Numerical Methods for Total Variation

To regularize the operator eqn $Kf = g$ with total variation regularization, we seek to minimize

$$T_{\alpha}(f) = \frac{1}{2} \|Kf-g\|^2 + \alpha \text{TV}(f)$$

Note $\text{TV}(f) = \int \sqrt{1+f^1} \, dx \, dy$, but $1 \cdot 1$ is not differentiable at the origin, so the approach of finding the min of $T_{\alpha}(f)$ by setting $\frac{dT_{\alpha}(f)}{df} = 0$ is not good. Instead, approximate $\text{TV}(f)$ by

$$J_\beta(f) = \int \left\{ \left( \frac{\partial f}{\partial x_1} \right)^2 + \cdots + \left( \frac{\partial f}{\partial x_n} \right)^2 + \beta^2 \right\}^{1/2} \, dx$$

and minimize

$$\frac{1}{2} \|Kf-g\|^2 + \alpha J_\beta(f)$$

Then one can apply a min. alg. such as Steepest Desc. or Conj. Gradient (after discretizing).

See the matlab routine bb-deblur.m

Experiment with different $\beta$ for fixed $S$,

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Plot an L-curve to optimize $S$. 