

Pries: M466-Groups, Rings, and Fields

Homework 4: Irreducible polynomials.

Due Wednesday 9/12 (or Friday if you need more time)

Read: Gallian 17.

Problems:

1.
 - a. Find a polynomial $f(x) \in \mathbb{Q}[x]$ which is reducible over \mathbb{Q} but has no roots.
 - b. Find a polynomial $f(x) \in (\mathbb{Z}/5)[x]$ which is reducible over $\mathbb{Z}/5$ but has no roots.
2.
 - a. What are the maximal ideals of $\mathbb{C}[x]$?
 - b. Find $f(x) \in \mathbb{R}[x]$ so that $(f(x))$ is a maximal ideal of $\mathbb{R}[x]$ but not of $\mathbb{C}[x]$.
 - c. Find $f(x) \in \mathbb{Q}[x]$ so that $(f(x))$ is a maximal ideal of $\mathbb{Q}[x]$ but not of $\mathbb{R}[x]$.
3. Let k be a field and $f(x) \in k[x]$. Suppose $0 \neq a \in k$. If $f(x + a)$ is irreducible over k , show that $f(x)$ is irreducible over k .
4. Construct a field with 25 elements.