

## Pries: 470 Euclidean and non-Euclidean Geometry

### Homework 4: Birkoff's Axioms, Similarity

Due Friday 2/10

#### Birkoff's axioms:

1. Let  $S_1$  and  $S_2$  be two convex sets. Prove that the intersection  $S_1 \cap S_2$  is convex. Give an example to show that the union  $S_1 \cup S_2$  does not have to be convex.
2. Given any angle, prove that there is a unique ray bisecting that angle.
3. Given a triangle  $\triangle ABC$  and a line  $\ell$  so that  $A, B, C \notin \ell$ , prove that  $\ell$  cannot intersect all three sides of the triangle.
4. Prove the angle-angle-side theorem.
5. Show that there is no angle-side-side theorem by finding two incongruent triangles which satisfy the angle-side-side hypotheses.
6. Draw a picture in the Poincaré upper half plane of a polygon with 4 sides and 3 right angles. Is it possible for the last angle to be a right angle?

#### Similarity

1. Consider a regular pentagon whose sides have length 1. Let  $x$  be the length of its diagonals. Use similar triangles to show that  $x/1 = 1/(x - 1)$  and then solve for  $x$ .
2. What is the area of a regular hexagon inscribed in a circle of radius 1? What is the area of a circle inscribed in this hexagon?
3. Use the previous problem to find an upper and lower bound for  $\pi$ .