Name

TOTAL [ ]

#	Max	Yours
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	
9	20	
10	20	

Math 369 **Fall 2006**
Final Exam

1. (a) Find the values of α for which the following system has no solution

$$\begin{cases} x_1 - x_2 + \alpha x_3 = -2 \\ -x_1 + 2x_2 - \alpha x_3 = 3 \\ \alpha x_1 + x_2 + x_3 = 2 \end{cases}$$

   (b) For which values of t will the following system have a solution? For those values of t in which there is a solution, what is the general solution set?

$$\begin{cases} x_1 + 4x_2 = -2 \\ x_1 + 2x_2 = t \\ -x_1 - 2x_2 = t \end{cases}$$

2. (a) Let $S_1 = \{v \in \Re^2 : v = (x, 1)^T, x \in \Re\}$
   a1) is S_1 closed under addition?
   a2) is S_1 closed under scalar multiplication?
   a3) Is S_1 a subspace of $\Re^2$?

   (b) Let $S_2 = \{v \in \Re^3 : v = (x_1, x_2, x_3)^T, x_1 = x_2\}$
   b1) is S_2 closed under addition?
   b2) is S_2 closed under scalar multiplication?
   b3) Is S_2 a subspace of $\Re^3$?

3. Let $V = P_3$ be the vector space of polynomials of degree 2 with real coefficients. Are the vectors

$$p_1(x) = x^2 - 2x + 3, p_2(x) = 2x^2 + x + 8, p_3(x) = x^2 + 8x + 7$$

linearly independent or dependent? If they are independent say why. If they are linearly dependent, exhibit a linear dependence relation among them.

4. (a) Let $T_1 : \Re^2 \longrightarrow \Re^3$, given by $T_1(x_1, x_2) = (x_1, x_2, x_1^2)$ Is T a linear transformation? If yes, then prove it and if it isn't then state a condition that it fails to satisfy.

   (b) Same question for $T_2 : \Re^3 \longrightarrow \Re^2$, given by $T_2(x_1, x_2, x_3) = (x_3, x_2 + x_1)$.

5. Let

$$\mathbf{b_1} = (1, 1, 0)^T, \mathbf{b_2} = (1, 0, 1)^T, \mathbf{b_3} = (0, 1, 1)^T$$

and let L be the linear transformation from $\Re^2$ to $\Re^3$ defined by

$$L(x_1, x_2) = x_1\mathbf{b_1} + x_2\mathbf{b_2} + (x_1 + x_2)\mathbf{b_3}.$$

a) show that $\{\mathbf{b_1}, \mathbf{b_2}, \mathbf{b_3}\}$ form a basis for $\Re^3$

b) find the matrix A representing L with respect to the bases $\{\mathbf{e_1}, \mathbf{e_2}\}$ and $\{\mathbf{b_1}, \mathbf{b_2}, \mathbf{b_3}\}$, where $\{\mathbf{e_1}, \mathbf{e_2}\}$ is the standard basis in $\Re^2$.

c) (Xcredit 10 points) Find the kernel and the range of L

6. Find an orthonormal basis for this subspace of $\Re^4$

$$\{\begin{pmatrix} x \\ y \\ z \\ y \end{pmatrix} \in \Re^4 \mid \ x - y - z + w = 0,\ x + z = 0\}$$

Verify your result.

7. Compute the singular value decomposition for $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

8. (a) Find all possible values of the scalar α that make the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 2 & -1 & \alpha \end{bmatrix}$ defective or show that no such values exist.

   (b) Let A and B be $n \times n$ matrices; suppose they are both diagonalizable, with the same diagonalizing matrix X. Prove that $AB = BA$

9. Is the matrix $A = \begin{bmatrix} 1-i & 2 \\ 2 & 3 \end{bmatrix}$ Hermitian? Is it normal?

10. Let $\mathbf{z} = (1 + i, 2i, 3 - i)^T$ and $\mathbf{w} = (2 - 4i, 5, 2i)^T$. Compute $||z||, ||w||$, $< z, w >$ and $< w, z >$