NAME: ______________________________

M 369 Linear Algebra EXAM 2, 11/18/06
Fall 2006 - Instructor: Dr. Iuliana Oprea

Clearly answer each of the questions below. Show your work. **Box your answers.** Grading will be also based on orderly and transparent presentation.

Problem 1. (15 pts)
Let **x, y**$\in \mathbf{R^3}$, **x**$= (2,4,3)^T$, **y**$= (1,1,1)^T$. Find the vector projection **p** of **x** onto **y** and verify that **p** and **(x - p)** are orthogonal.

Problem 2. (10 pts)
The following is an inner product on $\mathbf{R^2}$

$$< x, y >= x^T Jy, \text{ where } J = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

Find the angle between $\mathbf{e_1} = (1,0)^T$ and $\mathbf{e_2} = (0,1)^T$ with respect to this inner product.

Problem 3. (25 pts)
a) Find the least squares solution $\hat{\mathbf{x}}$ of

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 1 \end{bmatrix} x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

b) Calculate the residual $\mathbf{r}(\hat{\mathbf{x}})$.
c) (**X-credit 5 pts.**) Verify that $\mathbf{r}(\hat{\mathbf{x}}) \in N(A^T)$.

Problem 4. (20 pts.)
Find an orthonormal basis for

$$Span\{(1,0,1)^T, (2,1,1)^T\}.$$

Problem 5. (20 pts.)
Find the similarity transformation X that diagonalizes

$$A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}.$$

Check your result by direct calculation.

Problem 6. (10 pts)
Consider the vector space $\mathcal{C}[-1,1]$ with the inner product $< f, g >= 1/2 \int_{-1}^{1} f(x)g(x)dx$.
a) Do the functions 1 and x form an orthonormal set of vectors with respect to the inner product?
b) Find the vector projection of 1 onto x.

Problem 7. (X-credit, 5 pts.)
Let A be a $m \times n$ matrix with real entries. Show that if $x \in R(A)$ and $y \in N(A^T)$ then x and y are orthogonal to each other.

Problem 8. (X-credit, 5 pts.)
For which values of α is the following matrix diagonalizable?

$$A = \begin{bmatrix} 1 & \alpha \\ \alpha & 1 \end{bmatrix}.$$
