Pries: 470 Euclidean and non-Euclidean Geometry

Homework 10: Spherical geometry
Due Friday March 31

Spherical geometry:

1. Find the spherical Pythagorean theorem for a sphere of radius 2.
2. Find the spherical area of a circle of radius r on the sphere of radius 1. Hint: put the center of the circle at the north pole and use integration in spherical coordinates.
3. Spherical measurements: Let $P = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$, let $Q = (1/\sqrt{3}, -1/\sqrt{3}, -1/\sqrt{3})$, and let $R = (-1/\sqrt{3}, 1/\sqrt{3}, -1/\sqrt{3})$.
   a. Find the formula for s-line between P and Q, i.e. the plane containing P,Q and $\vec{0}$.
   b. Find the spherical angle $\angle PQR$.
   c. Find the spherical distance between P and Q.
   d. Find the spherical area of the triangle $\triangle PQR$.
   e. Find the point S on the sphere so that the 3-dimensional shape $PQRS$ in $\mathbb{R}^3$ is a regular tetrahedron.
4. Tilings in spherical geometry Let P, Q, R, and S be the same as in problem 3. Puffing out the tetrahedron $PQRS$ gives a regular tiling of the sphere. (Quickly) find:
   a. the number n of sides of each tile;
   b. the area A of each tile;
   c. the number k of tiles meeting at a vertex;
   d. the interior angle t of each tile;
   e. the number M of tiles;
   f. the number v of vertices;
   g. the number e of edges;
   h. the Euler characteristic $\chi = M - v + e$.
5. Let $\triangle ABC$ be a triangle on the unit sphere with a right angle at C, angle α at A. Let a be the spherical length of the side opposite A and let c be the spherical length opposite C. Show that $\sin(\alpha) = \sin(a)/\sin(c)$.
6. Suppose the sphere is covered with identical regular tiles so that each tile has n edges, k tiles meet at each vertex, and θ is the interior angle at each vertex. Find the spherical area of each tile in terms of n and k.