Pries: 470 Euclidean and non-Euclidean Geometry

Homework 1: Area in antiquity.
Due Friday 1/20

Area in antiquity:

1. In ancient Egypt, the formula $(a+c)(b+d)/4$ was used to approximate the area A of a quadrilateral whose successive sides have lengths a, b, c, d. Show that this approximation is accurate for rectangles and is always larger than the area for parallelograms, i.e. $A \leq (a+c)(b+d)/4$.

2. In an ancient Egyptian document (the Moscow papyrus), the area of a circle is approximated by the area of a square whose side has length 8/9 the diameter of the circle. Is this approximation too big or too small? Use this approximation to approximate π. Calculate the ratio of the approximation to the correct area of the circle.

3. The Pythagorean theorem states that a right triangle whose sides have lengths a and b and whose hypotenuse has length c satisfies the relationship $a^2 + b^2 = c^2$. Prove this theorem by drawing two squares whose sides have length $a + b$ and dividing each one into 4 such triangles and either 1 or 2 squares. Explain your reasoning.