

Pries: 470 Euclidean and non-Euclidean Geometry

Some harder homework problems (optional)

1. Let P be a projective plane, let p be a point of P , and let L be a line of P not containing p . Let A be the set of lines through p . Show that the function $f : A \rightarrow L$ defined by $f(M) = M \cap L$ is bijective.
2. Show that in a projective plane with finitely many points any two lines contain the same number of points.
3. Show that all of the distance-preserving bijections $f : \mathbb{R} \rightarrow \mathbb{R}$ can be written with a formula $f(x) = \pm x + t$.
4. Let $\ell : A_1x + B_1y + C_1 = 0$ and $m : A_2x + B_2y + C_2 = 0$ be two Euclidean lines. In class we showed that if $\ell = m$ then there are two points (x, y) which are solutions to both of these equations. Use linear algebra to show this implies there exists $\lambda \neq 0$ so that $A_1 = \lambda A_2$, $B_1 = \lambda B_2$ and $C_1 = \lambda C_2$.