Math Circle, an outreach program at the University of Utah

Renzo Cavalieri and David Hartenstine
Department of Mathematics
155 South 1400 East
University of Utah
Salt Lake City, Utah 84112–0090

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Math Circle at the University of Utah is a weekly two-hour program for area high school students. The participants are exposed to interesting or more advanced mathematics that they would not normally see in high school, and have the opportunity to explore these topics by working on problems with faculty and graduate students (and each other). In the Math Circle sessions, a lecture format is avoided and instead mathematics is developed through exploration, discovery and discussion in an informal atmosphere. The first author of this note is a graduate student at Utah and the second recently completed a postdoctoral appointment there, and is now at Western Washington University. Each of us has contributed to the planning and conducting of this program, under the direction of a faculty coordinator. This position has been (and currently is) held by Peter Trapa and also by Nick Korevaar. The Utah Math Circle originated in October 2001 in the department's VIGRE program and it constitutes a major part of VIGRE's outreach activity. The involvement of faculty, postdocs and graduate students with high school students exhibits the vertical integration that lies at the heart of the VIGRE program. (For more information about VIGRE at the University of Utah, see the website www.math.utah.edu/vigre.)

The idea of these mathematical interactions of high school students and academic mathematicians is not original to Utah. The first Math Circle was

formed in Hungary in the 19th century, and since that time, Math Circles have been set up all over the world. Several other American institutions have Math Circles, notably Berkeley and Harvard. The experience of those programs was influential in the initial development of Utah's Math Circle, and these more established Math Circles continue to be a source of ideas and inspiration. Historically, Math Circles have given rise to mathematics competitions, and have been instrumental in the development of some top mathematical minds. However, the purpose of the Math Circle at Utah is not to develop (or even necessarily recognize) prodigies, nor is it to train students for challenging contests, although several sessions have been devoted to the types of problems that appear on state and national exams.

The primary goal of the program is to generate, maintain and cultivate interest in mathematics in inclined and/or talented students. Math Circle also establishes a link between the high schools and our department. Through their participation in the program, students become familiar with our department and university. Several Math Circle students have gone on to attend the University of Utah or to major in mathematics or a related field elsewhere.

We feel that the program has been successful in creating an enrichment opportunity for mathematically-minded high school students. Student responses on evaluation forms have been uniformly positive. Some of the Math Circle sessions and activities have been successfully used in math clubs by high school teachers. Many Math Circle participants have signed up for our department's summer high school program. Furthermore, we believe that similar programs could be set up at other institutions.

So, what happens in Math Circle?

Some examples of topics covered are hyperbolic geometry, knot theory, algebraic curves, group theory by way of Rubik's cube, and genetic selection; a full list of topics including detailed notes from most sessions can be found at the website www.math.utah.edu/mathcircle/. Most session leaders (usually faculty members, but sometimes graduate students) conduct two consecutive sessions on a single topic. Using this framework provides enough time to achieve some depth with each topic. On the other hand, no matter how interesting the topic or dynamic the presenter, spending too much time on a single subject or with a single session leader risks losing the interest of some of the students. About every fifth session, there is a problem-solving contest.

The contest problems are related to the material from the preceding weeks, and the top scorers win their choice of mathematically-themed prizes (books, games, puzzles, etc.). These contests give the students the opportunity to apply what they have learned, and they enjoy the competition.

The best Math Circles are those in which the students experiment, discuss ideas, make conjectures, try to prove them and explain their discoveries and solutions of problems to their peers. Ideally, Math Circle is conducted in an environment in which students are encouraged to lead their own exploration into the mathematical world. There is not a canonical, standard recipe that guarantees a successful session. Some topics we thought would be difficult were picked up very easily. On the other hand, they had great difficulty with some topics that we thought would be accessible (countability is one example). After much trial and error, we did develop a rough format that often led to enjoyable and successful sessions. A session usually begins with the introduction of new material. After a sufficient amount of material has been developed, the students are given problems to work on, questions to think about, or other activities. During this time, the students can work in small groups and the presenter and the other facilitators are available to answer questions, give hints or pose additional questions, and work one-on-one with the students. Volunteers then present their solutions to the group. Following this, new material (building on the discussed problems) is presented, followed by more activities, and so on.

For example, an introduction to two-dimensional topology had as its ultimate aim the understanding of the classification theorem for compact, boundaryless surfaces. The mathematical content is conceptually more sophisticated than anything any of the participants have studied in their formal education. To capture the essence of a torus, the videogame "Asteroids", in which both spaceships and asteroids exit the screen both on the horizontal and vertical side to reappear from the opposite side, was invoked. The mathematization of something close to their intuition and recreational experience provoked the students' curiosity and amazement. This served as great motivation for some of the difficult conceptual work to come. One challenge was to define (or negotiate the region between a rigorous definition and a hazy, more intuitive notion) a topological surface and a homeomorphism. The participants were asked for examples of surfaces, the presenters added some of their own, and then the students were asked to decide which were homeomorphic and which were not. It then became clear that the task of classifying surfaces was not to be underestimated. The concept of identifica-

tion of sides of a polygon as a tool to generate surfaces was then introduced. As an activity, the students used paper and scissors to build cylinders and Möbius strips. This naturally led to the notion of orientability and a number of exciting discoveries for the students. It was surprising to them that there are surfaces with only one side and only one boundary circle. The Möbius strip offers a wealth of interesting and accessible problems. This experimentation led to many conjectures. Through identification of sides of a polygon a library of "familiar" surfaces was built: the sphere, torus, and the amazing projective plane. The next step was to motivate that any compact surface could be represented by a polygon, and, finally, through cutting and pasting, that any such polygon could be reduced to the canonical polygon representing a connected sum of tori and projective planes. This last part turned out to be quite hard for a fair number of younger students, but very exciting for the more advanced ones. As is often the case with Math Circle, the experience can still be very rewarding even if not all of the material is grasped by all of the students. Throughout this session, informal language was purposefully adopted; some standard mathematical concepts were even renamed. For example, non-orientability became the HSRP (Han Solo Reversing Property), by the fact that Starship Captain Han Solo has a chance to go on a mission and return to his original position upside down without ever having changed the direction of his spaceship. Care was also taken in striking a balance between mathematical rigor and intuitive notions. Some terms and concepts needed to be carefully defined, while for others it was sufficient to work with a more imprecise idea.

Who comes to Math Circle and how do we make it work?

The students are self-selected and most of them are in high school, although a few middle school students have participated as well. The distribution among the grades has been nearly uniform. Near the beginning of the school year, a brochure describing the program and a letter indicating how to apply (for a copy of the brochure and application materials, see the website) is mailed to heads of math departments at local schools and to school district mathematics specialists as well as to students who participated the previous year but did not graduate. Nearly all applicants are accepted into the program. This is hardly surprising, since the students are volunteering to give up some of their (in many cases, extremely limited) free time to work on

mathematics, their interest and ability in the subject is in most cases quite high.

A faculty member is the coordinator of the program. His responsibilities include planning and organizing the sessions, locating presenters, and generally being the official figure of the group. This person receives a teaching reduction (this is the largest cost to the department in running the program). In addition, there is at least one other department member who together with the coordinator constitute a core team. This group attends all of the Math Circle meetings, gets to know the regular participants personally, and gives the program continuity from presenter to presenter. They also have the experience to know when it is a good time to interrupt a presenter to give the students something to think about or to gauge student understanding. It is also very helpful to have a person with extensive high school teaching experience contribute to the program. Such a person's knowledge of what students have seen in school and what students would be interested in and able to handle mathematically adds another perspective to that of university faculty. Three or four postdocs and graduate students also attend the sessions. These assistants, along with the core team and the presenter, comprise a group of facilitators that assist the students as they work individually or in small groups. Administrative duties, such as designing brochures and contacting participants and schools among others, are handled by the department's VIGRE program coordinator.

Other than the faculty coordinator, no speaker has led more than three sessions in a single year. Varying the speakers allows the utilization of the differing areas of expertise of the faculty, and helped in the selection of topics. Since each leader has a different style, this changing cast of characters also keeps the program fresh and prevents Math Circle from resembling a weekly class. In addition, having many people from the department involved in Math Circle broadens the exposure of the students to the faculty. Students who attend all of the sessions in a year end up interacting with approximately ten different presenters.

The number of participants varies from week to week, but is usually between fifteen and twenty-five students. It seems that a group of about twenty is optimal. If the group is too large, the informal atmosphere can break down, there are not enough facilitators to assist students, and the group can become unruly. On the other hand, if the number of students is too small, it is harder to get discussions started and some of the participants seem more intimidated when the ratio of students to facilitators drops.

Some care is needed in the selection of the speakers and topics. When thinking of speakers to invite, it is important to keep in mind how they would be received by high school students. The leaders of the best sessions are those who can motivate the students with interesting, yet accessible, problems and guide the students to discover the mathematics involved. Abstract or technical topics presented without enough attention to these concerns have not been as well-received. This was also the case when the format tended toward a lecture style. Because even the most seasoned university faculty often do not have much experience with high school students generally, let alone the type of group this program has been fortunate to attract, it is very important to have assistants present who are familiar with what works well and what doesn't and who can interrupt if necessary to see if the students are keeping up. Since attendance in general varied from week-to-week, and many students are too busy with school and other activities to devote much time or thought to topics considered in Math Circle between sessions, it is best to make each session as self-contained as possible. Inevitably, this requires the review of material covered the previous week in the second session of a topic. Even so, it was often difficult for students who missed part one to keep up with part two. This is a situation for which having a large number of facilitators helped immensely: an assistant could work with those students who were absent the previous week and help them catch up.

Observations and Conclusions

We have learned a few things about what leads to successful and enjoyable sessions. Some of these are

1. The time spent lecturing needs to be minimized. Most of these students have gotten enough of that in school before coming to Math Circle. We tried to limit lecture portions of the sessions to no more than about 15 minutes.

2. It is best when students are engaged as active participants throughout the session and not just when working on or presenting solutions to problems. For example, this could be accomplished by motivating new material by asking questions, or getting them to think about things they already know in a new way.

3. Activities and problems need to be selected that are appropriate for the wide range of ability and mathematical experience found in the students. Some of the questions or problems should be accessible to all of the participants, while at the same time, it is important to keep the most advanced students challenged with more difficult problems.

4. Math Circle should be fun. The students are energized by mathematics. An informal atmosphere creates an environment in which students are not afraid to make mistakes or make their own conjectures.

5. The journey is often more important than the destination. While it is great to finish with a beautiful or surprising result (like the classification of surfaces), a session can suffer when the presenter speeds up (and loses many of the students) in order to get to it.

Many presenters have found that conducting a Math Circle is an invigorating yet exhausting experience. It is very satisfying to work with a group of students all of whom are interested in and excited by mathematics. The participants are very bright and often ask excellent, difficult questions. Many session leaders discovered that preparing for a Math Circle session is more time-consuming than preparing a week of an undergraduate course.

In running Math Circle, some features of our department are utilized that are not shared by all institutions. We still believe that similar programs could be set up at other types of institutions. We don't claim to have all the answers, but feel that an innovative program has been developed at Utah that is fostering interest in mathematics among high school students. A department needs to be large enough so that finding willing and able speakers is not impossible. If this is not the case, perhaps a joint program could be run by several neighboring institutions. Graduate students play a large role in Math Circle at Utah. Departments that do not have a graduate program could still conduct a similar program. Many institutions that do not have graduate programs in mathematics have them in education. Math Circle would be a great experience for students planning to teach secondary mathematics, and a Math Circle could be run as a joint program of the mathematics and education departments. Explicit financial costs, aside from the coordinator's teaching reduction, to run the program are minimal, consisting mainly of administrative costs and money for the prizes and snacks.