Name: **Spring 2006**
Math 340 Section 001 **Practice Exam 2**

1. Consider the differential equation
$$x^2y'' - x(x+2)y' + (x+2)y = 2x^3$$
   (a) show that $\{x, xe^x\}$ is a fundamental set of solutions for the homogeneous equation
   (b) Find the particular solution for the nonhomogeneous equation using the variation of parameters.

2. For the following equations write down the form of the particular solutions, using the method of the undetermined coefficients. Do not solve for the unknown coefficients:
   (a) $y'' - y = e^{2x} + xe^x$
   (b) $y'' + y = x^2 - \cos(x)$

3. (a) Let $Y(s) = \mathcal{L}(y(t))$ represent the Laplace transform of $y(t)$, where $y(t)$ is the solution of the initial value problem for the differential equation given below. Solve for $Y(s)$ and then find $y(t)$.
$$y'' - 4y' + 3y = t, \qquad y(0) = 0, y'(0) = 2$$
   (b) Solve the above initial value problem using the method the undetermined coefficients.

4. Find the inverse Laplace transforms of the following functions:
   (a) $Y(s) = \dfrac{1}{(s^2+4)(s+1)}$
   (b) $Y(s) = \dfrac{e^{-s+5}}{s^2+6s+10}$

5. Rewrite the following functions in terms of the functions $H(t-c)$. The final answer should be such that the Laplace transform can immediately be take.
   a. $f(t) = \begin{cases} 4, & \text{if } 0 \le t < 2 \\ -1 & \text{if } 2 \le t < 4 \\ 2, & \text{if } 4 \le t \end{cases}$
   b. $f(t) = \begin{cases} t+1, & \text{if } 0 \le t < 1 \\ t & \text{if } 1 \le t \end{cases}$

6. Find the null space of the matrix $A_{3\times 4}$ given below.
$$A = \begin{bmatrix} 2 & 5 & 8 & -14 \\ 6 & 21 & 38 & 5 \\ 0 & 6 & 12 & 0 \end{bmatrix}$$

7. a. Problem 5, Section **7.5**
   b. Problem 21, Section **7.5**