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M 369 Linear Algebra Quiz 1,
Spring 2007 - Instructor: Dr. Iuliana Oprea

Problem 1 (10 pts)

Consider the vectors $\vec{v}_1 = (1, 2, -1)^T$, $\vec{v}_2 = (2, 1, 0)^T$, $\vec{v}_3 = (-1, 0, 1)^T$.

a) are the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ linearly independent or dependent? If they are independent say why; if they are dependent find the linear dependence relation.

Use the definition:

Solve $c_1 \vec{v}_1 + c_2 \vec{v}_2 + c_3 \vec{v}_3 = \vec{0}$ for c_1, c_2, c_3 .

$$c_1 \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + c_2 \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + c_3 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow$$

$$\begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 2 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \end{array} \right) \Rightarrow$$

$$\Rightarrow c_1 = c_2 = c_3 = 0 \Rightarrow \text{Li}$$

b) Write the vector $\vec{b} = (1, -7, 5)^T$ as a linear combination of the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$.

$$\vec{b} = c_1 \vec{v}_1 + c_2 \vec{v}_2 + c_3 \vec{v}_3$$

$$\begin{pmatrix} 1 \\ -7 \\ 5 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + c_2 \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + c_3 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -7 \\ 5 \end{pmatrix} \Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 2 & 1 & 0 & -7 \\ -1 & 0 & 1 & 5 \end{array} \right) \Rightarrow$$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{array} \right) \Rightarrow \begin{matrix} c_1 = -5 \\ c_2 = 3 \\ c_3 = 0 \end{matrix} \Rightarrow \boxed{\vec{b} = -5\vec{v}_1 + 3\vec{v}_2}$$