Name:

M369 Linear Algebra (section 2) : Quiz 2

1. (5 points)
Let A be an $m \times n$ matrix (i.e., m rows and n columns). Recall that the rank of a matrix is the dimension of its row space.

T/F The rank of A can never be larger than n.

T/F The rank of A can never be larger than m.

T/F If $m \neq n$ then the dimension of the row space of A is *not* equal to the dimension of the column space of A.

T/F The rank of A plus the nullity of A is equal to n.

T/F The matrix A and its transpose A^T have the same rank.

2. XC 5pts

Let $B = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{pmatrix}$

Determine a basis for and the dimension of, the following subspaces:

a) Column Space (B)

b) Row Space (B)

c) Null space (B)

RREF : $\begin{pmatrix} \boxed{1} & 0 & -1 & -2 \\ 0 & \boxed{1} & 2 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ $\Rightarrow$ a) $\left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}; \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \right\}$, dim = 2

b) $\{(1,2,3,4),(0,1,2,3)\}$,
dim = 2

RREF(A) $\Rightarrow$

c) $A\vec{x} = \vec{0} \Rightarrow$

$\Rightarrow \begin{cases} x_3 = s, \ x_4 = t \\ x_1 = s + 2t \\ x_2 = 2s + 3t \end{cases}$

$\Rightarrow \left\{ \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ 0 \\ 1 \end{pmatrix} \right\}$, dim = 2.