ColoState Fall 2013 Math 340 Final

Mon. 12/16/2013

Name: ______________________ CSUID: ______________________

Section: ________

Problem	Score
1	
2	
3	
4	
5	
6	
7	
Total	

Exam Policy

(i) **No** calculator, textbook, homework, notes, or any other references should be used. Please write down all necessary steps, partial credit will be given if deserved.

(ii) You could use two double-sided Cheat Sheets for this exam.

Good luck!

(20 points) *Problem 1.* True or False, circle your answer (2 points for each item, no partial credit).

(i) (T) (F) The function $x(t) = \tan(t)$ is a solution to the ODE $x'(t) = 1 + x(t)^2$.

(ii) (T) (F) The equation $(xy - 1)dx + (x^2y - xy^2)dy = 0$ is exact.

(iii) (T) (F) Suppose $\lambda_i (i = 1, 2)$ are distinct eigenvalues of a matrix $\mathbf{A}$ and $\mathbf{v}_i (i = 1, 2)$ are eigenvectors corresponding respectively to λ_i. Then $e^{\lambda_i t}\mathbf{v}_i (i = 1, 2)$ are two linearly independent solutions of the ODE system $\mathbf{x}'(t) = \mathbf{A}\mathbf{x}(t)$.

(iv) (T) (F) For the given matrix $\mathbf{A} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & -1 \\ 0 & 1 & -2 \end{bmatrix}$, the geometric multiplicity of the eigenvalue $\lambda = -2$ is three.

(v) (T) (F) For the given matrix $\mathbf{A} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & -1 \\ 0 & 1 & -2 \end{bmatrix}$, it has two different complex eigenvalues.

(vi) (T) (F) Consider the ODE system $\begin{cases} x'(t) = -2x - y \\ y'(t) = x - 2y \end{cases}$. The equilibrium point at the origin is a center.

(vii) (T) (F) The ODE $x''(t) + 10x'(t) + 16x(t) = 0$ describes an underdamped harmonic motion.

(viii) (T) (F) The curve $(x(t), y(t)) = (\cos(t), 2\sin(t))$ is a solution curve of the autonomous ODE system $\begin{cases} x'(t) = -2y + x(x^2 + 4y^2 - 4) \\ y'(t) = \frac{1}{2}x - y(x^2 + 4y^2 - 4) \end{cases}$

(ix) (T) (F) Since the Laplace transform of $f(t)$ is $\mathcal{F}(s)$, the Laplace transform of $f'(t)$ is $s\mathcal{F}(s)$.

(x) (T) (F) The inverse Laplace transform of $\dfrac{s}{s^2 + 1}$ is $\sin(t)$.

(10 points) *Problem 2.* Consider a mass-spring system:

- The mass weighs 2 kg;
- The mass stretches the spring by 0.06 m before the motion starts;
- A damping force numerically equal two times the instantaneous velocity acts on the system;
- The gravity acts on the system also;
- The mass is given an initial tap at the mass-spring equilibrium with an upward velocity 0.1 m/s.

Set up but do not solve an ODE and two initial conditions to describe the motion.

(15 points) *Problem 3.* It is known that the ODE $xydx + (2x^2 + 3y^2 - 20)dy = 0$ is not exact but has an integrating factor that depends only on y.

(i) Find the integrating factor.

(ii) Find the general solution to the ODE.

(10 points) *Problem 4.* True or False, circle your answer (2 points for each item, no partial credit). Given a function $f(t, x) = \sqrt[3]{t}\sqrt{x}$ and consider the ODE $x'(t) = f(t, x)$.

(i) (T) (F)
$f(t, x)$ and $\frac{\partial f}{\partial x}$ are both continuous on the whole (t, x) plane.

(ii) (T) (F)
$f(t, x)$ and $\frac{\partial f}{\partial x}$ are both continuous in the region $R = \{(t, x) : t > 0, x > 0\}$.

(iii) (T) (F)
The initial value problem $x' = f(t, x), x(0) = 1$ has a unique solution.

(iv) (T) (F)
The initial value problem $x' = f(t, x), x(1) = 0$ has a unique solution.

(v) (T) (F)
The initial value problem $x' = f(t, x), x(1) = 1$ has a unique solution.

(15 points) *Problem 5.* Given an ODE system $\mathbf{x}'(t) = A\mathbf{x}$, where $A = \begin{bmatrix} 2 & 1 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 2 \end{bmatrix}$, find the general solution.

(15 points) *Problem 6.* Given an ODE $x''(t) - 3x'(t) + 2x(t) = e^t$,

(i) Find a fundamental set of solutions of the corresponding homogeneous ODE.

(ii) Find one particular solution of the given ODE.

(iii) Find the general solution of the given ODE.

(15 points) *Problem 7.* Apply both Laplace and inverse Laplace transforms to solve the IVP: $y'' - 3y' - 4y = e^{-t}, y(0) = 0, y'(0) = -1$.