ColoState Fall 2013 Math 340 Exam 1

Thu. 10/10/2013

Name: ____________________ CSUID: ____________________

Section: ________

Problem	Score
1	
2	
3	
4	
5	
6	
7	
Total	

Exam Policy

(i) **No** calculator, textbook, homework, or any other references should be used. Please write down all necessary steps, partial credit will be given if deserved.

(ii) You could use one 2-sided Cheat Sheet for this exam.

Good luck!

(10 points) *Problem 1.* True or False, circle your answer (2 points for each item, no partial credit).

(i) (T) (F) The constant functions $x(t) = k\pi$ (k any integer) are all solutions of the ODE $x'(t) = \sin(x)\ln(t)$.

(ii) (T) (F) The function $x(t) = t^2 + \dfrac{1}{t^2}$ (for $t > 0$) is a solution to the initial value problem $tx' + 2x = 4t^2, x(1) = 1$.

(iii) (T) (F) The ODE $\sin(y)dx + (1 - x\sin(y))dy = 0$ is exact.

(iv) (T) (F) The two vectors $\mathbf{u} = (1, 2, -5), \mathbf{v} = (-1, -2, 3)$ are linearly independent.

(v) (T) (F) If $\mathbf{x}_1, \mathbf{x}_2$ are both solutions of a nonhomogeneous linear system $\mathbf{Ax} = \mathbf{b}$, then $\mathbf{x}_1 - \mathbf{x}_2$ is a solution of the homogeneous linear system $\mathbf{Ax} = \mathbf{0}$.

(15 points) *Problem 2.* Find the solution of $y'(x) = 2\cos(x)\sqrt{y}$ that satisfies the initial condition $y(\pi/4) = 2$.

(15 points) *Problem 3.* Given a linear ODE $y'(x) = y\tan(x) + \sin(x)$,

(i) Find the general solution.

(ii) Find the particular solution satisfying $y(0) = 2$.

(15 points) *Problem 4.* It is known that the ODE $2y dx + (x + \sqrt{y}) dy = 0$ is not exact but has an integrating factor in the form $\mu(y)$.

(i) Find the integrating factor $\mu(y)$.

(ii) Use the integrating factor to find a general solution of the ODE.

(15 points) *Problem 5.*

(i) Show that the ODE $x'(t) = e^{-t^2}\sqrt[3]{x}$ has a unique solution satisfying the initial condition $x(1) = 1$.

(ii) Consider another initial value problem (IVP): $x'(t) = e^{-t^2}\sqrt[3]{x},\ x(1) = 0$. Does the Uniqueness Theorem guarantee a unique solution for this IVP? Justify your answer.

(15 points) *Problem 6.* Consider a given autonomous ODE $x'(t) = (x+1)(x-4)$.

(i) Find all equilibrium points.

(ii) Classify each equilibrium point as either unstable or asymptotically stable.

(iii) Sketch the equilibrium solutions in the tx-plane. These equilibrium solutions divide the tx-plane into regions. Sketch at least one solution trajectory in each of these regions.

(15 points) *Problem 7.* Consider a given matrix $\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ -1 & -3 & -1 \end{bmatrix}$.

(i) Apply elementary row operations to simplify the matrix to the **row echelon form**.

(ii) Write the solutions of the linear system $\mathbf{A}\mathbf{x} = \mathbf{0}$ in the parametric form.

(iii) Find a basis for $\mathrm{null}(\mathbf{A})$.