Pries: M466-Groups, Rings, and Fields

Homework 7: Minimal polynomial and degree.
Due Wednesday 10/3

Read: Gallian 21, Reid 3.1, Wilkons 4.4-4.5
(Review of vector spaces is in Gallian 19.)

Problems:

1. Let $\alpha = \sqrt{1+\sqrt{13}}$. Find $m_{\alpha,\mathbb{Q}(\sqrt{13})}(x)$. Find $m_{\alpha,\mathbb{Q}}(x)$.

2. Let $L = \mathbb{Q}(\sqrt{11}, \sqrt{2})$. Let $\alpha = \sqrt{2} + \sqrt{11}$. Prove that $L = \mathbb{Q}(\alpha)$. Some steps are:

   (i) Show $\sqrt{11} \notin \mathbb{Q}(\sqrt{2})$ and $[L : \mathbb{Q}(\sqrt{2})] = 2$.

   (ii) Find $[L : \mathbb{Q}]$.

   (iii) Find the minimal polynomial $m_{\alpha,\mathbb{Q}}(x)$.

   (iv) Prove that $L = \mathbb{Q}(\alpha)$.

3. Let M, K, L be fields with $M \subset K \subset L$. If the degree $[L : M]$ is prime, prove that $M = K$ or $M = L$.

4. Let $L = \mathbb{Q}(\alpha)$ where the minimal polynomial $m_{\alpha,\mathbb{Q}}$ has degree 5. Prove that the minimal polynomial of α^2 over $\mathbb{Q}$ also has degree 5.

5. Let $f(x) = x^4 - 3$. Find the splitting field L of $f(x)$ and find a basis of L over $\mathbb{Q}$.