Pries: 470 Euclidean and non-Euclidean Geometry

Homework 11: Spherical transformations and hyperbolic distance
Due Friday April 7

Spherical transformations:

1. Let v be a vector of length 1. Let $R_{v,\theta}$ be the rotation of the sphere around the axis v with angle θ. Let $[-1]$ be the antipodal map.

   a. What are the fixed points of $R_{v,\theta}$?

   b. There is one choice of angle θ so that $R_{v,\theta}[-1]$ has fixed points. What is θ and what are the fixed points?

2. Let $a^2 + b^2 = 1$.

   a. Find an orthogonal matrix N that takes $(0, 0, 1)$ to $v = (a, b, 0)$.

   b. Find a matix M so that NMN^{-1} is the rotation of the sphere with axis v and angle θ. Explain why this works.

3. a. Find the image of the great circle L under stereographic projection when L is the intersection of the sphere with the plane $AX + BY = 0$.

   b. Suppose $C \neq 0$ and L is a great circle with equation $AX + BY + CZ = 0$. What kind of shape is the image of L under stereographic projection?

Hyperbolic geometry:

1. Find the hyperbolic distance between $(-2, 2)$ and $(-2, 1/7)$.

2. Let $P = (4, 4)$ and $Q = (5, 3)$. Find the hyperbolic line between P and Q.

3. Let $P = (-1, y)$ and $Q = (1, y)$. Find and compare these three numbers.

   a. The Euclidean distance between P and Q.

   b. The hyperbolic arclength of the straight line segment between P and Q (this is NOT a hyperbolic line segment).

   c. The hyperbolic distance between P and Q (use a computer).