

## Pries: 470 Euclidean and non-Euclidean Geometry

### Homework 13: Cross ratio

Due Friday May 5 (last one!)

#### Cross ratio:

1. Find the map  $f$  so that  $f(1) = 1$ ,  $f(-i) = 0$ , and  $f(-1) = \infty$ . Is this an isometry of the Poincare upper half plane?
2. There are 6 isometries  $f$  which permute the set  $\{0, 1, \infty\}$ . Use the cross-ratio to find the formula for  $f(z)$  in each of these cases.
3.
  - a. Find the isometry taking  $P = 3 + 4i$  to  $P' = 7i$  and taking  $\infty$  to  $\infty$ .
  - b. Show it has no fixed points in the Poincare upper half plane.
4. Let  $P$  and  $Q$  be two points in the Poincare upper half plane. In class, we proved a formula for the hyperbolic distance  $d$  between  $P$  and  $Q$ . Here's how it went.
  - a. Let  $M$  and  $N$  be the two endpoints of the hyperbolic line containing  $P$  and  $Q$  (so that  $P$  is between  $M$  and  $Q$ ). Here  $M$  and  $N$  are on the  $x$ -axis.
  - b. Let  $f$  be an isometry so that  $f(P) = i$ ,  $f(M) = 0$ , and  $f(N) = \infty$ . Then  $f(Q) = yi$  where  $\ln(y) = d$ .
  - c. We used the cross-ratio to find another formula for  $y$ : namely  $y = \text{CR}(Q, P, M, N)$ . So  $d = |\ln(\text{CR}(Q, P, M, N))|$ .
  - d. Your work starts here: Let's say that  $P = -4 + 6i$  and  $Q = 4 + 6i$ . In this case, what is the isometry  $f$  and what is the hyperbolic distance  $d$ ?