ColoState Spring 2017 Math 340 Exam 1

Thu. 03/02/2017

Name: _____ CSUID: _____

Section: _____

Problem	Score
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Exam Policy

(i) **No** calculator, textbook, homework, or any other references should be used. Please write down all necessary steps, partial credit will be given if deserved.

(ii) You could use one letter-size 2-sided Cheat Sheet for this exam.

Good luck!

(10 points) *Problem 1.* True or False, circle your answer (2 points for each item, no partial credit).

(i) (T) (F) The constant function $x = -1$ is a solution to the ODE $x'(t) = (x-1)\ln(x+2)$.

(ii) (T) (F) The ODE $(x + 2y^2)dx + 2xydy = 0$ is exact.

(iii) (T) (F) For a given logistic population model $\dfrac{dP(t)}{dt} = \left(1 - \dfrac{P}{K}\right)P$, the rate at which the population is increasing is at its greatest when the population is at one-half of its carrying capacity K.

(iv) (T) (F) If $y(t)$ is a solution of the ODE $\cos(t)x'(t) - \sin(t)x(t) = 0$, then $\sqrt{2}\, y(t)$ is also a solution of the ODE.

(v) (T) (F) If $y_1(t), y_2(t)$ are both solutions of the ODE

$$\cos(t)x'(t) - \sin(t)x(t) = e^t,$$

then $y_1(t) - y_2(t)$ is a solution of the ODE

$$\cos(t)x'(t) - \sin(t)x(t) = 0.$$

(10 points) *Problem 2.* Consider an initial value problem (IVP)

$$\begin{cases} x'(t) = f(t,x) = \tan(x)/(1+t^2), \\ x(1) = \pi/4. \end{cases}$$

Determine whether the following statements are true or false. Circle your answer (2 points for each item, no partial credit).

(i) (T) (F) $f(t,x)$ is continuous on the entire tx-plane.

(ii) (T) (F) There exists a rectangle containing the point $(1, \pi/4)$ in which $f(t,x)$ is continuous.

(iii) (T) (F) There exists a rectangle containing the point $(1, \pi/4)$ in which $\frac{\partial f}{\partial x}(t,x)$ is positive.

(iv) (T) (F) The IVP has a unique solution.

(v) (T) (F) If we change the initial condition to $x(1) = \pi/2$, then the new IVP still has a unique solution.

(10 points) *Problem 3.* An object with mass $m = 1kg$ is released from rest at a height 10 meter above the ground. Assume the air resistance can be ignored.

(i) Establish an ODE for the height $y(t)$ of the object.

(ii) Write down initial conditions for the object's motion.

(iii) How long will it take for the object to reach the ground?

Note: The gravitational constant $g = 9.8m/s^2$.

(10 points) *Problem 4*. Find the solution of $x'(t) = \cos^2(x)/(1+t^2)$ that satisfies $x(0) = \frac{\pi}{4}$.

(15 points) *Problem 5.* Given a linear ODE: $\cos(t)x'(t) - \sin(t)x(t) = e^t$,

(i) Find a general solution.

(ii) Find the specific solution satisfying $x(0) = 2017$.

(15 points) *Problem 6.* It is known that the ODE

$$Pdx + Qdy = (xy - 2)dx + (x^2 - xy)dy = 0$$

is not exact but has an integrating factor that depends only on x.

(i) Find such an integrating factor.

(ii) Find a general solution of the ODE.

(15 points) *Problem 7.* Given an ODE $x'(t) = (x-1)\ln(x+2)$,

(i) Find all equilibrium solutions and determine the stability of each.

(ii) Sketch three representative solution curves on the tx-plane.
Requirement: They should not be straight lines.

(15 points) *Problem 8.* Given a linear system $\mathbf{Ax} = \mathbf{b}$ as follows

$$\begin{cases} x_1 + 2x_2 - x_3 + x_4 = 4 \\ 2x_1 + 5x_2 - x_3 = 5 \\ 3x_1 + 6x_2 - 3x_3 + 3x_4 = 12 \end{cases}$$

(i) Apply elementary row operations to simplify the augmented matrix to the **reduced row echelon form**.

(ii) Write the solutions of the linear system in a parametric form.