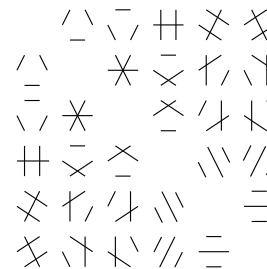


# Mathematics Seminar



## Rocky Mountain Algebraic Combinatorics Seminar

---

### Weisfeiler–Leman for Group Isomorphism: Action Compatibility

Michael Levet  
UC Boulder

The Weisfeiler–Leman (WL) algorithm is a key combinatorial subroutine in Graph Isomorphism, that (for fixed  $k \geq 2$ ) computes an isomorphism invariant coloring of the  $k$ -tuples of vertices. Brachtler & Schweitzer (LICS 2020) recently adapted WL to the setting of groups. Using a classical Ehrenfeucht-Fraïssé pebble game, we will show that Weisfeiler–Leman serves as a polynomial-time isomorphism test for several families of groups previously shown to be in  $\mathbf{P}$  by multiple methods. These families of groups include:

- Coprime extensions  $H \rtimes N$ , where  $H$  is  $O(1)$ -generated and the normal Hall subgroup  $N$  is Abelian (Qiao, Sarma, & Tang, STACS 2011).
- Groups without Abelian normal subgroups (Babai, Codenotti, & Qiao, ICALP 2012).

In both of these cases, the previous strategy involved identifying key group-theoretic structure that could then be leveraged algorithmically, resulting in different algorithms for each family. A common theme among these is that the group-theoretic structure is mostly about the action of one group on another. Our main contribution is to show that a single, combinatorial algorithm (Weisfeiler-Leman) can identify those same group-theoretic structures in polynomial time.

We also show that Weisfeiler–Leman requires only a constant number of rounds to identify groups from each of these families. Combining this result with the parallel WL implementation due to Grohe & Verbitsky (ICALP 2006), this improves the upper bound for isomorphism testing in each of these families from  $\mathbf{P}$  to  $\mathbf{TC}^0$ .

This is joint work with Joshua A. Grochow.

Weber 223  
4.15pm, Friday, Apr 8, 2022  
(Refreshments 3:30–4 pm)  
Colorado State University

---

This is a joint Denver U / UC Boulder / UC Denver / U of Wyoming / CSU seminar that meets biweekly.  
Anyone interested is welcome to join us at a local restaurant for dinner after the talks.



Department of Mathematics  
Fort Collins, Colorado 80523