**Part I: Short Answer** Give a short answer to the following questions in the space provided. You do not need to show work.

1. $x$, 10, $x + 9$, $2x + 5$ and $x^2 - 55$ represent different positive integers, listed in order from least to greatest. What is the value of $x$?

   ANSWER:____________________

2. For all numbers $a$ and $b$, let the operation $\diamond$ be defined by $a \diamond b = b^2 - ab$. What is the value of $(1 \diamond 2) \diamond 3$?

   ANSWER:____________________

3. When four consecutive integers are added together, the result is $-22$. What is the greatest product that can be obtained by multiplying two of these 4 integers together?

   ANSWER:____________________

4. Let $f(x) = 2x^2 - 2x - 12$. What values of $t$ are solutions of the equation $f(t - 2) = 0$?

   ANSWER:____________________

5. Jane is planning her next business trip. Each of her outfits consists of shoes, either a skirt or a pair of slacks, a blouse and a jacket or no jacket at all. If she packs 3 pairs of shoes, 3 skirts, 3 pairs of slacks, 6 blouses and 2 jackets, how many outfits will she have available on her trip?

   ANSWER:____________________

**Part II: Long Answer** Answer the following questions as completely as possible. Show all work for partial credit.

1. Let $a$, $b$, and $c$ be integers such that

$$a + b\sqrt{2} + c\sqrt{3} = 0.$$

   Show that $a = b = c = 0$.

2. Consider the "bogus cancellation"

$$\frac{16}{64} = \frac{1\not{6}}{\not{6}4} = \frac{1}{4}$$

which miraculously works. Let $a$, $b$, $c \in \{1, 2, 3, \ldots, 9\}$ be digits and assume that $a \neq b$. Find all possible triples $(a, b, c)$ such that the bogus cancellation

$$\frac{ab}{bc} = \frac{a\not{b}}{\not{b}c} = \frac{a}{c}$$

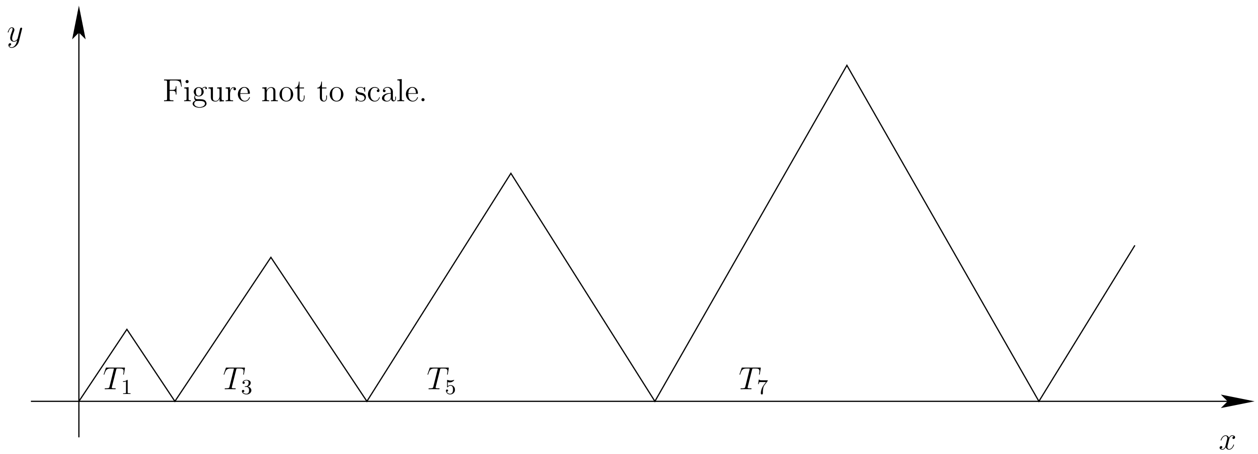
works. (Here, $ab$ and $bc$ are *not* products, but rather the two digits numbers made of the digits $a$, $b$, $c$.)

3. In coordinate three-space, consider nine points

$$P_1 = (x_1, y_1, z_1), P_2 = (x_2, y_2, z_2), P_3 = (x_3, y_3, z_3), \ldots, P_9 = (x_9, y_9, z_9)$$

and assume that all of their coordinates $x_1, \ldots, x_9, y_1, \ldots, y_9, z_1, \ldots, z_9$ are integers. Show that for some choice of $i \neq j$, the midpoint $M$ of the segment $\overline{P_i P_j}$ must also have integer coordinates.

4. For $i = 1, 3, 5, \ldots$, let $T_i$ be an equilateral triangle with sides of length $i$. For each triangle, identify one side as its *base*, and call the vertex opposite the base its *apex*. Arrange the triangles in the $xy$-plane as shown, with all of their bases on the $x$-axis:




Show that there is a parabola containing the apexes of all of the triangles.

5. In the plane, assume that a circle $S$ with center $O$ is inscribed in a quadrilateral $ABCD$. (That is, each of the segments $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ are tangent to $S$.) Show that the measures of angles $\angle AOB$ and $\angle COD$ sum to $180°$.

6. Let $n$ be a natural number. Your vehicle is a gas-guzzling Buzzer B1 which needs one gallon of gas to go 10 miles and comes equipped with an empty 20-gallon gas tank. Your goal is to drive your Buzzer once around a 100-mile track. The challenge: 10 gallons of gas are divided into $n$ different gas cans, which are placed at random points (known to you) around the track. Whenever you get to a gas can, you add its contents to your tank. Show that if you may choose where on the track you start, then you can always achieve your goal.