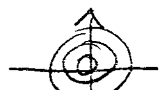
6. $\begin{vmatrix}1-\lambda & 5\\ -1 & -1-\lambda\end{vmatrix} = \lambda^2+4 \Rightarrow \lambda_{1,2}=\pm 2i \Rightarrow \underline{\alpha=0,\ \beta=2}$ (5p)

eigenvectors: $\vec{V}_{1,2} = \begin{pmatrix}-1\\1\end{pmatrix} \pm i\begin{pmatrix}0\\2\end{pmatrix} \Rightarrow \vec{u} = \begin{pmatrix}-1\\1\end{pmatrix},\ \vec{w} = \begin{pmatrix}0\\2\end{pmatrix}$ (10p)

$\Rightarrow \vec{x} = \left[c_1\begin{pmatrix}-1\\1\end{pmatrix}\cos 2t - \begin{pmatrix}-2\\0\end{pmatrix}\sin 2t\right] + c_2\left[\begin{pmatrix}-1\\1\end{pmatrix}\sin 2t + \begin{pmatrix}-2\\0\end{pmatrix}\cos 2t\right]$ (5p)

Center: 

7. $k=\frac{F}{y}=\frac{mg}{y}$ (Hooke's law) $\Rightarrow k=\frac{(0.05kg)(9.8\,m/s^2)}{(0.2m)} = 2.45\frac{N}{M}$

$\Rightarrow my''+\gamma y'+ky=0$ becomes $0.05y''+\gamma y'+2.45y=0$, or

$\boxed{y''+20\ y'+49y=0.}$

The characteristic equation $\lambda^2+20\ \lambda+49=0$ has the zeros $\lambda_{1,2}=\frac{-20\pm\sqrt{400\mu^2-196}}{2}$.

The system is critically damped if it has one single repeated root, i.e. if

$400\ ^2-196=0 \Rightarrow \gamma^2=\frac{196}{400} \Rightarrow \gamma=\frac{14}{20}$

$\Rightarrow \gamma=7/10$ and the equation becomes

$y''+14y'+49=0$, with $\lambda_1=\lambda_2=-7$

$\Rightarrow$ the general solution is $\boxed{y(t)=(C_1+C_2t)e^{-7t}}$ (15p)

If the mass is displaced downward, $y(0)=-0.15\,m \Rightarrow$

$\Rightarrow C_1=-0.15$. Differentiate: $y'(t)=C_2e^{-7t}-7(C_1+C_2t)e^{-7t}$

If the mass is released from the rest, then $\underline{y'(0)=0}$

$\Rightarrow 0=C_2-7C_1 \Rightarrow C_1=-1.05$ and

$y(t)=-(0.15+1.05t)e^{-7t}$