### Table 1
Electromagnetic wave spectrum

<table>
<thead>
<tr>
<th>Energy (eV)</th>
<th>Frequency (Hz)</th>
<th>Wavelength (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4 \times 10^{-11}$</td>
<td>$10^4$</td>
<td>$10^4$</td>
</tr>
<tr>
<td>$4 \times 10^{-10}$</td>
<td>$10^5$</td>
<td>$10^3$</td>
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<tr>
<td>$4 \times 10^{-9}$</td>
<td>$10^6$</td>
<td>$10^2$</td>
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<tr>
<td>$4 \times 10^{-8}$</td>
<td>$10^7$</td>
<td>$10^1$</td>
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<tr>
<td>$4 \times 10^{-7}$</td>
<td>$10^8$</td>
<td>$10^0$</td>
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<tr>
<td>$4 \times 10^{-6}$</td>
<td>$10^9$</td>
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<tr>
<td>$4 \times 10^{-5}$</td>
<td>$10^{10}$</td>
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<tr>
<td>$4 \times 10^{-4}$</td>
<td>$10^{11}$</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>$4 \times 10^{-3}$</td>
<td>$10^{12}$</td>
<td>$10^{-4}$</td>
</tr>
<tr>
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<td>$10^{13}$</td>
<td>$10^{-5}$</td>
</tr>
<tr>
<td>$4 \times 10^{-1}$</td>
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<td>$10^{-6}$</td>
</tr>
<tr>
<td>$4 \times 10^0$</td>
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<td>$10^{-7}$</td>
</tr>
<tr>
<td>$4 \times 10^1$</td>
<td>$10^{16}$</td>
<td>$10^{-8}$</td>
</tr>
<tr>
<td>$4 \times 10^2$</td>
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<td>$10^{-9}$</td>
</tr>
<tr>
<td>$4 \times 10^3$</td>
<td>$10^{18}$</td>
<td>$10^{-10}$</td>
</tr>
<tr>
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<td>$10^{-11}$</td>
</tr>
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<td>$10^{20}$</td>
<td>$10^{-12}$</td>
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<tr>
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<td>$10^{-13}$</td>
</tr>
<tr>
<td>$4 \times 10^7$</td>
<td>$10^{22}$</td>
<td>$10^{-14}$</td>
</tr>
</tbody>
</table>

- AM radio waves
- Short radio waves
- FM radio waves and TV
- Microwaves and radar
- Infrared light
- Visible light
- Ultraviolet light
- X-ray
- Gamma ray
- Cosmic ray
1. Nature of x-rays

X-rays are part of the EM spectrum.

(see Table 1.)

The mag. & electric fields are func. of time & space & can be repr. by:

\[ \phi(x,t) = \phi_0 \cos(\omega t - kx) \]

\( \phi = \) elec. field, \( x = \) dist. wave has tran.
\( t = \) time, \( \omega = \) ang. freq.
\( k = \) wave num. = \( \frac{2\pi}{\lambda} \), \( \lambda = \) wavelength

\[ \begin{align*}
\phi(x,t) & \uparrow \\
T & \longrightarrow \ t \\
fixed \ x
\end{align*} \]

\[ \begin{align*}
\phi(x,t) & \uparrow \\
\lambda & \longrightarrow \ x \\
fixed \ t
\end{align*} \]

\[ T_c = \frac{\lambda}{c}, \quad f = \frac{1}{T} \]

\( c = \) sp. of EM prop. = \( 3 \times 10^8 \text{ m/s} \) in a vacuum.

The eqn for \( \phi \) is a soln of the wave eqn:

\[ \frac{\partial^2 \phi}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} \]

where \( c = \sqrt{\mu \epsilon} \), \( \mu = \) mag. perm. = \( 1.257 \times 10^{-6} \) henry/m in free space.

\( \epsilon = \) perm. = \( 8.854 \times 10^{-12} \) Farad/m in free sp.
X-rays can also be treated as particles traveling at the speed of light and carrying an energy given by \( E = hf \)

\( h = \text{Planck's const.} = 4.13 \times 10^{-18} \text{ keV-sec} \)

The particles are called photons.

A photon with an energy level greater than a few eV can ionize atoms and molecules (i.e., knock electrons out of their orbitals) and is called ionizing radiation.

An X-ray photon with wavelength 1 nm has energy

\[
E = hf = \left(4.13 \times 10^{-15} \text{ eV-s}\right) \left(3 \times 10^8 \text{ m/s}\right) = 1.2 \times 10^2 \text{ eV}
\]

Through X-rays are ionizing radiation.

X-rays can interact with orbital electrons and with the nuclei of atoms. There are 5 ways that they can interact:

1. Coherent scattering

   ![Diagram of coherent scattering]

   X-ray photon is deflected, losing little energy. No ionization occurs.

2. Photoelectric effect

   ![Diagram of photoelectric effect]

   Photon collides with an inner electron, photon is absorbed, electron escapes. Atom is now a positive ion. Characteristic radiation is emitted carrying energy equal to diff. in energy.
3. Compton scattering

![Diagram of Compton scattering]

- X-ray
- Scat. x-ray photon
- Electron
- Atom is ionized

4. Pair production
- High-energy photon is absorbed by nucleus and converted to an electron and a positron

5. Photo disintegration
- One or more nuclear particles (protons or neutrons) are ejected from nucleus

The photoelectric effect is the most desirable for X-ray imaging, since photon is absorbed producing little scatter, rad., which is a health hazard and a form of image noise.

Compton scattering produces 2 major problems:
- Background noise + major health hazard

Intensity of beam - can be changed by varying the # of photons or the energy of the photon

Attenuation:
- Assume beam has intensity I at cross-sectional area A.
- Assume all atoms in material are the same and have cross-sectional area of σ, and there are n atoms per unit vol. in the material.

\[ n = \frac{\#}{\text{vol}} = \frac{\#}{m^3} \Rightarrow \text{[nA]} = \frac{\#}{m^3} \]
Then number of atoms encountered by X-ray beam is $nA$
and the area they occupy is $A\sigma$.

$$[n] = \frac{n}{m^2}$$
$$[nA] = a$$

So,

The probability that a photon interacts with an atom is

$$A\sigma dx = \frac{\text{occupied by atoms}}{\text{area of beam}} = n\sigma dx$$

So, the X-ray energy removed in thickness $dx$ is

$$dI = -n\sigma I \, dx$$

Int. = \left( \frac{\text{# atoms}}{\text{area} \cdot \text{length}} \right) \cdot \text{length}

or

$$\frac{dI}{dx} = -n\sigma I$$

Solve:

$$I = I_0 e^{-n\sigma x}$$

Let $\beta = n\sigma$

Show fig. 6

Photoel. eff. & Comp. sc. are the main causes of atten. in diagnostic X-rays.
Diagnostic X-rays
The first x-ray machines were just 2-D proj.s

(Draw pic's)

Basic components

\[ \text{x-ray tube} \]

\[ \text{filter} \]

\[ \text{diaphragm} \]

\[ \text{patient} \]

\[ \text{grid} \]

\[ \text{film} \text{ (records intensity)} \]

\[ \text{lower mass density} \Rightarrow \text{greater intensity} \]

\[ \text{dark image} \Rightarrow \text{low density} \]

Limitations:
1. Patients are 2-D
2. Conventional x-ray cannot differentiate different types of soft tissue
3. No quantitative info about tissue densities

Newer techniques: X-ray tomography
(CAT scans) also called CT
1st practical scanner: 1972
11/22

Nonhomogeneous media:

At any point \( \beta = \beta(x, y) \)

\[
\Delta x \rightarrow 
\begin{array}{c|c|c|c|c}
\beta_1 & \beta_2 & \beta_3 & \beta_4 & \beta_5 \\
\hline
I_0 e^{-\beta x} & I_0 e^{-\beta_2 x} & I_0 e^{-\beta_3 x} & I_0 e^{-\beta_4 x} & I_0 e^{-\beta_5 x}
\end{array}
\]

\[
I = I_0 e^{-\beta x}
\]

Now

\[
I = I_0 e^{-(\beta_1 + \beta_2 + \cdots + \beta_5) \Delta x}
\]

If we write this in terms of an integral:

\[
I = I_0 e^{\int \mathcal{A} \beta(x, y_1) \, dx}
\]

Since \( y_1 \) is arbitrary,

\[
I(y) = I_0 e^{\int \mathcal{A} \beta(x, y_1) \, dx}
\]

\[
\ln\left( \frac{I}{I_0} \right) = -\int_{y_1}^{y} \beta(x, y_1) \, dx
\]

Let \( f(y) = -\ln\left( \frac{I}{I_0} \right) \) (Note: \( I \leq I_0 \), so \( \ln(I/I_0) \) is always positive)

Then \( f \) is called the proj. func.

Another Q: Can we find \( \beta \)?
Ex:

Tumor
\[ \beta_1 = 3 \]
\[ \beta_2 = 3 \]

Draw proj. func. from 2 angles
to show this will allow us to localize the "tumor", knowing that the tumor & background are homogenous.

Forward Problem: draw projection func.
Inverse Prob. : find tumor given proj. func.

Note: Don't need \( \beta \) to get \( f \) in IP. \( I(F) \) is measured data.

Ex:

Give them the proj.
func.

Exercises:

\[ \text{inhomog. has attenuation coeff. of twice the background} \]

\[ f \text{ determines the locations of the inhomog.'s exactly. (Also their attenuation coeff.'s)} \]

Suppose \( \beta_1 = 1, \beta_2 = 2 \)

Note from the formula that

\[ f(y) = -\ln \left( \frac{I}{I_0} \right) = (\beta_1 + \beta_1 + \beta_2 + \beta_1) \Delta x \]
Exercises

1. Are microwaves ionizing radiation? Why or why not?
   Is ultraviolet light ionizing radiation? Why or why not?

2. Assume all inhomogeneities have an attenuation factor of twice the background.
   a) Give (draw) the projection functions for the following body:
b) Find the location of the inhomogeneities.
Is your answer unique?

C) Find the location of the inhomogeneities.
Is your answer unique?