Math Circle
An Outreach Program at the University of Utah

By Renzo Cavalieri and David Hartenstine

Math Circle at the University of Utah is a weekly two-hour program for high school (and some middle school) students. The participants are exposed to interesting mathematics or more advanced topics that they would not normally see in high school, and have the opportunity to explore these by working on problems with faculty and graduate students (and each other).

University of Utah Math Circle participants, May 2002.

In Math Circle, a lecture format is avoided and instead mathematics is developed through exploration, discovery, and discussion. The first author of this note is a graduate student at Utah and the second recently completed a postdoctoral appointment there. Each of us has contributed to the planning and conducting of this program, under the direction of a faculty coordinator. This position has been (and currently is) held by Peter Trapa and also by Nick Korevaar.

The Utah Math Circle originated in October 2001 in the outreach component of the department's VIGRE program. The involvement of faculty, postdocs, and graduate students with high school students exhibits the vertical integration that lies at the heart of the VIGRE program. (For more information about VIGRE program at Utah, visit http://www.math.utah.edu/vigre.)

The idea of these mathematical interactions between high schoolers and mathematicians is not original to Utah. The first Math Circle met in Hungary in the 1800s, and Math Circles have been set up in many places, including Berkeley and Harvard. Those programs have been particularly influential in the development of Utah's Math Circle.

The primary goal of Math Circle is to generate and cultivate interest in mathematics in younger students. The program also establishes a link between the high schools and our department. Some of the Math Circle sessions and activities have been successfully used in math clubs by high school teachers.

We feel that the program has been successful in creating an enrichment opportunity for mathematically-minded high school students. Student responses on evaluation forms have been uniformly positive. Many Math Circle participants have signed up for our department's summer high school program. Several Math Circlers have gone on to attend the University of Utah or to major in mathematics or a related field elsewhere.

So, what happens in Math Circle?

Some examples of topics covered are hyperbolic geometry, algebraic curves, group theory by way of Rubik's cube, and genetic selection; a full list of topics including detailed notes from most sessions can be found online at http://www.math.utah.edu/mathcircle/. Most session leaders (usually faculty members, but sometimes graduate students) conduct two consecutive sessions on a single topic. This framework provides enough time to achieve some depth.

On the other hand, no matter how interesting the topic or dynamic the presenter, spending too much time on a single subject or with a single leader risks losing the interest of some of the students. About every fifth session, there is a contest, with problems related to the preceding weeks' material, and the top scorers win their choice of mathematically-themed prizes (books, games, puzzles, etc.). These contests give the students the opportunity to apply what they have learned, and they enjoy the competition.

The best Math Circles are those in which the students experiment, discuss ideas, make conjectures, try to prove them, and explain their discoveries and solutions of problems to their peers. Ideally, Math Circle is conducted in an environment in which students are encouraged to lead their own exploration into the mathematical world.

There is no standard recipe that guarantees a successful session, but after much trial and error, we did develop a rough format that often led to enjoyable and successful sessions. After beginning by developing new material, the students are given problems to work on, questions to think about, or other activities. During this time, the students can work in small groups and the presenter and the other facilitators are available to answer questions, give hints or pose additional questions, and work one-on-one with the students. Volunteers then present their so-

lutions to the group. Following this, new material (building on the discussed problems) is presented, followed by more activities, and so on.

For example, an introduction to two-dimensional topology had as its ultimate aim the understanding of the classification theorem for compact surfaces with no boundary. The mathematical content is conceptually more sophisticated than anything the participants have seen in their formal education. To capture the essence of a torus, the videogame "Asteroids," in which both spaceships and asteroids exit the screen both on the horizontal and vertical side to reappear from the opposite side, was invoked. The mathematization of something from their recreational experience provoked the students' curiosity and amazement. This served as great motivation for some of the difficult conceptual work to come.

One challenge was to define (or negotiate the region between a rigorous definition and a hazy, more intuitive notion of) a topological surface and a homeomorphism. The participants were asked for examples of surfaces, the presenters added some of their own, and then the students were asked to decide which were homeomorphic and which were not. It then became clear that the task of classifying surfaces was not to be underestimated.

The concept of identification of sides of a polygon as a tool to generate surfaces was then introduced. As an activity, the students used paper and scissors to build cylinders and Möbius strips. This naturally led to the notion of orientability and a number of exciting discoveries. It was surprising to the students that there are surfaces with only one side and only one boundary circle.

One-on-one interaction is a big part of the Marh Circle experience.

The Möbius strip offers a wealth of interesting and accessible problems. This experimentation led to many conjectures. Through identification of sides of a polygon a library of "familiar" surfaces was built: the sphere, torus, and the amazing projective plane. The next step was to convince the students that any compact surface could be represented by a polygon, and, finally, through cutting and pasting, that any such polygon could be reduced to the canonical polygon representing a connected sum of tori and projective planes. This last part turned out to be quite hard for a fair number of younger students, but very exciting for the more advanced ones.

As is often the case with Math Circle, the experience can still be very rewarding even if not all of the material is grasped by all of the students. Throughout this session, informal language was purposefully adopted; some standard mathematical concepts were even renamed. For example, non-orientability became the HSRP (Han Solo Reversing Property), by the fact that Starship Captain Han Solo has a chance to go on a mission and return to his original position upside down without ever having changed the direction of his spaceship.

Care was also taken in striking a balance between mathematical rigor and intuitive notions. Some terms and concepts needed to be carefully defined, while for others it was sufficient to work with a more imprecise idea.

Who comes to Math Circle and how does it work?

The students are self-selected. Near the beginning of the school year, a program brochure and a letter indicating how to apply are mailed to heads of math departments at local schools and to school district mathematics specialists as well as to students who participated the previous year and are still in high school.

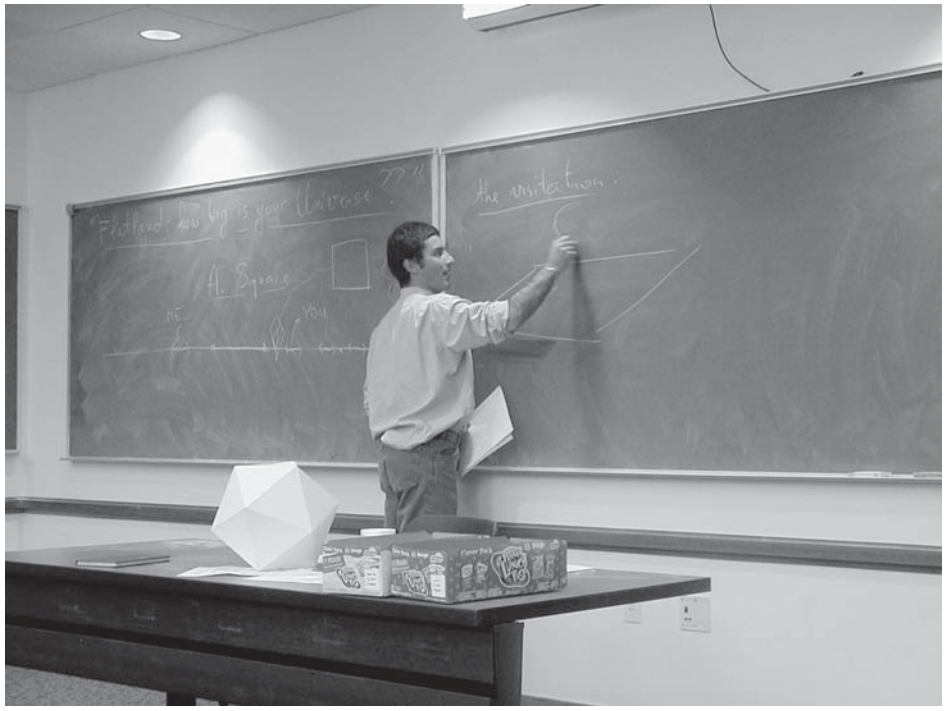
A faculty member is the coordinator of the program. His re-

sponsibilities include planning the sessions, locating presenters, and generally being the official figure of the group. This person receives a teaching reduction (this is the largest cost to the department in running the program). There is also at least one other department member who together with the coordinator constitute a core team. This group attends all of the Math Circle meetings, gets to know the regular participants personally, and gives the program continuity from presenter to presenter. They know when it's a good time to interrupt to give the students something to think about or to gauge student understanding.

It is also very helpful to have a person with extensive high school teaching experience contribute to the program. Such a person's knowledge of what students have seen in school and what students would be interested in and able to handle mathematically adds another perspective to that of university faculty. Three or four postdocs and graduate students also attend the sessions. These assistants, along with the core team and the presenter, assist the students as they work individually or in small groups. Administrative duties, such as designing brochures and contacting participants and schools among others, are handled by the department's VIGRE program coordinator.

Other than the faculty coordinator, no speaker has led more than three sessions in a single year. Varying the speakers allows the utilization of the differing areas of expertise of the faculty, and helps in the selection of topics. This changing cast of characters also keeps the program fresh and prevents Math Circle from resembling a weekly class. In addition, having many people from the department involved in Math Circle broadens the exposure of the students to the faculty.

The number of participants varies from week to week, but is usually between fifteen and twenty-five students. It seems that a group of about twenty is optimal. If the group is too large, the informal atmosphere can break down, there are not enough facilitators to assist students, and the group can become unruly. On the other hand, if the number of students is too small, it is harder to get discussions started.



Renzo Cavalieri leading a Math Circle on dimension and Flatland.

Observations and Conclusions

First and foremost, Math Circle should be fun. It is best when students are actively engaged throughout the session and not just when working on or presenting solutions to problems. Activities and problems need to be selected that are appropriate for the wide range of ability and mathematical experience found in the students. Some of the questions or problems should be accessible to all of the participants, while at the same time, it is important to keep the most advanced students challenged.

Since attendance in general varied from week-to-week, and many students are too busy with school and other activities to devote much thought to Math Circle between sessions, it is best to make each session as self-contained as possible. An informal atmosphere creates an environment in which students are not afraid to make mistakes or make their own conjectures.

Many presenters have found that conducting a Math Circle is an invigorating experience. It is very satisfying to work with a group of students all of whom are interested in and excited by mathematics. The participants are very bright and often ask excellent, unexpected, and thought-provoking questions.

Finally, the journey is often more important than the destination. While it is great to finish with a beautiful or surprising result (like the classification of surfaces), a session can suffer when the presenter speeds up (and loses many of the students) in order to get to such a goal.

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Photographs taken by Sarah Strong.