Pries: 470 Euclidean and non-Euclidean Geometry

Homework 8: Euclidean Transformations

Due Friday March 10

More on Euclidean Transformations:

1. Find the images of $P = (0, 0)$, $Q = (0, 2)$, and $R = (2, 1)$ under the clockwise rotation by 45 degrees that fixes $(3, 3)$:

   a. using the rotation matrix.

   b. using complex numbers.

2. A set S of points has *rotational symmetry* if it is invariant under some rotation. How many rotational symmetries do these sets have?

   a. an isoceles triangle.

   b. a regular pentagon.

   c. a regular hexagon.

   d. a circle.

3. Let $m = \tan(\theta)$. Let γ_ℓ be the reflection over the line $\ell : y = mx$.

   a. Write γ_ℓ as the composition of three transformations.

   b. Now $\gamma_\ell = T_C \circ \rho_B$ for some C and some unit vector B. Use the composition rules to find C and B.

   c. Find a formula for $\gamma_\ell(z)$ using complex numbers.

4. There is some isometry $T = T_C \circ \rho_B \circ \gamma$ that takes $\triangle ABC$ to $\triangle DEF$ where $A = (6, 2)$, $B = (7, 2)$, $C = (7, 4)$, $D = (-4, 3)$, $E = (-7/2, 3 + \sqrt{3}/2)$, and $F = (-4 - \sqrt{3}, 4)$. What are C and B?

5. Let γ_ℓ be the reflection over the line ℓ.

   a. Find two lines ℓ and m so that $\gamma_\ell \circ \gamma_m = \rho_{\pi/4}$.

   b. Find two lines ℓ and m so that $\gamma_\ell \circ \gamma_m = T_{(4,4)}$.

6. Find the images of $(0, 0)$, $(0, 2)$ and $(2, 1)$ under the glide reflection G_{AB} where $A = (-2, 0)$ and $B = (0, 1)$.