

Pries: 470 Euclidean and non-Euclidean Geometry

Homework 5: lines in analytic Euclidean geometry

Due Friday 2/17

Lines in analytic Euclidean geometry:

1. Find the parametric formula for the line segment starting at $(0, b)$ when $t = 0$ and ending at $(1, m + b)$ when $t = 1$.
2. Find the coordinates of the point of intersection of the lines $ax+by+c=0$ and $dx+ey+g=0$ in terms of the variables a, b, c, d, e, f (assuming they are distinct, non-parallel lines).
3. Let Z be the ellipse $x^2 + 4y^2 = 1$. Let G be the subset of points for which $x^2 + 4y^2 < 1$ and let H be the subset of points for which $x^2 + 4y^2 > 1$.
 - a) Show that H is not convex (explicitly find points).
 - b) Use parametric line segments to prove that G is convex.
 - c) Use parametric line segments to prove that if A is a point in G and B is a point in H then \overline{AB} intersects Z . Hint: define a function $g(t) : \overline{AB} \rightarrow \mathbb{R}$ whose value at a point $P_t = (x_t, y_t)$ on the line segment is $x_t^2 + 4y_t^2 - 1$.
4. Find the two points of intersection of the hyperbola $x^2 - y^2 = 1$ with the line with slope m through $(-1, 0)$. Use this to find a parametrization (bijection) between the hyperbola $x^2 - y^2 = 1$ and almost all points on the y -axis.
5. Finish the proof that the dot product is linear in the second variable by showing $\langle P, Q + R \rangle = \langle P, Q \rangle + \langle P, R \rangle$.
6. Modify the proof of Cauchy-Schwarz to show that $|\langle P, Q \rangle| = |P||Q|$ if and only if the vector P is a scalar multiple of Q .
7. Prove the Pythagorean theorem using vectors by showing $|P + Q|^2 = |P|^2 + |Q|^2$ if and only if $\langle P, Q \rangle = 0$.