

COLOSTATE FALL 2016 MATH 340 EXAM 1

Thu. 10/06/2016

NAME: _____ CSUID: _____

SECTION: _____

Problem	Score
1	
2	
3	
4	
5	
6	
7	
Total	

Exam Policy

- (i) **No** calculator, textbook, homework, or any other references should be used. Please write down all necessary steps, partial credit will be given if deserved.
- (ii) You could use **one** letter-size 2-sided Cheat Sheet for this exam.

Good luck!

(10 points) *Problem 1.* True or False, circle your answer (2 points for each item, no partial credit).

- (i) (T) (F) The constant function $y(x) \equiv \pi/2$ is a solution to $y'(x) = 2x \cos^2(y)$.
- (ii) (T) (F) The initial value problem $tx'(t) = x + 3t^2, x(0) = 1$ does not have a solution.
- (iii) (T) (F) The ODE $(x + 3x^3 \sin(y))dx + x^4 \cos(y)dy = 0$ is exact.
- (iv) (T) (F) For a given logistic population model $\frac{dP(t)}{dt} = \left(1 - \frac{P}{K}\right)P$, the rate at which the population is increasing is at its greatest when the population is at one-third of its carrying capacity K .
- (v) (T) (F) Let $\mathbf{u} = (2, 2, 1), \mathbf{v} = (1, 1, 2), \mathbf{w} = (4, 4, 2)$ be three vectors. Then the subspace spanned by $\mathbf{u}, \mathbf{v}, \mathbf{w}$ has dimension 3.

(15 points) *Problem 2.* A ball is thrown upwards into the air at an initial height $y_0 = 20$ ft with an initial velocity $v_0 = 8$ ft/s. Ignore the air resistance.

- (i) Find the maximal height reached by the ball.
- (ii) At what time does this event (*reaching its maximal height*) occur?
- (iii) When does the ball hit the ground? Answer to this part should be simplified.

Hint: $g = 32$ ft/s².

(15 points) *Problem 3.* Consider the 1st order ODE:

$$x'(t) - \tan(t)x(t) = \sin(t).$$

- (i) Find its general solution.
- (ii) Find the particular solution satisfying $x\left(\frac{\pi}{4}\right) = \frac{3}{4}\sqrt{2}$.

(15 points) *Problem 4.* It is known that the ODE

$$Pdx + Qdy = (xy - 2)dx + (x^2 - xy)dy = 0$$

is not exact but has an integrating factor that depends only on x .

- (i) Find such an integrating factor.
- (ii) Find the general solution of the given ODE.

(15 points) *Problem 5.* There are two parts in this problem.

(i) Show that $x_0(t) \equiv 0$ and $x_1(t) = \frac{t^4}{16}$ are both solutions to the initial value problem $x'(t) = t\sqrt{x}, x(0) = 0$.

(ii) Explain why this fact does not contradict the Uniqueness Theorem.

(15 points) *Problem 6.* Consider an autonomous ODE $x'(t) = 1 - x^4$.

- (i) Find all equilibrium points.
- (ii) Classify each of the equilibrium points as asymptotically stable or asymptotically unstable. Show your work.
- (iii) Sketch three representative solution curves in the tx -plane.
(*Remark: They should not be straight lines.*)

(15 points) *Problem 7.* Given a linear system

$$\begin{cases} x_1 + 2x_2 + 2x_3 - 2x_4 = 3 \\ x_1 + 2x_2 - 2x_3 + 4x_4 = 2 \\ -x_1 - 3x_2 = 2 \end{cases}$$

- (i) Apply row operations to simplify the augmented matrix to the reduced row echelon form (RREF).
- (ii) Write the solutions of the linear system in the parametric form.