of a spherical balloon, by how much does its surface area decrease?

ANSWER: \_\_\_\_\_

4. We say that a prime number  $p$  is *the sum of two squares* if  $p = x^2 + y^2$  for some integers  $x$  and  $y$ . How many primes  $p$  with  $2 \leq p \leq 37$  are *not* the sum of two squares?

ANSWER: \_\_\_\_\_

5. Define a function  $f$  with complex values by  $f(z) = (z - i)^2$ . What is  $f(f(f(-1)))$ ?

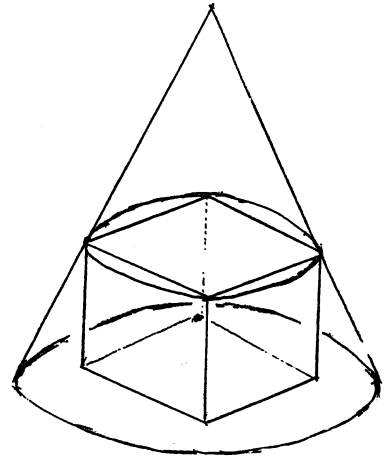
ANSWER: \_\_\_\_\_

**Part II: Long Answer** Answer the following questions as completely as possible. Show all work for partial credit.

6. Let  $n$  be a natural number and  $\theta$  any angle. Prove that

$$\left(\cos \frac{\theta}{2}\right) \left(\cos \frac{\theta}{4}\right) \left(\cos \frac{\theta}{8}\right) \cdots \left(\cos \frac{\theta}{2^n}\right) = \frac{\sin \theta}{2^n \sin(\theta/2^n)}.$$

7. A cube is inscribed in a right circular cone. (See figure.) If the radius of the cone is  $\sqrt{2}$  meters and its height is 3 meters, what is the volume of the cube?



8. An ordinary deck of cards consists of 13 clubs, 13 diamonds, 13 hearts and 13 spades. Cards are drawn at random from an ordinary deck of cards, one by one, without replacement. What is the probability that no heart is drawn before the ace of spades is drawn?

9. For any natural numbers  $n \geq 3$ , show that there exist odd, positive integers  $x_n$  and  $y_n$  such that

$$7x_n^2 + y_n^2 = 2^n.$$

10. Find the greatest integer  $N$  less than

$$\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \cdots + \frac{1}{\sqrt{10,000}}.$$

11. Fifty people are in line to enter a theater with 50 seats. Each person has a ticket for an assigned seat. The first person in line is very rude, and upon entering the theater, sits in a seat at random. The other 49 people are very polite. Each enters the theater in turn and sits in his/her assigned seat if it is available. But if it is occupied, (s)he sits in an available seat at random rather than disturb the person already sitting in her/his seat.  
What is the probability that the last (50th) person will sit in his/her assigned seat?