Pries: 470 Euclidean and non-Euclidean Geometry

Homework 6: Angles and review problems
Angle problems are due Wed 2/22. Review problems are for the midterm on Friday 2/24.

Angles:

1. Let $\vec{u} = (\sqrt{2}/2, \sqrt{2}/2)$ and let $\vec{v} = (\sqrt{2}/2, -\sqrt{2}/2)$.
   (i) Show that $\vec{u}$ and $\vec{v}$ are perpendicular to each other and have length 1. (Then U and V are called an orthonormal set of vectors).
   (ii) Show that for any $\vec{w} = (x, y)$ then $\vec{w} = (\vec{w} \cdot \vec{u})\vec{u} + (\vec{w} \cdot \vec{v})\vec{v}$.
2. Let $A = (0, 0)$, $B = (1, 0)$, and $C = (\sqrt{3}, 1)$.
   (i) What is $\angle BAC$?
   (ii) Compare this angle with the approximation $\arctan(w) = w - w^3/3 + w^5/5$ when $w = 1/\sqrt{3}$ and show the absolute value of the error is bounded by $w^7/7$.
   (iii) Repeat with $C = (1, \sqrt{3})$ and explain what happens.
3. Let $\vec{u} = (\cos(\alpha), \sin(\alpha))$ and let $\vec{v} = (\cos(\beta), \sin(\beta))$. Show that the angle sum of $\vec{u}$ and $\vec{v}$ is $(\cos(\alpha + \beta), \sin(\alpha + \beta))$ using trig formulas.

Review Problems: do not hand in

HW1, #3

HW2, Parallel #2

HW3, Euclid #1

HW3, Birkoff #1

HW3, Birkoff #3

HW4, Birkoff #2

HW4, Similarity #1

HW5, #3c

HW5, #7

HW6, #1