Pries: 470 Euclidean and non-Euclidean Geometry

Homework 7: Euclidean Transformations
Due Friday March 3

Euclidean Transformations:

1. Let $C \in \mathbb{R}^2$. Let $T_C : \mathbb{R}^2 \to \mathbb{R}^2$ be the translation map $T_C(P) = P + C$.
   a. Show that T_C is a bijection by showing it is 1-to-1 and onto by method 1.
   b. Show that T_C is a bijection by finding the inverse.
   c. Show that T_C is distance preserving using vectors.
   d. Let T_{AB} be the translation that takes A to B. Prove $T_{CA}T_{BC}T_{AB}$ is the identity. There is a non-messy approach to all of these!

2. If T is any distance-preserving transformation, show that T preserves circles, i.e. show that the image under T of a set of points in a circle is still a circle.

3. Prove the 6th composition rule: $\gamma \circ \rho_B = \rho_{\gamma(B)} \circ \gamma$.

4. Find the images of $P = (0, 0)$, $Q = (0, 2)$, and $R = (2, 1)$ under the following isometries (distance-preserving transformations)
   a. The reflection across the line $y = x$.
   b. The reflection across the line $x + y = 5$.
   c. The rotation by 180 degrees that fixes $(-1, 1)$.
   d. (optional) The clockwise rotation by 45 degrees that fixes $(2, 2)$.

5. Let S be a set of points. A line ℓ is called a *line of symmetry for S* if the set S is preserved when you reflect over ℓ.
   a. Draw all lines of symmetry of an isoceles triangle.
   b. Draw all lines of symmetry of an equilateral triangle..
   c. How many lines of symmetry does a regular pentagon have?
   d. How many lines of symmetry does a regular hexagon have?
   e. How many lines of symmetry does a circle have?

6. Find a formula for the reflection over the line $y = x + 1$.
   Hint: either use projections or composition of transformations.