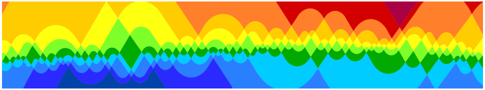


Recent Results on Venn Diagrams

Frank Ruskey[1]

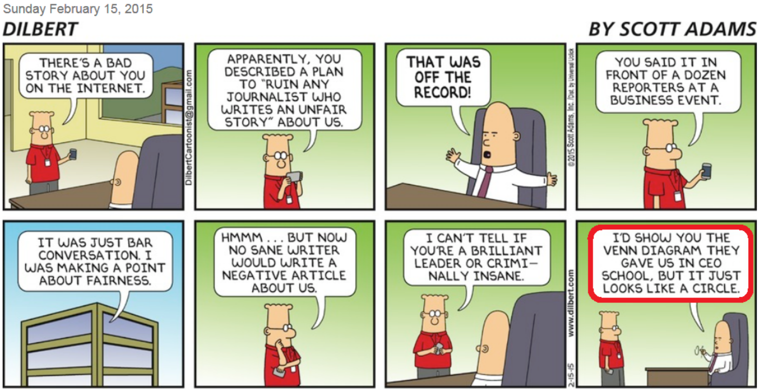
[1]Department of Computer Science, University of Victoria, CANADA.

CoCoa 2015, Fort Collins, Colorado

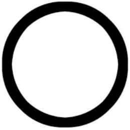
The Plan

1. Basic definitions.
2. Winkler's conjecture and recent connectivity result.
3. Symmetric Venn diagrams, the GKS result
4. Simple symmetric Venn diagrams, computer searches
5. Venn diagrams made from polyominoes (time permitting)

Venn diagram examples; famous and otherwise ($n = 1$).



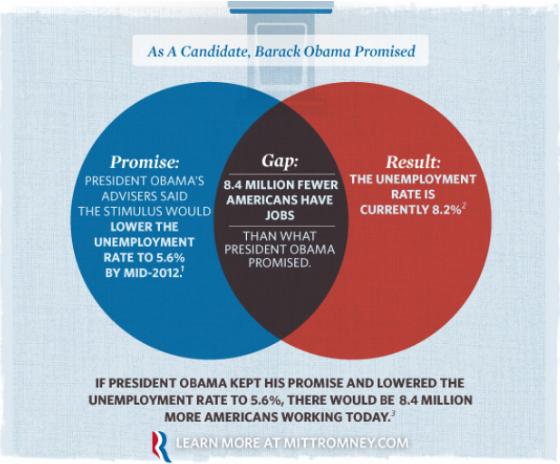
$n =$ number of curves $= 1$



Venn diagram examples; famous and otherwise ($n = 2$).

Mitt Romney doesn't understand Venn diagrams

The Romney campaign have been making "venn diagrams". Oh dear.

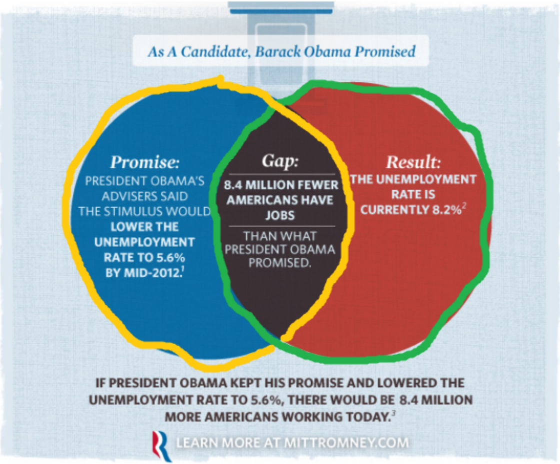


From the "NewStatesman.com" July 2012.

Venn diagram examples; famous and otherwise ($n = 2$).

Mitt Romney doesn't understand Venn diagrams

The Romney campaign have been making "venn diagrams". Oh dear.



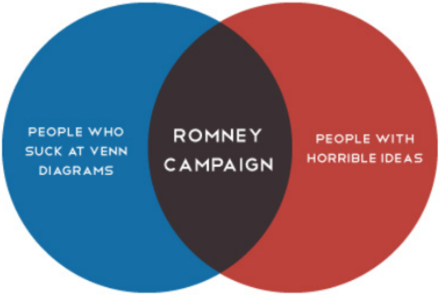
From the "NewStatesman.com" July 2012.

Venn diagram examples; famous and otherwise ($n = 2$).

Mitt Romney doesn't understand Venn diagrams

The Romney campaign have been making "venn diagrams". Oh dear.

A Venn Diagram, you see, is designed to show all possible logical relationships between a finite collection of sets. Put more simply, you label the left circle with one factor, the right circle with another, and the center with something that has properties of both. For example, this is a Venn Diragram:



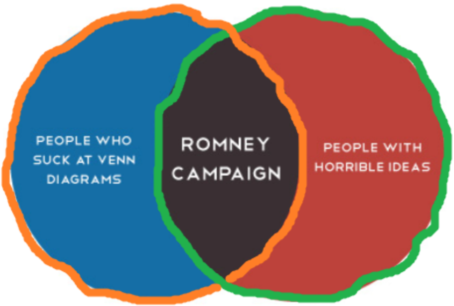
From the "Upworthy.com".

Venn diagram examples; famous and otherwise ($n = 2$).

Mitt Romney doesn't understand Venn diagrams

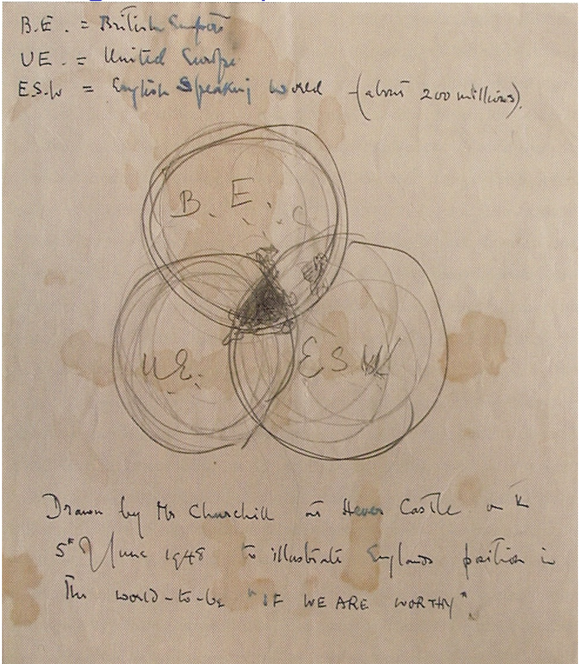
The Romney campaign have been making "venn diagrams". Oh dear.

A Venn Diagram, you see, is designed to show all possible logical relationships between a finite collection of sets. Put more simply, you label the left circle with one factor, the right circle with another, and the center with something that has properties of both. For example, this is a Venn Diragram:

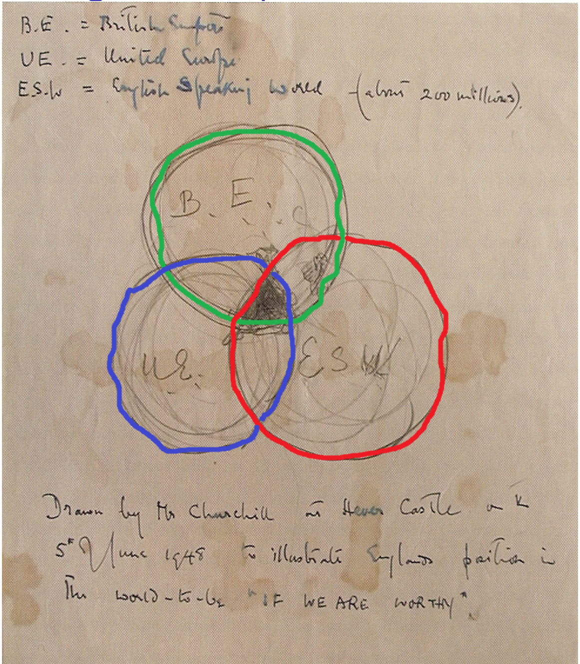


From the “Upworthy.com”.

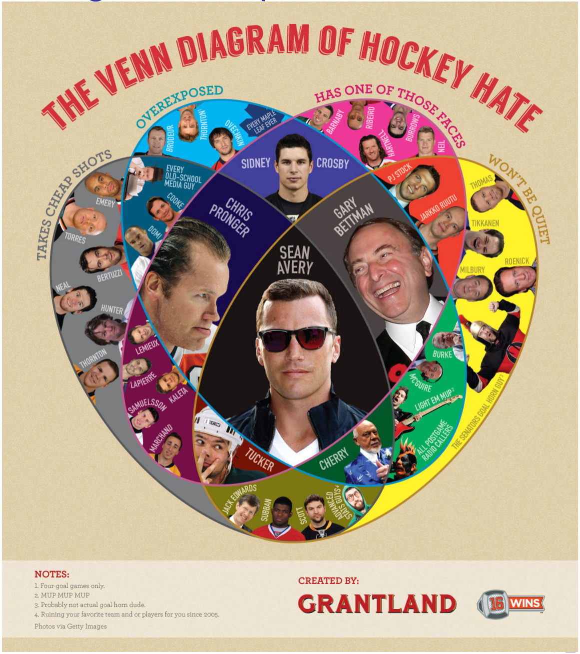
Venn diagram examples; famous and otherwise ($n = 3, 4$).



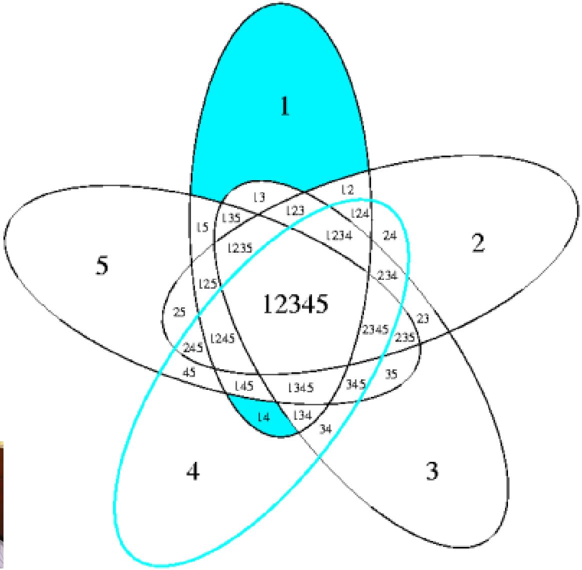
Venn diagram examples; famous and otherwise ($n = 3, 4$).



Venn diagram examples; famous and otherwise ($n = 3, 4$).



An irreducible Venn diagram ($n = 5$)



What is a Venn diagram?

- Made from simple closed curves $C_1, C_2, \ldots, C_n$.
- Only finitely many intersections.
- Each such intersection is transverse (no "kissing").
- Let X_i denote the interior or the exterior of the curve C_i and consider the 2^n intersections $X_1 \cap X_2 \cap \cdots \cap X_n$.
- *Euler diagram* if each such intersection is connected.
- *Venn diagram* if Euler and no intersection is empty.
- *Independent family* if no intersection is empty.

What is a Venn diagram?

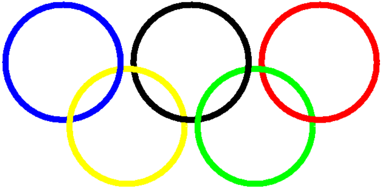
- Made from simple closed curves $C_1, C_2, \ldots, C_n$.
- Only finitely many intersections.
- Each such intersection is transverse (no "kissing").
- Let X_i denote the interior or the exterior of the curve C_i and consider the 2^n intersections $X_1 \cap X_2 \cap \cdots \cap X_n$.
- *Euler diagram* if each such intersection is connected.
- *Venn diagram* if Euler and no intersection is empty.
- *Independent family* if no intersection is empty.

What is a Venn diagram?

- Made from simple closed curves $C_1, C_2, \ldots, C_n$.
- Only finitely many intersections.
- Each such intersection is transverse (no "kissing").
- Let X_i denote the interior or the exterior of the curve C_i and consider the 2^n intersections $X_1 \cap X_2 \cap \cdots \cap X_n$.
- *Euler diagram* if each such intersection is connected.
- *Venn diagram* if Euler and no intersection is empty.
- *Independent family* if no intersection is empty.

What is a Venn diagram?

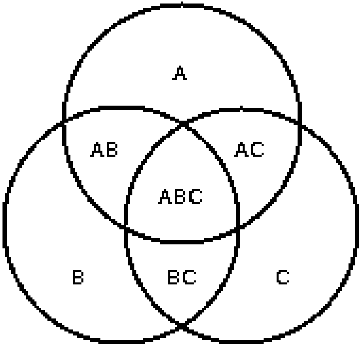
- Made from simple closed curves $C_1, C_2, \ldots, C_n$.
- Only finitely many intersections.
- Each such intersection is transverse (no "kissing").
- Let X_i denote the interior or the exterior of the curve C_i and consider the 2^n intersections $X_1 \cap X_2 \cap \cdots \cap X_n$.
- *Euler diagram* if each such intersection is connected.
- *Venn diagram* if Euler and no intersection is empty.
- *Independent family* if no intersection is empty.



Euler but not Venn

What is a Venn diagram?

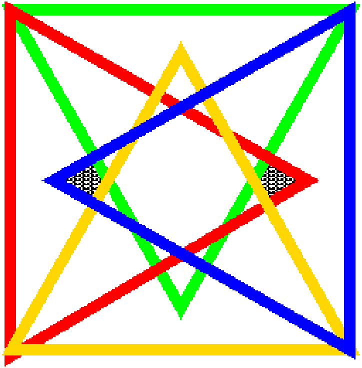
- Made from simple closed curves $C_1, C_2, \ldots, C_n$.
- Only finitely many intersections.
- Each such intersection is transverse (no "kissing").
- Let X_i denote the interior or the exterior of the curve C_i and consider the 2^n intersections $X_1 \cap X_2 \cap \cdots \cap X_n$.
- *Euler diagram* if each such intersection is connected.
- *Venn diagram* if Euler and no intersection is empty.
- *Independent family* if no intersection is empty.



Venn (and Euler)

What is a Venn diagram?

- Made from simple closed curves $C_1, C_2, \ldots, C_n$.
- Only finitely many intersections.
- Each such intersection is transverse (no "kissing").
- Let X_i denote the interior or the exterior of the curve C_i and consider the 2^n intersections $X_1 \cap X_2 \cap \cdots \cap X_n$.
- *Euler diagram* if each such intersection is connected.
- *Venn diagram* if Euler and no intersection is empty.
- *Independent family* if no intersection is empty.



Neither Venn nor Euler

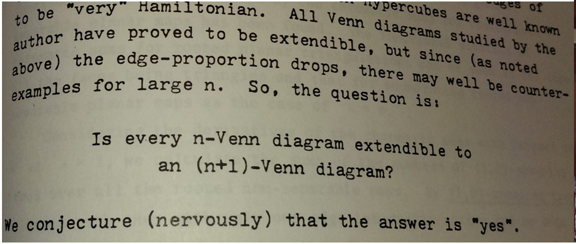
Winkler's conjecture

- An n-Venn diagram is *reducible* if there is some curve whose removal leaves an $(n-1)$-Venn diagram.
- An n-Venn diagram is *extendible* if the addition of some curve results in an $(n+1)$-Venn diagram.
- Not every Venn diagram is reducible. Every reducible diagram is extendible.
- **Conjecture:** Every *simple* n-Venn diagram is extendible to a *simple* $(n+1)$-Venn diagram.
- Reference: Peter Winkler, Venn diagrams: Some observations and an open problem, Congressus Numerantium, 45 (1984) 267–274.
- The conjecture is true if the simplicity condition is removed (Chilakamarri, Hamburger, and Pippert (1996)).
- The conjecture is true if $n \leq 5$. Determined by Bultena; there are 20 non-isomorphic (spherical) diagrams to check.

Winkler's conjecture

- An n-Venn diagram is *reducible* if there is some curve whose removal leaves an $(n-1)$-Venn diagram.
- An n-Venn diagram is *extendible* if the addition of some curve results in an $(n+1)$-Venn diagram.
- Not every Venn diagram is reducible. Every reducible diagram is extendible.
- **Conjecture:** Every *simple* n-Venn diagram is extendible to a *simple* $(n+1)$-Venn diagram.
- Reference: Peter Winkler, Venn diagrams: Some observations and an open problem, Congressus Numerantium, 45 (1984) 267–274.
- The conjecture is true if the simplicity condition is removed (Chilakamarri, Hamburger, and Pippert (1996)).
- The conjecture is true if $n \leq 5$. Determined by Bultena; there are 20 non-isomorphic (spherical) diagrams to check.

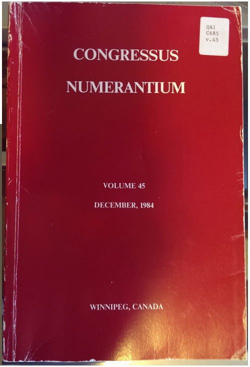
Winkler's conjecture



to be "very" Hamiltonian. All Venn diagrams studied by the author have proved to be extendible, but since (as noted above) the edge-proportion drops, there may well be counter-examples for large n. So, the question is:

Is every n-Venn diagram extendible to an (n+1)-Venn diagram?

We conjecture (nervously) that the answer is "yes".



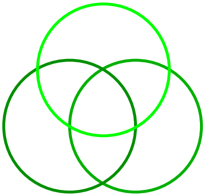
Puzzled

Where Sets Meet (Venn Diagrams)

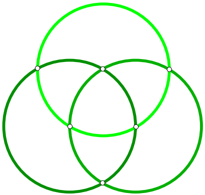
Welcome to three new puzzles. Solutions to the first two will be published next month; the third is as yet unsolved.

3. Prove *or disprove* that to any Venn diagram of order *n* another curve can be added, making it a Venn diagram of order *n*+1; remember, only simple crossings allowed.

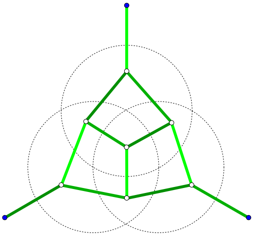
Venn diagrams and their duals



Venn diagram



Venn graph



Venn dual

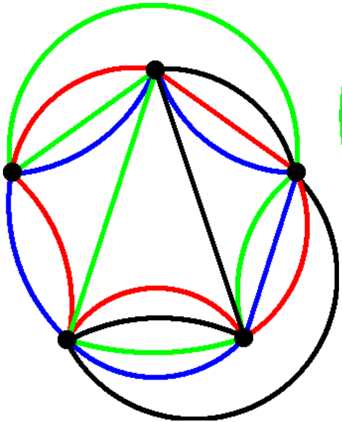
Basic facts

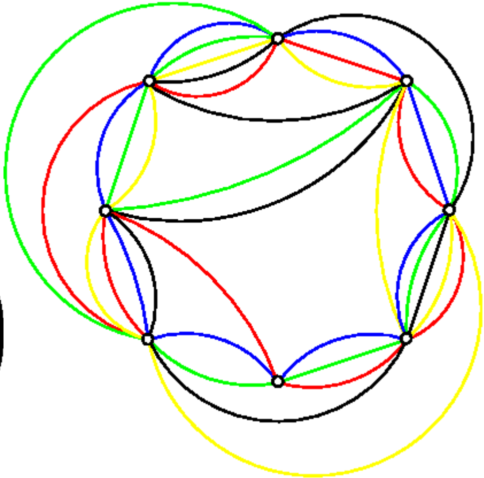
- If v is the number of vertices (intersection points) then

$$\left\lceil \frac{2^n - 2}{n-1} \right\rceil \leq v \leq 2^n - 2$$

  Open: Venn diagrams meeting the lower bound for $n > 8$.
- The dual is a spanning planar subgraph of the hypercube. If the Venn diagram is simple, then the dual is maximal (every face is a quadrilateral).
- There is a natural *directed* dual graph.
- A Venn diagram is drawable with all curves convex if and only if the directed dual has only one source and one sink (Bultena, Grünbaum, R., 1999).
- If a Venn diagram is convexly drawable, then $v \geq \binom{n}{n/2}$.
- Venn diagrams exist for all n.

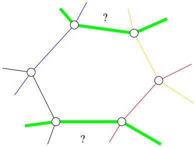
Minimum vertex Venn diagrams

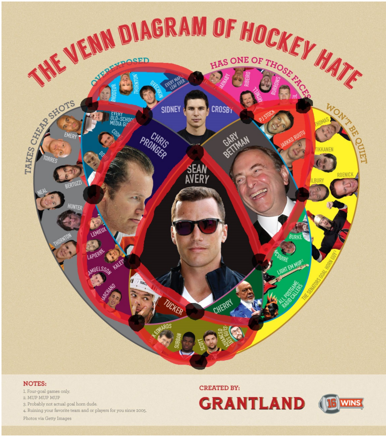




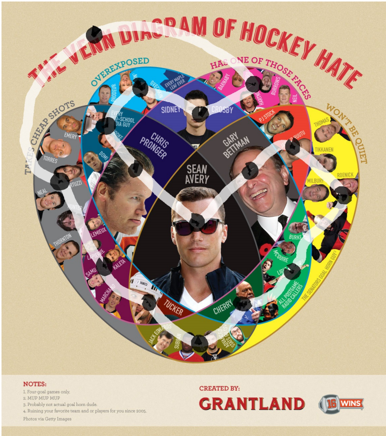
Basic facts, cont.

- Every Venn dual is 3-connected, every Venn graph is 3-connected. (Chilakamarri, Hamburger, Pippert, 1996)
- Every simple Venn graph is 4-connected. (Pruesse, R., 2015, arXiv).
  - As a consequence, by a theorem of Tutte, *every Venn diagram (graph) is Hamiltonian*.
  - Proof applies more generally to any collection of simple closed curves in general position *if* no curve has two edges on the same face (a key property of Venn diagrams).



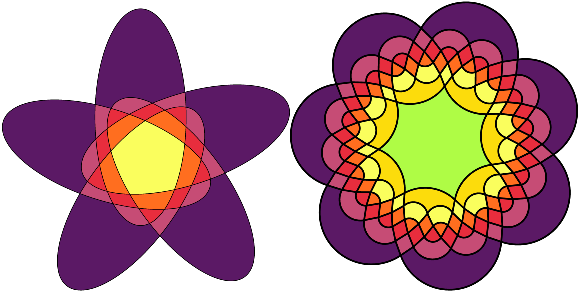


Our result



Winkler conjecture

Tutte's Theorem for Winkler's conjecture?

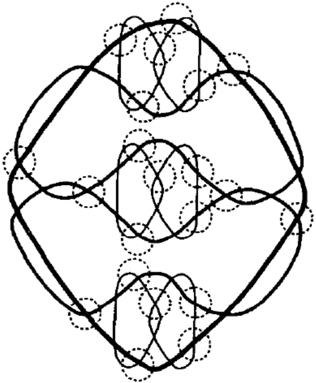


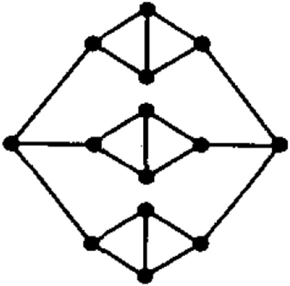
Problem: Venn diagram duals are only 3-connected in general, because Venn diagrams have 3-faces. In fact

Theorem

For $n \geq 3$, any n-Venn diagram has at least 8 3-faces.

A 3-connected non-Hamilton collection of curves

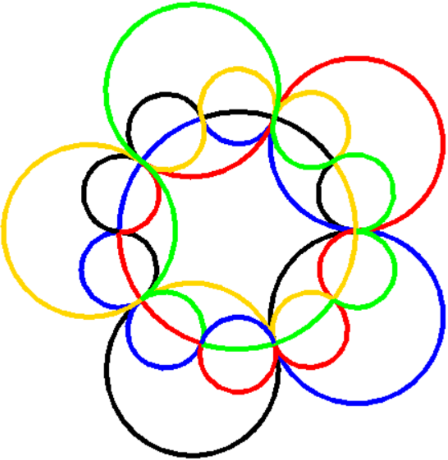


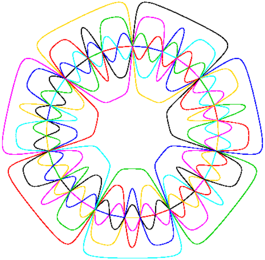


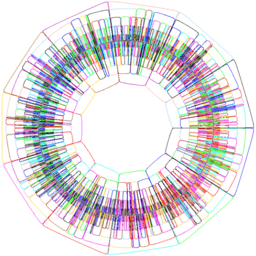
Iwamoto & Touissant (1994) *Finding Hamiltonian circuits in arrangements of Jordan curves is NP-complete*.

What about non-simple Venn diagrams?

They are only 2-connected in general:







Examples of a general family on prime numbers of curves.

Open problems

- Is every *non-simple* Venn graph Hamiltonian?
- Does every Venn diagram dual have a perfect matching?
- Is every monotone Venn diagram extendible? Recall: Monotone = drawable with all curves convex.

Symmetric Venn Diagrams

Theorem

Symmetric n-Venn diagrams exist if and only if n is prime.

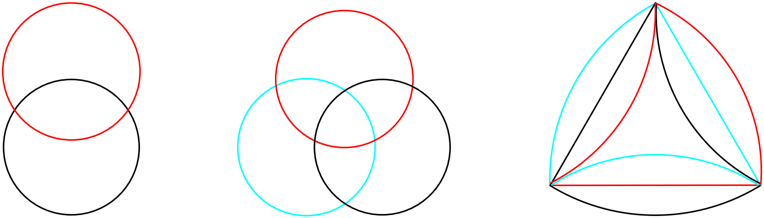
Proof.

Necessity: (D. W. Henderson, *Venn diagrams for more than four classes*, American Mathematical Monthly, **70** (1963) 424–426).

$$n \mid \binom{n}{k} \text{ for all } 0 < k < n.$$

Sufficiency: (Jerrold Griggs, Charles E. Killian and Carla D. Savage, *Venn Diagrams and Symmetric Chain Decompositions in the Boolean Lattice*, Electronic Journal of Combinatorics, Volume 11 (no. 1), #R2, (2004)). (The GKS construction). $\square$

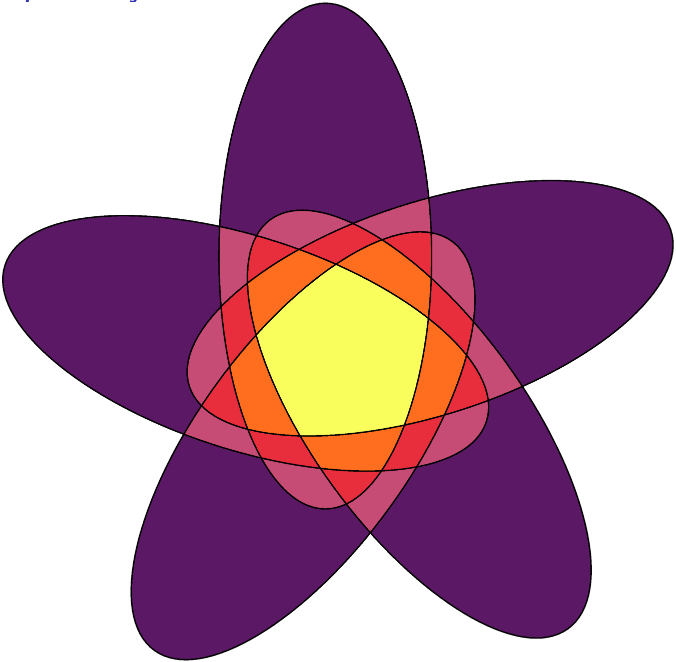
Small symmetric Venn diagrams



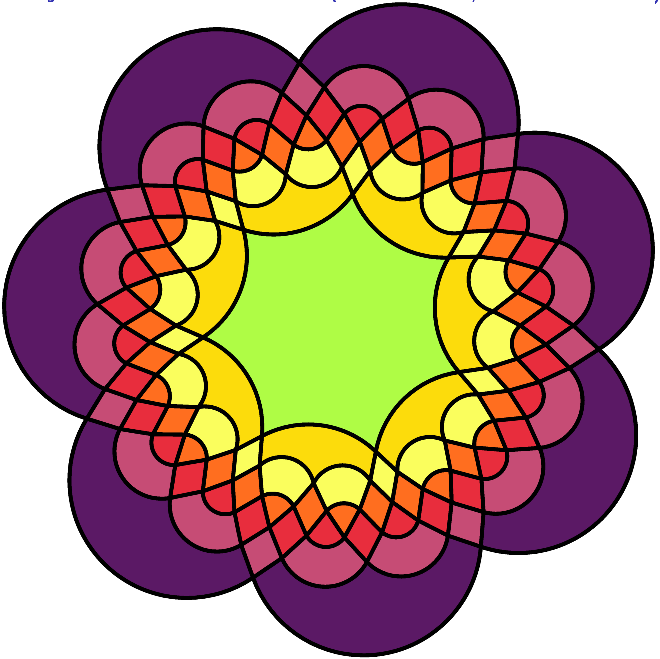
(a) (b) (c)

(a) $n = 2$ Only one diagram.
(b) $n = 3$ Only one *simple* diagram.
(c) $n = 3$ And one *non-simple* diagram.

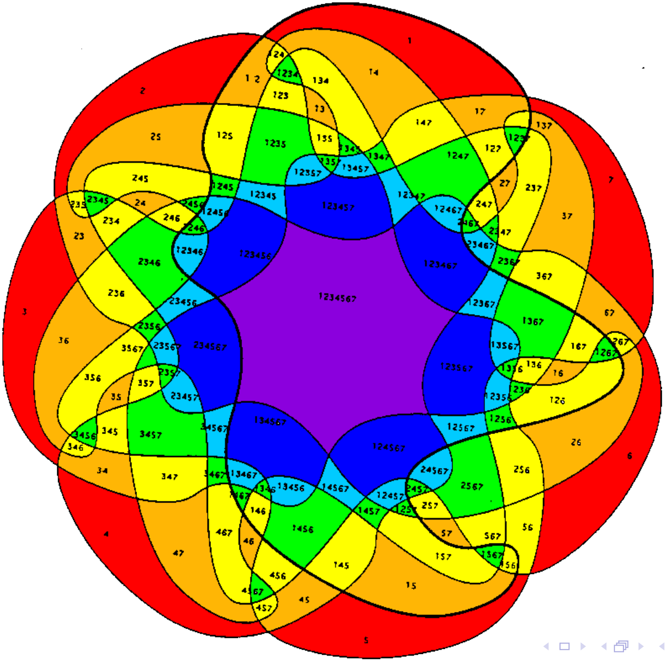
5 ellipses, by Grünbaum



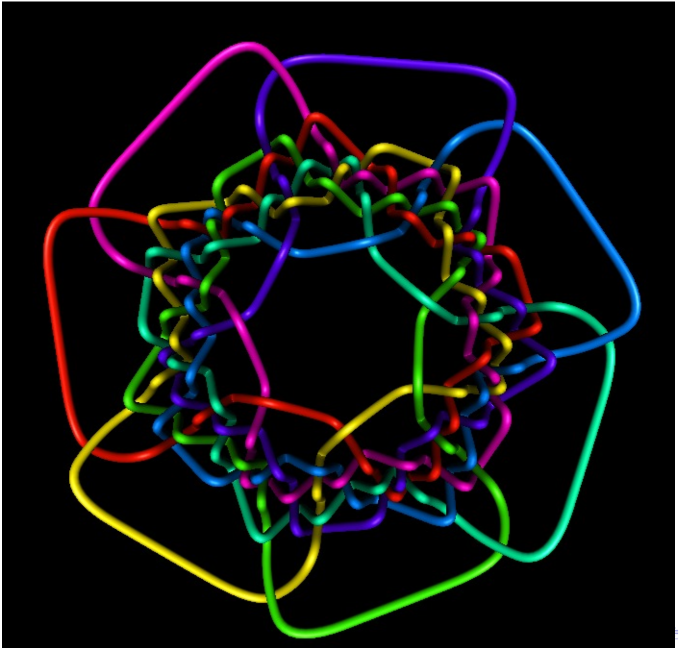
First symmetric 7-Venn (Edwards/Grünbaum)



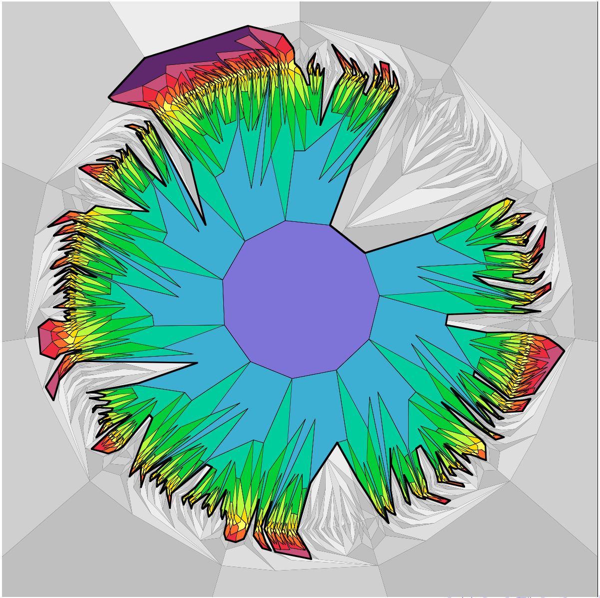
A non-convex 7-Venn diagram, by Grünbaum



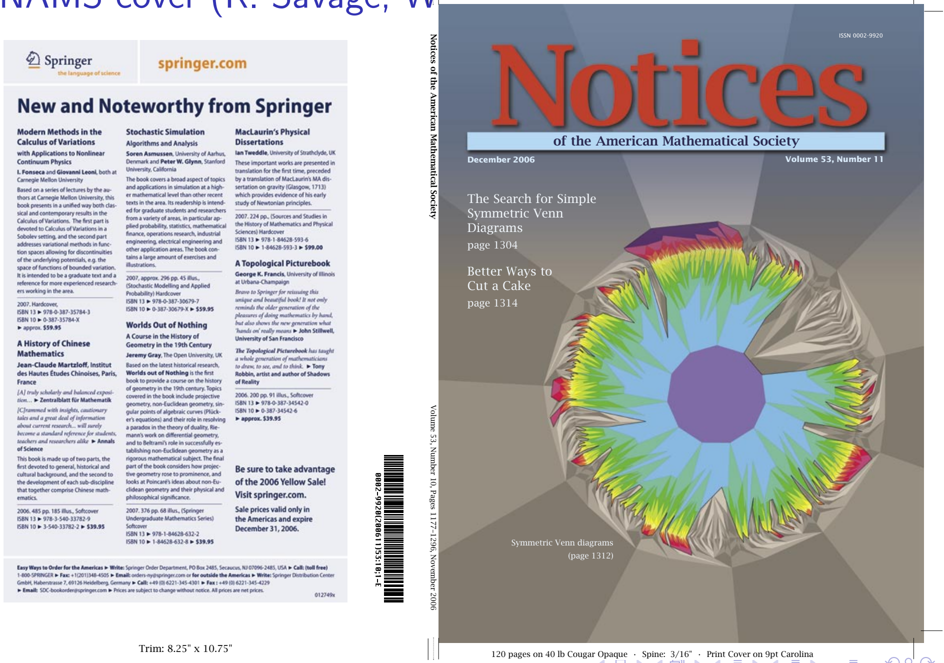
“Victoria”, rendered as a link



A "half-simple" 11-Venn diagram (rendered by Wagon)



NAMS cover (R. Savage, Wagon)



Springer
the language of science

springer.com

New and Noteworthy from Springer

Modern Methods in the Calculus of Variations

with Applications to Nonlinear Continuum Physics

I. Fonseca and **Giovanni Leoni**, both at Carnegie Mellon University

Based on a series of lectures by the authors at Carnegie Mellon University, this book presents in a unified way both classical and contemporary results in the Calculus of Variations. The first part is devoted to Calculus of Variations in a Sobolev setting, and the second part addresses variational methods in function spaces allowing for discontinuities of the underlying potentials, e.g. the space of functions of bounded variation. It is intended to be a graduate text and a reference for more experienced researchers working in the area.

2007. Hardcover,
ISBN 13 ► 978-0-387-35784-3
ISBN 10 ► 0-387-35784-X
► approx. **$59.95**

A History of Chinese Mathematics

Jean-Claude Martzloff, Institut des Hautes Études Chinoises, Paris, France

[A] truly scholarly and balanced exposition... ► **Zentralblatt für Mathematik**

[C]rammed with insights, cautionary tales and a great deal of information about current research... will surely become a standard reference for students, teachers and researchers alike ► **Annals of Science**

This book is made up of two parts, the first devoted to general, historical and cultural background, and the second to the development of each sub-discipline that together comprise Chinese mathematics.

2006. 485 pp. 185 illus., Softcover
ISBN 13 ► 978-3-540-33782-9
ISBN 10 ► 3-540-33782-2 ► **$39.95**

Stochastic Simulation

Algorithms and Analysis

Soren Asmussen, University of Aarhus, Denmark and **Peter W. Glynn**, Stanford University, California

The book covers a broad aspect of topics and applications in simulation at a higher mathematical level than other recent texts in the area. Its readership is intended for graduate students and researchers from a variety of areas, in particular applied probability, statistics, mathematical finance, operations research, industrial engineering, electrical engineering and other application areas. The book contains a large amount of exercises and illustrations.

2007, approx. 296 pp. 45 illus., (Stochastic Modelling and Applied Probability) Hardcover
ISBN 13 ► 978-0-387-30679-7
ISBN 10 ► 0-387-30679-X ► **$59.95**

Worlds Out of Nothing

A Course in the History of Geometry in the 19th Century

Jeremy Gray, The Open University, UK

Based on the latest historical research, **Worlds out of Nothing** is the first book to provide a course on the history of geometry in the 19th century. Topics covered in the book include projective geometry, non-Euclidean geometry, singular points of algebraic curves (Plücker's equations) and their role in resolving a paradox in the theory of duality, Riemann's work on differential geometry, and to Beltrami's role in successfully establishing non-Euclidean geometry as a rigorous mathematical subject. The final part of the book considers how projective geometry rose to prominence, and looks at Poincaré's ideas about non-Euclidean geometry and their physical and philosophical significance.

2007. 376 pp. 68 illus., (Springer Undergraduate Mathematics Series) Softcover
ISBN 13 ► 978-1-84628-632-2
ISBN 10 ► 1-84628-632-8 ► **$39.95**

MacLaurin's Physical Dissertations

Ian Tweddle, University of Strathclyde, UK

These important works are presented in translation for the first time, preceded by a translation of MacLaurin's MA dissertation on gravity (Glasgow, 1713) which provides evidence of his early study of Newtonian principles.

2007. 224 pp., (Sources and Studies in the History of Mathematics and Physical Sciences) Hardcover
ISBN 13 ► 978-1-84628-593-6
ISBN 10 ► 1-84628-593-3 ► **$99.00**

A Topological Picturebook

George K. Francis, University of Illinois at Urbana-Champaign

Bravo to Springer for reissuing this unique and beautiful book! It not only reminds the older generation of the pleasures of doing mathematics by hand, but also shows the new generation what 'hands on' really means ► **John Stillwell, University of San Francisco**

The Topological Picturebook has taught a whole generation of mathematicians to draw, to see, and to think. ► **Tony Robbin, artist and author of Shadows of Reality**

2006. 200 pp. 91 illus., Softcover
ISBN 13 ► 978-0-387-34542-0
ISBN 10 ► 0-387-34542-6
► **approx. $39.95**

Be sure to take advantage of the 2006 Yellow Sale! Visit springer.com.

Sale prices valid only in the Americas and expire December 31, 2006.

Easy Ways to Order for the Americas ► **Write:** Springer Order Department, PO Box 2485, Secaucus, NJ 07096-2485, USA ► **Call: (toll free)** 1-800-SPRINGER ► **Fax:** +1(201)348-4505 ► **Email:** orders-ny@springer.com or **for outside the Americas** ► **Write:** Springer Distribution Center GmbH, Haberstrasse 7, 69126 Heidelberg, Germany ► **Call:** +49 (0) 6221-345-4301 ► **Fax :** +49 (0) 6221-345-4229 ► **Email:** SDC-bookorder@springer.com ► Prices are subject to change without notice. All prices are net prices.

012749x

Trim: 8.25" x 10.75"

Notices of the American Mathematical Society

Volume 53, Number 10, Pages 1177–1296, November 2006

ISSN 0002-9920

Notices

of the American Mathematical Society

December 2006 Volume 53, Number 11

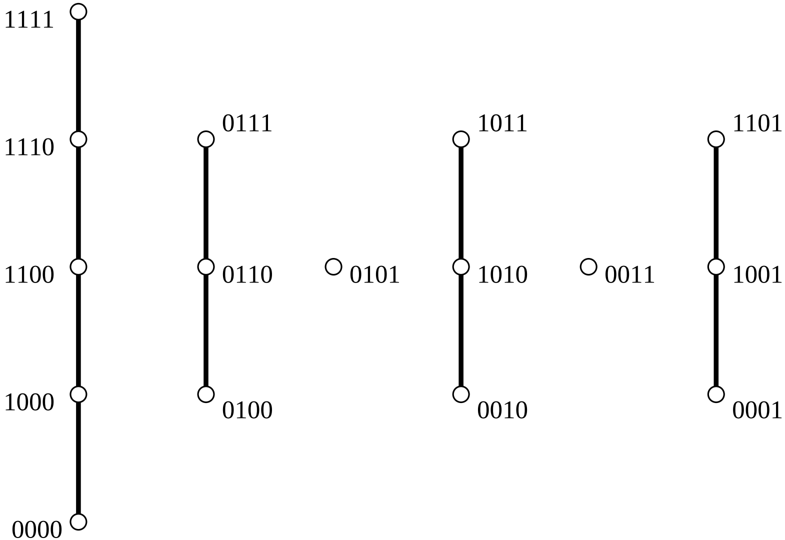
The Search for Simple Symmetric Venn Diagrams
page 1304

Better Ways to Cut a Cake
page 1314

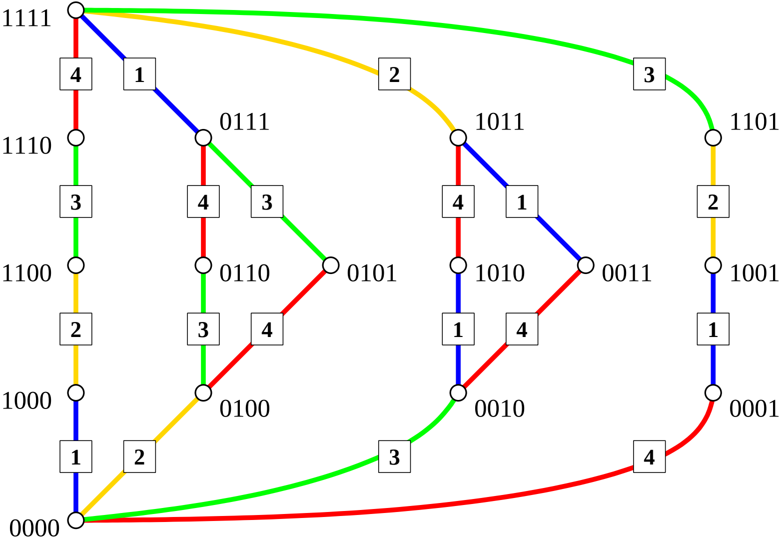
Symmetric Venn diagrams
(page 1312)

120 pages on 40 lb Cougar Opaque · Spine: 3/16" · Print Cover on 9pt Carolina

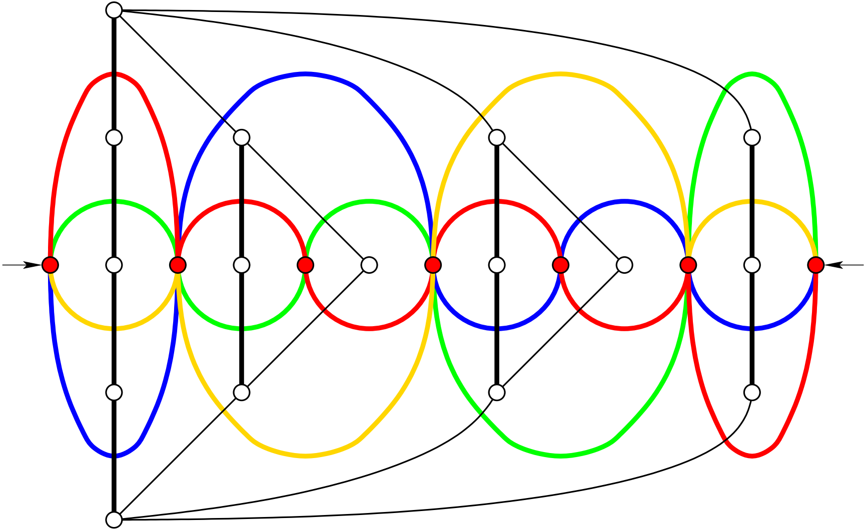
Symmetric chain decompositions give Venn diagrams



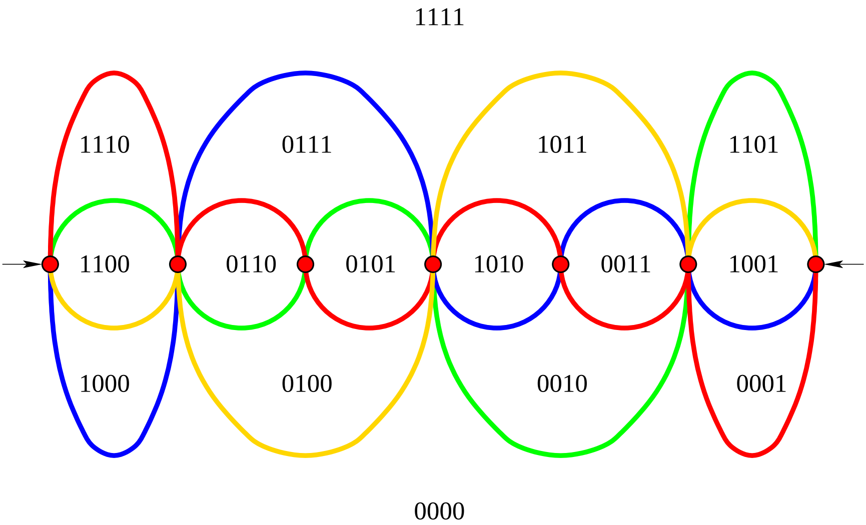
Symmetric chain decompositions give Venn diagrams



Symmetric chain decompositions give Venn diagrams



Symmetric chain decompositions give Venn diagrams



The Greene-Kleitman rule

Parentheses matching with $0 = ($ and $1 =)$.

1 1 1 1 **0 0 1 1** 1 **0 1 0 1** 1 1 1 **0 1** 1
1 1 1 1 **0 0 1 1** 1 **0 1 0 1** 1 1 1 **0 1** 0
1 1 1 1 **0 0 1 1** 1 **0 1 0 1** 1 1 0 **0 1** 0
1 1 1 1 **0 0 1 1** 1 **0 1 0 1** 1 0 0 **0 1** 0
1 1 1 1 **0 0 1 1** 1 **0 1 0 1** 0 0 0 **0 1** 0
1 1 1 1 **0 0 1 1** 0 **0 1 0 1** 0 0 0 **0 1** 0
1 1 1 0 **0 0 1 1** 0 **0 1 0 1** 0 0 0 **0 1** 0
1 1 0 0 **0 0 1 1** 0 **0 1 0 1** 0 0 0 **0 1** 0
1 0 0 0 **0 0 1 1** 0 **0 1 0 1** 0 0 0 **0 1** 0
0 0 0 0 **0 0 1 1** 0 **0 1 0 1** 0 0 0 **0 1** 0

The Greene-Kleitman rule

Parentheses matching with 0 = (and 1 =).

1 1 1 1 **0 0 1 1** 1 **0 1 0 1** 1 1 1 **0 1** 1

1 1 1 1 **0 0 1 1** 1 **0 1 0 1** 1 1 1 **0 1** 0

1 1 1 1 **0 0 1 1** 1 **0 1 0 1** 1 1 0 **0 1** 0

1 1 1 1 **0 0 1 1** 1 **0 1 0 1** 1 0 0 **0 1** 0

1 1 1 1 **0 0 1 1** 1 **0 1 0 1** 0 0 0 **0 1** 0

1 1 1 1 **0 0 1 1** 0 **0 1 0 1** 0 0 0 **0 1** 0

1 1 1 0 **0 0 1 1** 0 **0 1 0 1** 0 0 0 **0 1** 0

1 1 0 0 **0 0 1 1** 0 **0 1 0 1** 0 0 0 **0 1** 0

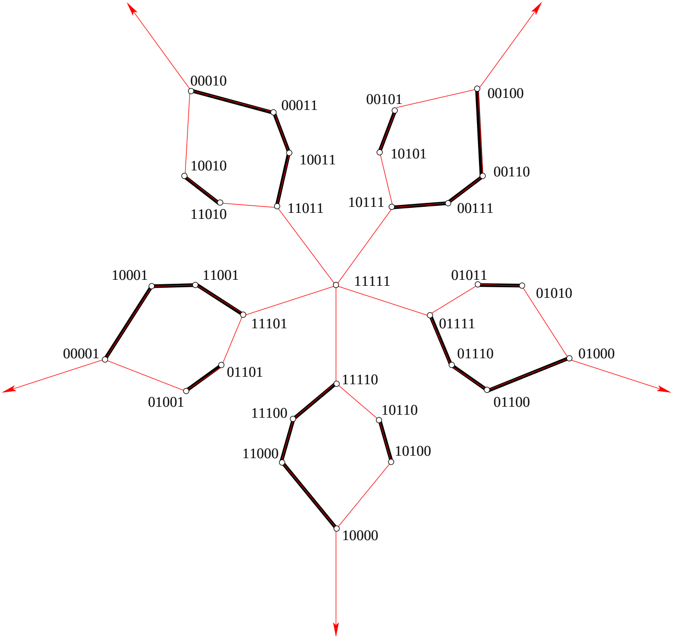
1 0 0 0 **0 0 1 1** 0 **0 1 0 1** 0 0 0 **0 1** 0

0 0 0 0 **0 0 1 1** 0 **0 1 0 1** 0 0 0 **0 1** 0

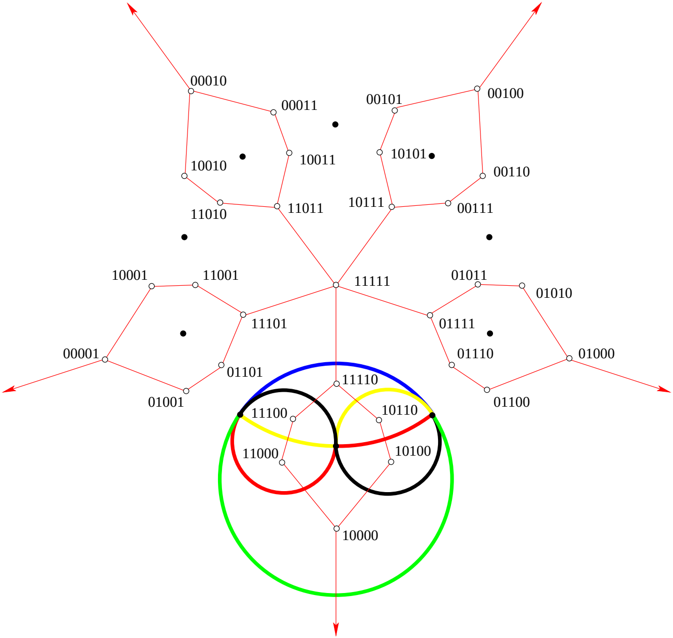
The Greene-Kleitman rule

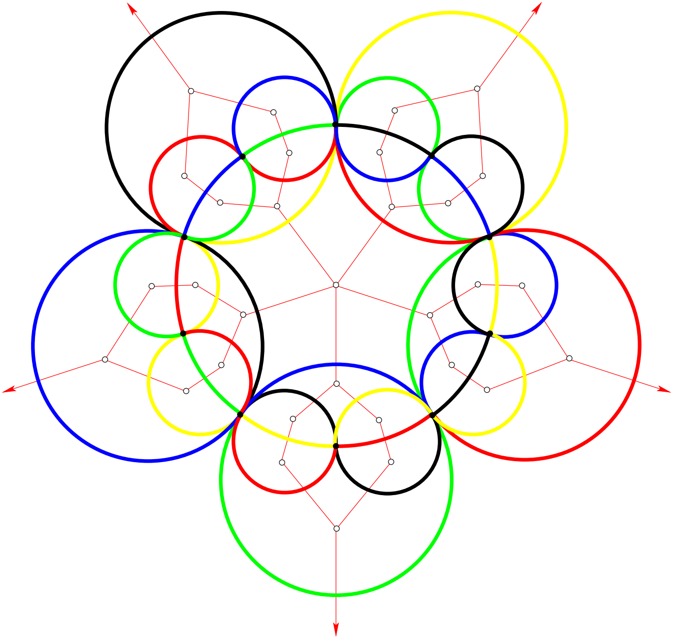
Parentheses matching with $0 = ($ and $1 =)$.

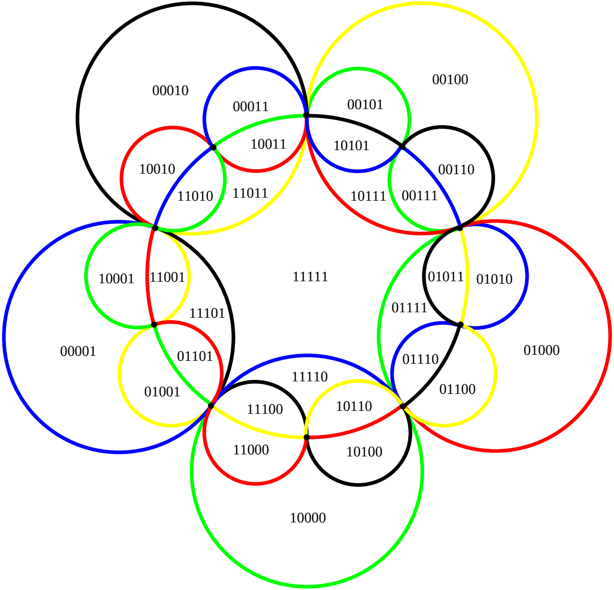
1 1 1 1 **0 0 1 1** 1 **0 1 0 1** 1 1 1 **0 1** 1
1 1 1 1 **0 0 1 1** 1 **0 1 0 1** 1 1 1 **0 1** 0
1 1 1 1 **0 0 1 1** 1 **0 1 0 1** 1 1 0 **0 1** 0
1 1 1 1 **0 0 1 1** 1 **0 1 0 1** 1 0 0 **0 1** 0
1 1 1 1 **0 0 1 1** 1 **0 1 0 1** 0 0 0 **0 1** 0
1 1 1 1 **0 0 1 1** 0 **0 1 0 1** 0 0 0 **0 1** 0
1 1 1 0 **0 0 1 1** 0 **0 1 0 1** 0 0 0 **0 1** 0
1 1 0 0 **0 0 1 1** 0 **0 1 0 1** 0 0 0 **0 1** 0
1 0 0 0 **0 0 1 1** 0 **0 1 0 1** 0 0 0 **0 1** 0
0 0 0 0 **0 0 1 1** 0 **0 1 0 1** 0 0 0 **0 1** 0



00010
00011
10011
10010
11010
11011
00101
00100
10101
00110
10111
00111
10001
11001
11111
01011
01010
11101
01111
00001
01110
01000
01101
11110
01001
01100
10110
11100
11000
10100
10000


00010
00011
00101
00100
10010
10011
10101
00110
11010
11011
10111
00111
10001
11001
11111
01011
01010
11101
01111
00001
01110
01000
01101
01001
11110
01100
11100
10110
11000
10100
10000



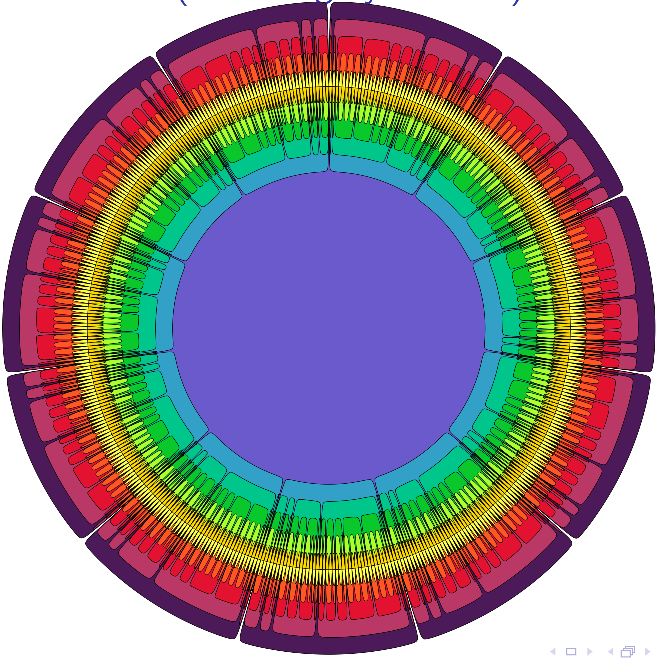


00010
00011
00101
00100
10011
10101
10010
00110
11010
11011
10111
00111
10001
11001
11111
01011
01010
11101
01111
00001
01101
01110
01000
11110
01100
01001
11100
10110
11000
10100
10000

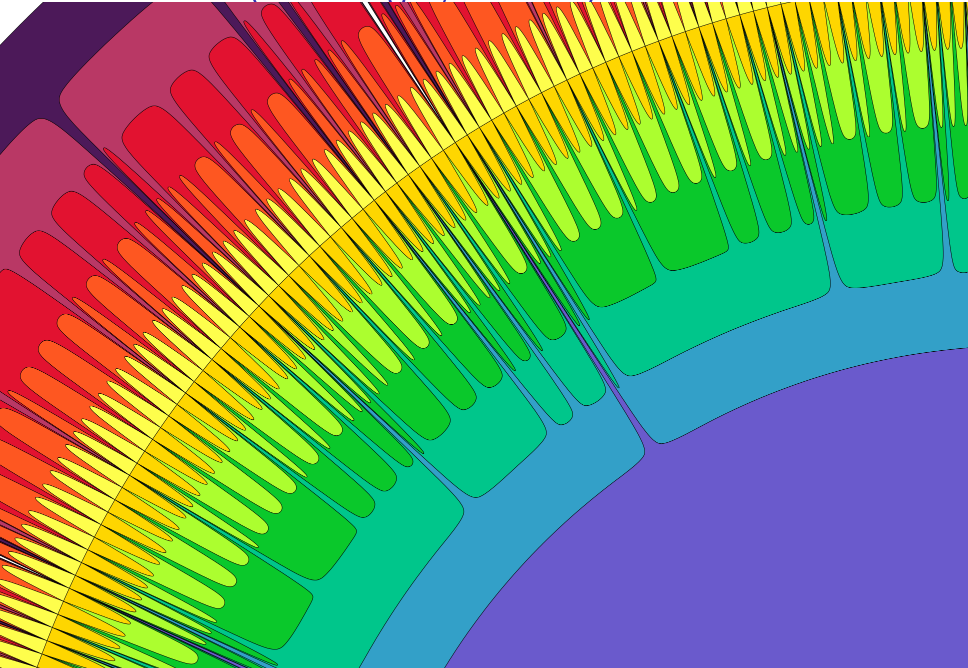
Choosing necklace representatives

- Break the bitstring into *blocks* of 1s followed by 0s and list their sizes as a sequence, the *block code*.
- E.g., 111000 1100 10 10000 10 has block code (6,4,2,5,2).
- Rotate block code to its *unique* lex minimum and act on the bitstring similarly. E.g., (2,5,2,6,4) is lex minimum and gives 10 10000 10 111000 1100.
- Apply Greene-Kleitman, ignoring the initial 1 and final 0.
- Key observation: block code is invariant under Greene-Kleitman!
  1 **0** . **1** 0 . **0 0 0 1 0** . **1 1 1** 0 **0 0** . **1 1** 0 0

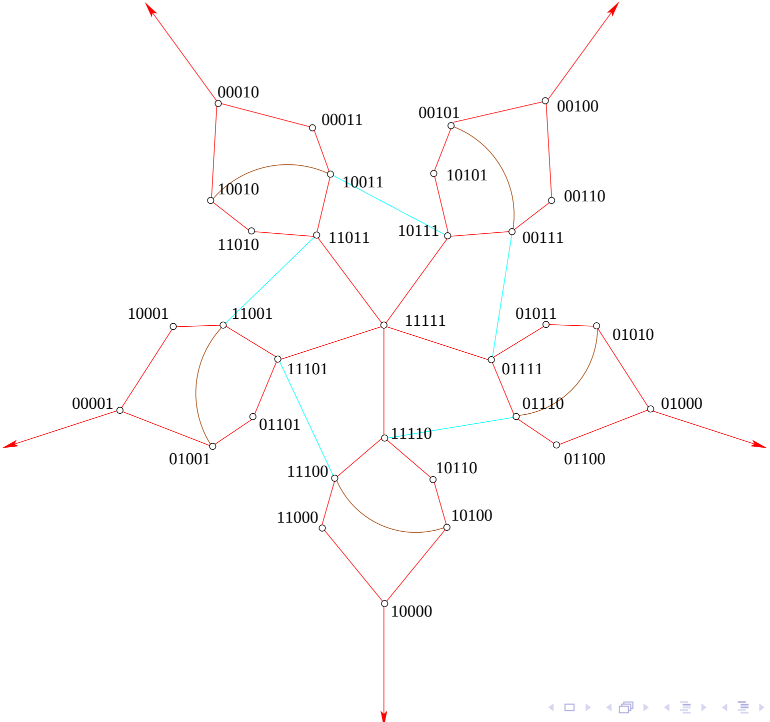
GKS 11-Venn (rendering by Weston)

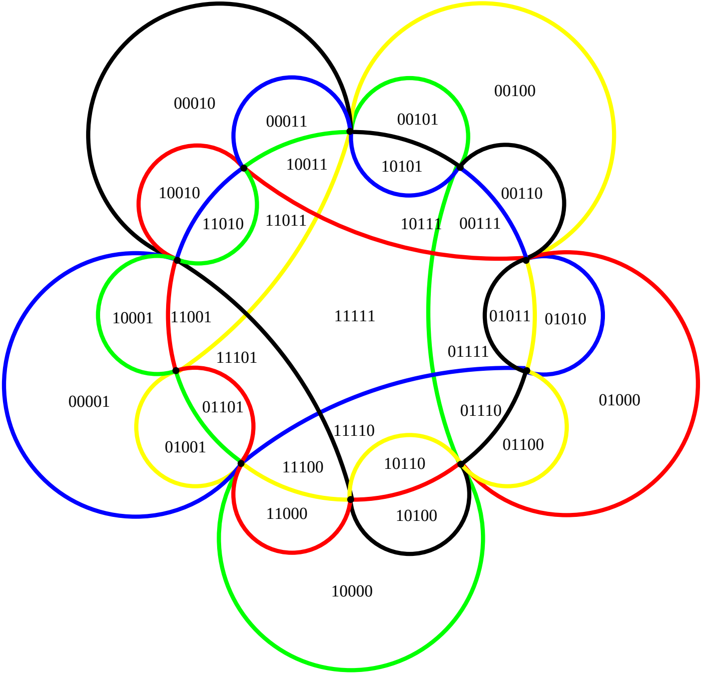


GKS 11-Venn (rendering by Weston)



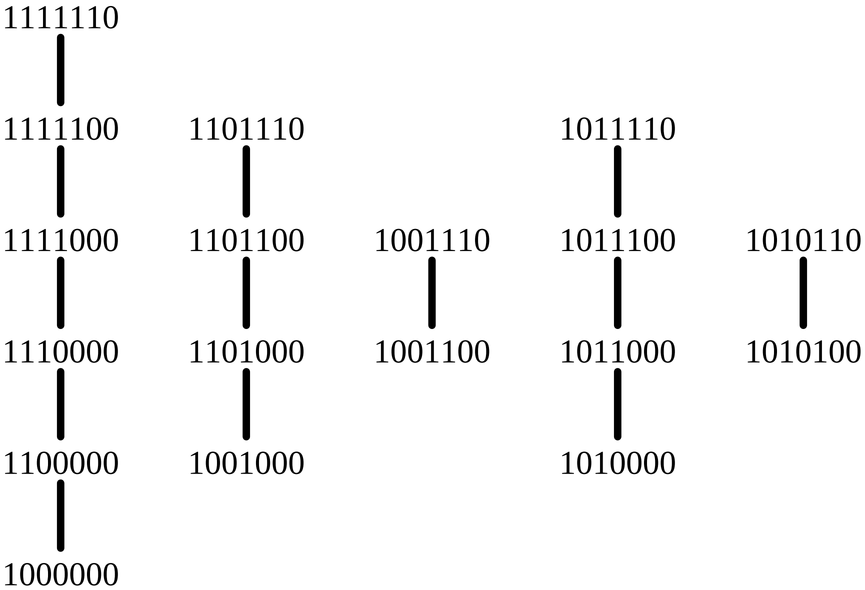
Simplify, simplify!


00010
00011
00101
00100
10011
10010
10101
00110
10111
00111
11010
11011
10001
11001
11111
01011
01010
11101
01111
00001
01110
01000
01101
11110
01001
01100
11100
10110
11000
10100
10000

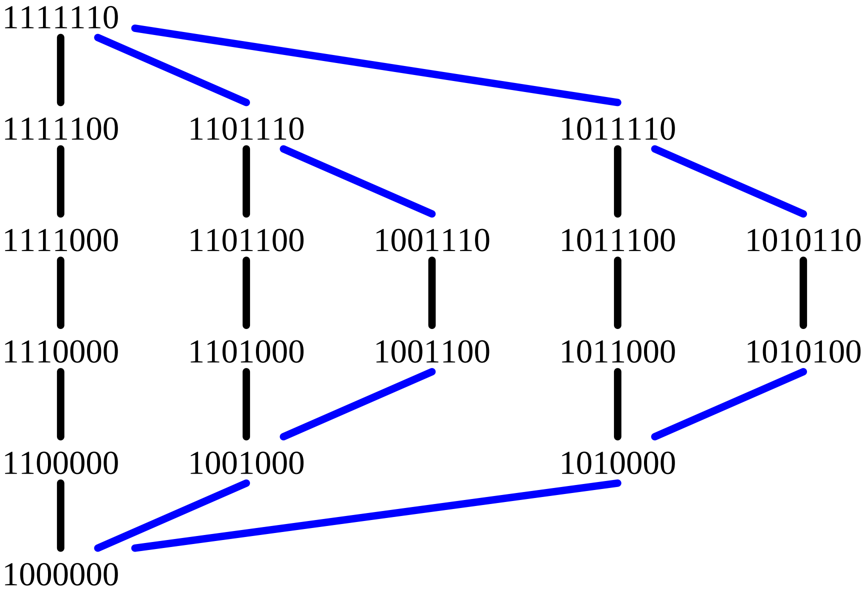


00010
00011
00101
00100
10011
10101
10010
00110
11010
11011
10111
00111
10001
11001
11111
01011
01010
11101
01111
00001
01101
01110
01000
11110
01001
01100
10110
11100
11000
10100
10000

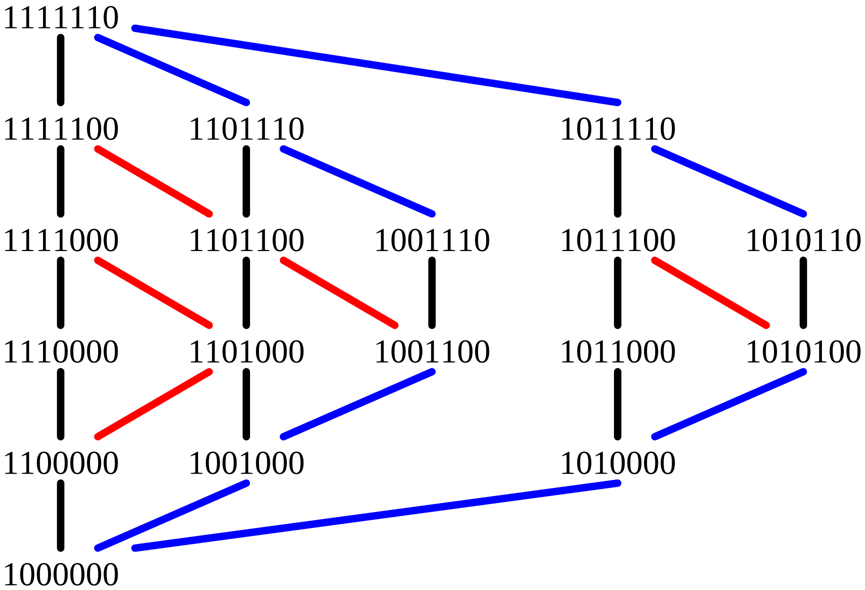
1/7-th of a Venn diagram



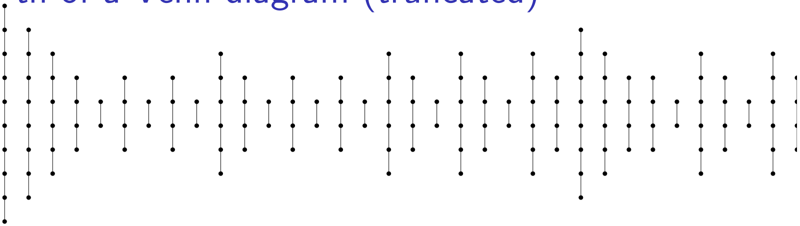
1/7-th of a Venn diagram



1/7-th of a Venn diagram



1/11-th of a Venn diagram (truncated)

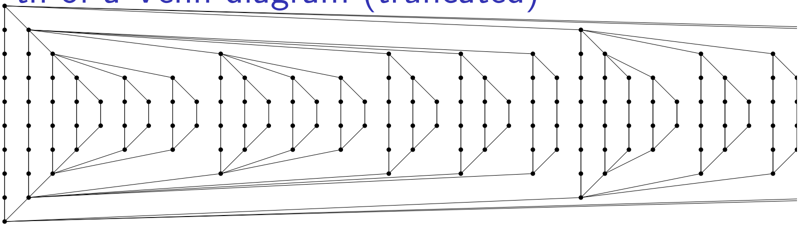


The chains

Half-simple Venn diagrams: Number of vertices is $> (2^n - 2)/2$.

Killian,R,Savage,Weston (2004)

1/11-th of a Venn diagram (truncated)

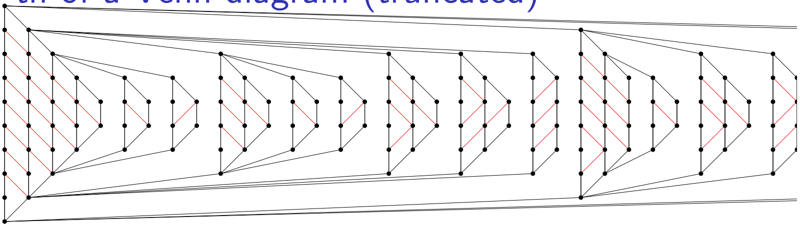


The opposing trees

Half-simple Venn diagrams: Number of vertices is $> (2^n - 2)/2$.

Killian,R,Savage,Weston (2004)

1/11-th of a Venn diagram (truncated)

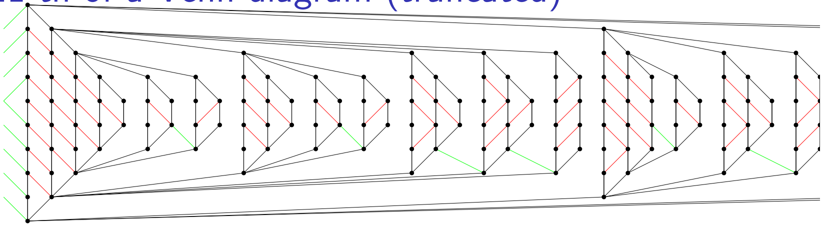


Quadrangulating edges

Half-simple Venn diagrams: Number of vertices is $> (2^n - 2)/2$.

Killian,R,Savage,Weston (2004)

1/11-th of a Venn diagram (truncated)



More can be added by hand

Half-simple Venn diagrams: Number of vertices is $> (2^n - 2)/2$.

Killian,R,Savage,Weston (2004)

History

- Henderson (1963) observed that if there is a symmetric n-Venn diagram, then n must be a prime number.
- From [GR75] regarding symmetric 7-Venn diagrams: "The present author's search for such a diagram has been unsuccessful ... at present it seems that no such diagram exists."
- In [GR92b] Branko draws two symmetric 7-Venn diagrams, one of which is non-convex and the other from (non-convex) pentagons. Around the same time symmetric 7-Venn diagrams are also found by Edwards.
- From [GR92b]: "Conjecture 3: For every positive prime n there exists symmetric Venn diagrams with n sets."
- Hamburger (2002) found a *non-simple* 11-Venn diagram.
- Griggs, Killian and Savage (2004) proved the existence of *non-simple* symmetric n-Venn diagrams whenever n is prime.
- Khalegh Mamakani and R. (2014) find a symmetric *simple* 11-Venn diagram and then a 13-Venn diagram (published in Discrete and Computational Geometry).

History

- Henderson (1963) observed that if there is a symmetric n-Venn diagram, then n must be a prime number.
- From [GR75] regarding symmetric 7-Venn diagrams: "The present author's search for such a diagram has been unsuccessful ... at present it seems that no such diagram exists."
- In [GR92b] Branko draws two symmetric 7-Venn diagrams, one of which is non-convex and the other from (non-convex) pentagons. Around the same time symmetric 7-Venn diagrams are also found by Edwards.
- From [GR92b]: "Conjecture 3: For every positive prime n there exists symmetric Venn diagrams with n sets."
- Hamburger (2002) found a *non-simple* 11-Venn diagram.
- Griggs, Killian and Savage (2004) proved the existence of *non-simple* symmetric n-Venn diagrams whenever n is prime.
- Khalegh Mamakani and R. (2014) find a symmetric *simple* 11-Venn diagram and then a 13-Venn diagram (published in Discrete and Computational Geometry).

History

- Henderson (1963) observed that if there is a symmetric n-Venn diagram, then n must be a prime number.
- From [GR75] regarding symmetric 7-Venn diagrams: "The present author's search for such a diagram has been unsuccessful ... at present it seems that no such diagram exists."
- In [GR92b] Branko draws two symmetric 7-Venn diagrams, one of which is non-convex and the other from (non-convex) pentagons. Around the same time symmetric 7-Venn diagrams are also found by Edwards.
- From [GR92b]: "Conjecture 3: For every positive prime n there exists symmetric Venn diagrams with n sets."
- Hamburger (2002) found a *non-simple* 11-Venn diagram.
- Griggs, Killian and Savage (2004) proved the existence of *non-simple* symmetric n-Venn diagrams whenever n is prime.
- Khalegh Mamakani and R. (2014) find a symmetric *simple* 11-Venn diagram and then a 13-Venn diagram (published in Discrete and Computational Geometry).

History

- Henderson (1963) observed that if there is a symmetric n-Venn diagram, then n must be a prime number.
- From [GR75] regarding symmetric 7-Venn diagrams: "The present author's search for such a diagram has been unsuccessful ... at present it seems that no such diagram exists."
- In [GR92b] Branko draws two symmetric 7-Venn diagrams, one of which is non-convex and the other from (non-convex) pentagons. Around the same time symmetric 7-Venn diagrams are also found by Edwards.
- From [GR92b]: "Conjecture 3: For every positive prime n there exists symmetric Venn diagrams with n sets."
- Hamburger (2002) found a *non-simple* 11-Venn diagram.
- Griggs, Killian and Savage (2004) proved the existence of *non-simple* symmetric n-Venn diagrams whenever n is prime.
- Khalegh Mamakani and R. (2014) find a symmetric *simple* 11-Venn diagram and then a 13-Venn diagram (published in Discrete and Computational Geometry).

History

- Henderson (1963) observed that if there is a symmetric n-Venn diagram, then n must be a prime number.
- From [GR75] regarding symmetric 7-Venn diagrams: "The present author's search for such a diagram has been unsuccessful ... at present it seems that no such diagram exists."
- In [GR92b] Branko draws two symmetric 7-Venn diagrams, one of which is non-convex and the other from (non-convex) pentagons. Around the same time symmetric 7-Venn diagrams are also found by Edwards.
- From [GR92b]: "Conjecture 3: For every positive prime n there exists symmetric Venn diagrams with n sets."
- Hamburger (2002) found a *non-simple* 11-Venn diagram.
- Griggs, Killian and Savage (2004) proved the existence of *non-simple* symmetric n-Venn diagrams whenever n is prime.
- Khalegh Mamakani and R. (2014) find a symmetric *simple* 11-Venn diagram and then a 13-Venn diagram (published in Discrete and Computational Geometry).

History

- Henderson (1963) observed that if there is a symmetric n-Venn diagram, then n must be a prime number.
- From [GR75] regarding symmetric 7-Venn diagrams: "The present author's search for such a diagram has been unsuccessful ... at present it seems that no such diagram exists."
- In [GR92b] Branko draws two symmetric 7-Venn diagrams, one of which is non-convex and the other from (non-convex) pentagons. Around the same time symmetric 7-Venn diagrams are also found by Edwards.
- From [GR92b]: "Conjecture 3: For every positive prime n there exists symmetric Venn diagrams with n sets."
- Hamburger (2002) found a *non-simple* 11-Venn diagram.
- Griggs, Killian and Savage (2004) proved the existence of *non-simple* symmetric n-Venn diagrams whenever n is prime.
- Khalegh Mamakani and R. (2014) find a symmetric *simple* 11-Venn diagram and then a 13-Venn diagram (published in Discrete and Computational Geometry).

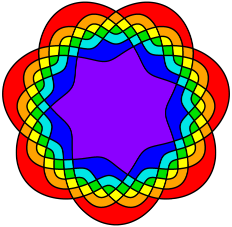
History

- Henderson (1963) observed that if there is a symmetric n-Venn diagram, then n must be a prime number.
- From [GR75] regarding symmetric 7-Venn diagrams: "The present author's search for such a diagram has been unsuccessful ... at present it seems that no such diagram exists."
- In [GR92b] Branko draws two symmetric 7-Venn diagrams, one of which is non-convex and the other from (non-convex) pentagons. Around the same time symmetric 7-Venn diagrams are also found by Edwards.
- From [GR92b]: "Conjecture 3: For every positive prime n there exists symmetric Venn diagrams with n sets."
- Hamburger (2002) found a *non-simple* 11-Venn diagram.
- Griggs, Killian and Savage (2004) proved the existence of *non-simple* symmetric n-Venn diagrams whenever n is prime.
- Khalegh Mamakani and R. (2014) find a symmetric *simple* 11-Venn diagram and then a 13-Venn diagram (published in Discrete and Computational Geometry).

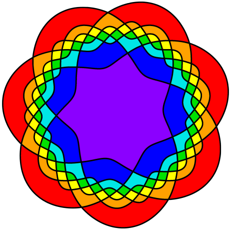
History

- Henderson (1963) observed that if there is a symmetric n-Venn diagram, then n must be a prime number.
- From [GR75] regarding symmetric 7-Venn diagrams: "The present author's search for such a diagram has been unsuccessful ... at present it seems that no such diagram exists."
- In [GR92b] Branko draws two symmetric 7-Venn diagrams, one of which is non-convex and the other from (non-convex) pentagons. Around the same time symmetric 7-Venn diagrams are also found by Edwards.
- From [GR92b]: "Conjecture 3: For every positive prime n there exists symmetric Venn diagrams with n sets."
- Hamburger (2002) found a *non-simple* 11-Venn diagram.
- Griggs, Killian and Savage (2004) proved the existence of *non-simple* symmetric n-Venn diagrams whenever n is prime.
- Khalegh Mamakani and R. (2014) find a symmetric *simple* 11-Venn diagram and then a 13-Venn diagram (published in Discrete and Computational Geometry).

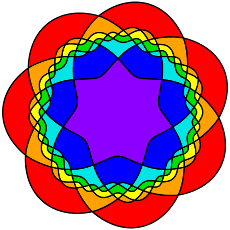
The 6 polar symmetric convex Venn diagrams



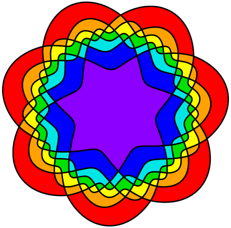
Adelaide



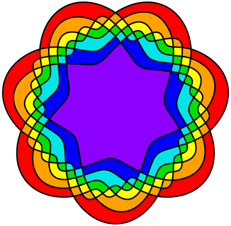
Hamilton



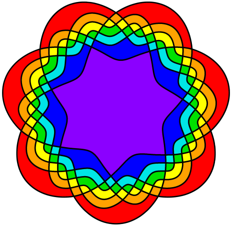
Manawatu



Palmerston North

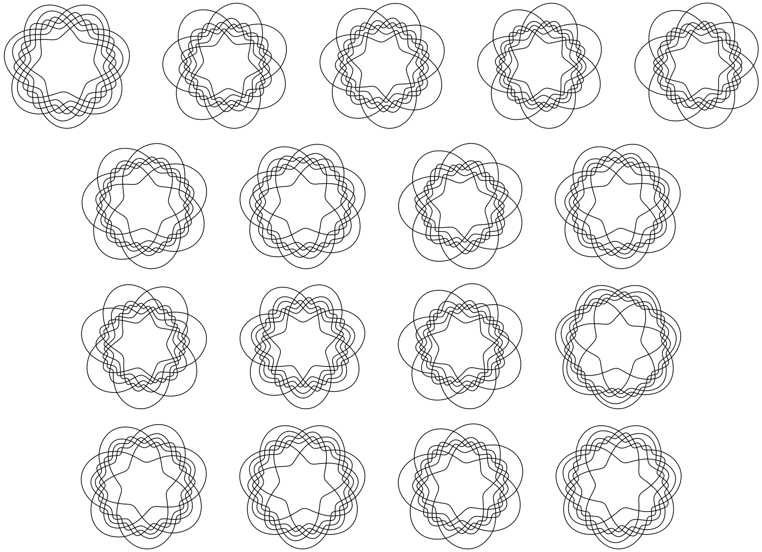


Massey



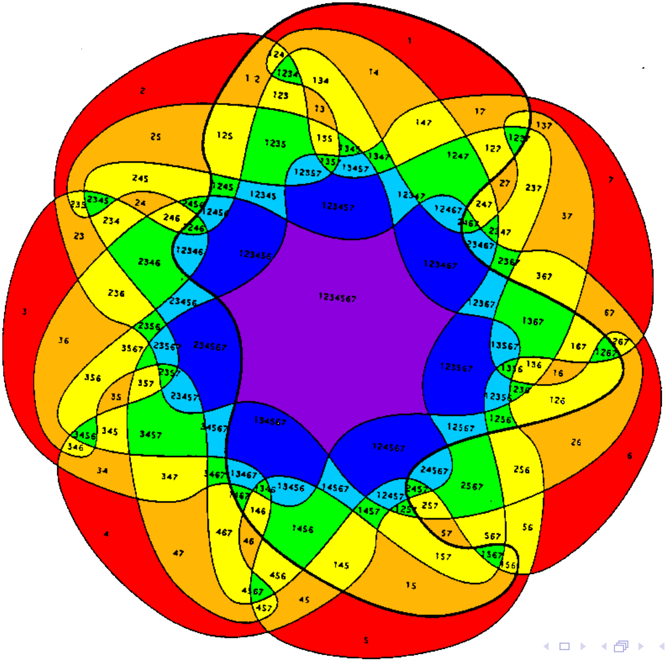
Victoria

The 17 remaining symmetric convex 7-Venn diagrams

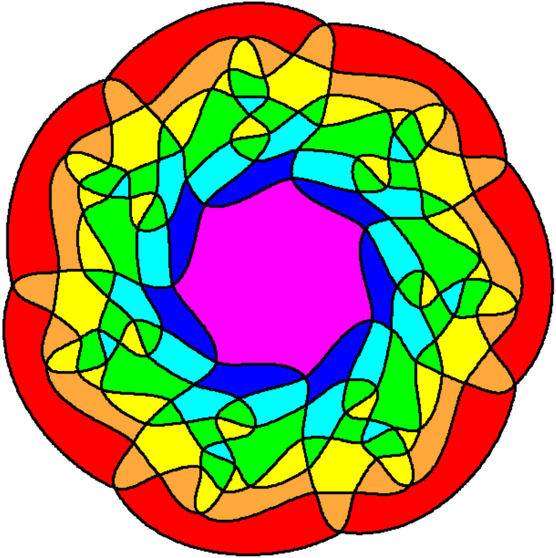


From Cao, Mamakani, and R. (2010)

A non-convex symmetric 7-Venn diagram, by Grünbaum



Another non-convex symmetric 7-Venn diagram



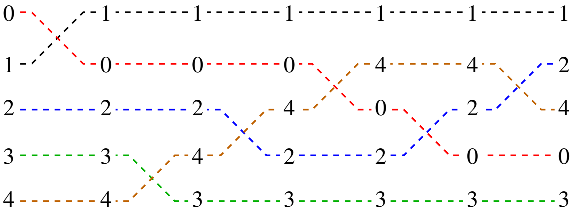
Open: How many simple non-convex 7-Venn diagrams? Or non-simple but convex? Or non-simple and non-convex?

Searching for simple symmetric Venn diagrams

Again we restrict ourselves to monotone=convex diagrams.

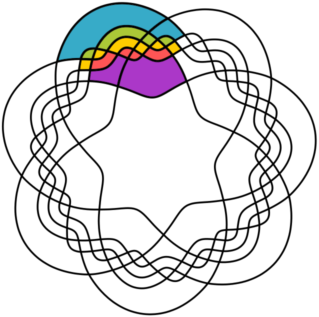
Representing Monotone Venn diagrams

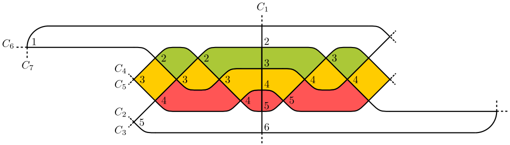
- One fifth of Grünbaum's 5 ellipses:



- 1 4 3 2 3 2
- In total the diagram is represented by 143232 143232 143232 143232 143232.
- The representation is not unique (e.g., swap 1 and 4 above to get 213232).
- Call this a *crossing sequence*.

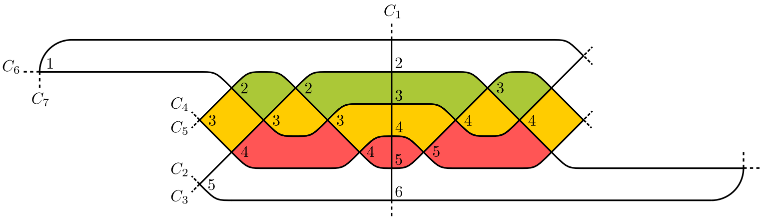
Crosscut symmetry





Crosscut: Curve segment that sequentially crosses all other curves once.
Crosscut symmetry: Reflective symmetry across the crosscut (except top and bottom).
Strategy: Limit the search to diagrams that have crosscut symmetry.

Crosscut symmetry



Curve intersections are palindromic (except C_1). E.g., the intersections with C_5 are

$$L_{5,1} = [C_4, C_6, C_3, C_6, C_4, C_1, C_4, C_6, C_3, C_6, C_4]$$

The crossing sequence:

$$\underbrace{1, 3, 2, 5, 4}_{\rho}, \underbrace{3, 2, 3, 4}_{\alpha}, \underbrace{6, 5, 4, 3, 2}_{\delta}, \underbrace{5, 4, 3, 4}_{\alpha^{r+}}$$

Crosscut symmetry theorem

Theorem

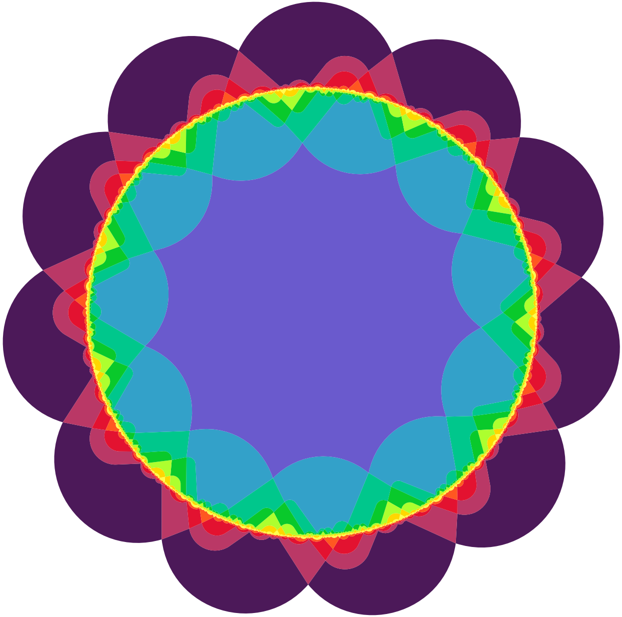
A simple monotone rotationally symmetric n-Venn diagram is crosscut symmetric if and only if it can be represented by a crossing sequence of the form $\rho, \alpha, \delta, \alpha^{r+}$ *where*

- ρ *is* $1, 3, 2, 5, 4, \ldots, n-2, n-3$.
- δ *is* $n-1, n-2, \ldots, 3, 2$.
- α *and* α^{r+} *are two sequences of length* $(2^{n-1} - (n-1)^2)/n$ *such that* α^{r+} *is obtained by reversing* α *and adding* 1 *to each element; that is,* $\alpha^{r+}[i] = \alpha[|\alpha| - i + 1]$.

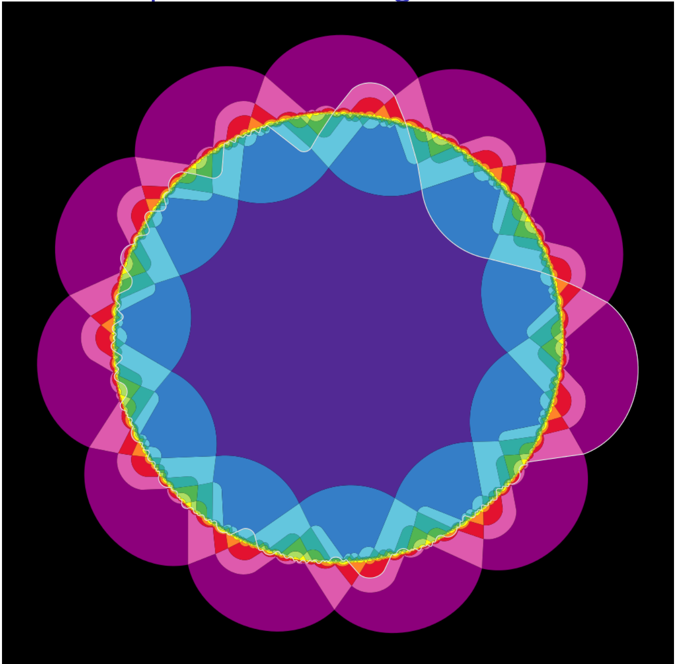
Below is the α sequence for Newroz.

[3234345432343454345456545656765432543465456767876565434576546587654576568765465765 67]

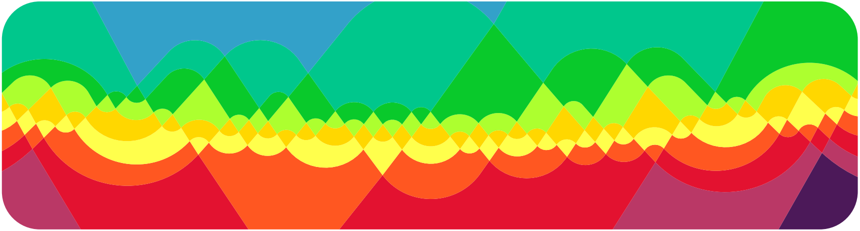
The first simple 11-Venn diagram “Newroz”



The first simple 11-Venn diagram “Newroz”



Blow-up



Polar and Crosscut symmetry?

Theorem

Unless $n \in \{2, 3, 5, 7\}$ there is no symmetric Venn diagram with both polar and crosscut symmetry.

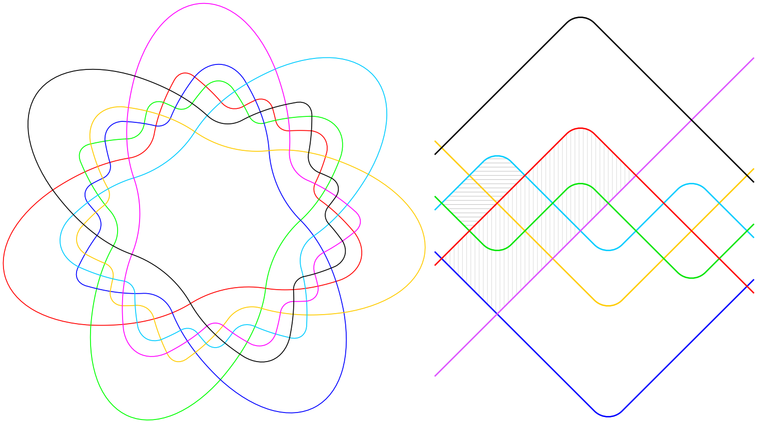
Proof summary:

- Consider a cluster in such a Venn diagram.
- Let R_k be the number of k-regions to the left of the crosscut.
- $R_k = (\binom{n-1}{k} + (-1)^{k+1})/n$.
- By the symmetries, each $m = (n-1)/2$ region (these lie along the "equator") is incident to at least one $(m-1)$-point.
- Thus $R_m \leq R_{m-1} + 1$, and so m can't be too large

Our 15 minutes of fame

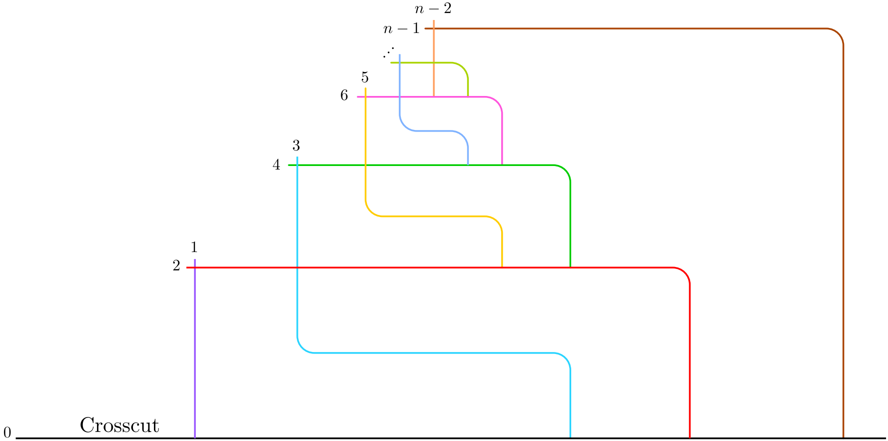
- Write-up in New Scientist Magazine: teaser; longer; gallery.
- In Wired UK.
- And on Physics Central.
- Appears in the AMS Math in the Media magazine (August 2012), and is the image of the month there.
- Commented on here: Gizmodo.
- Getting some attention on reddit.
- A very well written blog entry: Cartesian Product.
- On tumblr.
- It generated some comments on slashdot.
- We were the August 20 entry in the Math Munch.
- Comments in Farsi.
- Comments in Dutch.
- On Pirate Science.

Another symmetric 7-Venn diagram with crosscut symmetry



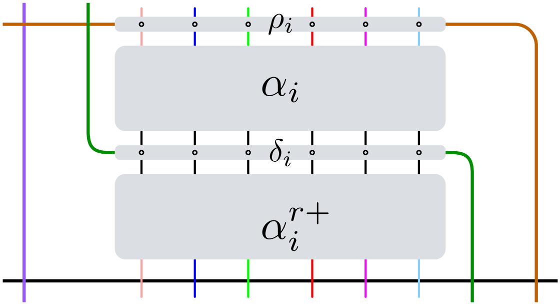
Note the smaller structures with crosscut symmetry. Here $\alpha_H = 3, 2, 4, 3$.

Iterated crosscuts in general



Note: labels are all off by 1.

Iterated crosscuts in general



$\rho, \alpha, \delta, \alpha^{r+}$ occurs again!

Using α_H as a "seed".

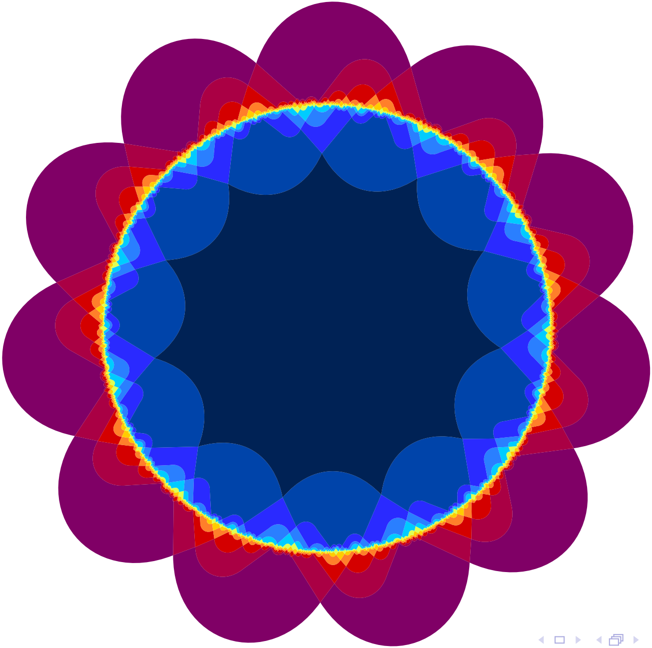
And restricting the search to consider only iterated crosscuts, yields an 11-Venn diagram.

$$\alpha_E = \overbrace{{\color{red}3}, {\color{red}2},}^{\rho_2} \overbrace{{\color{red}4}, {\color{red}3},}^{\delta_2} \overbrace{5, 4, 3, 2,}^{\rho_3} \overbrace{4, 3, 5, 4,}^{\alpha_3} \overbrace{6, 5, 4, 3,}^{\delta_3} \overbrace{5, 4, 6, 5,}^{\alpha_3^{r+}}$$

$$\overbrace{7, 6, 5, 4, 3, 2,}^{\rho_4} \overbrace{3, 4, 3, 4, 5, 4, 5, 6, 5, 4, 3, 6, 5, 6, 5, 4, 5, 4, 7, 6, 5, 4, 6, 5, 7, 6,}^{\alpha_4}$$

$$\overbrace{8, 7, 6, 5, 4, 3,}^{\delta_4} \overbrace{7, 8, 6, 7, 5, 6, 7, 8, 5, 6, 5, 6, 7, 6, 7, 4, 5, 6, 7, 6, 5, 6, 5, 4, 5, 4.}^{\alpha_4^{r+}}$$

An iterated crosscut 11-Venn diagram (not Newroz)



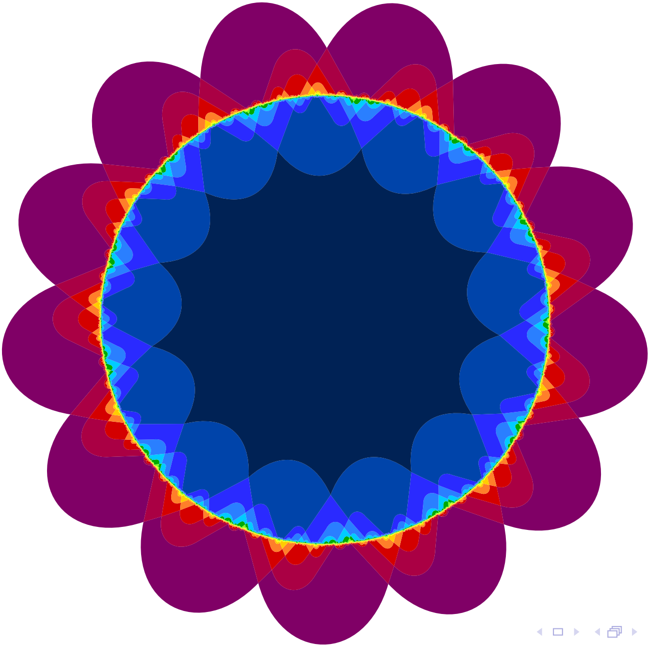
Sequence for 11, size 4: $\alpha_E =$

3, 2, 4,   3, 5, 4, 3, 2, 4,   3, 5, 4, 6, 5, 4, 3, 5,   4, 6, 5, 7, 6, 5, 4, 3, 2,   3, 4,
3, 4, 5,   4, 5, 6, 5, 4, 3,   6, 5, 6, 5, 4, 5, 4, 7,   6, 5, 4, 6, 5, 7, 6, 8, 7,   6, 5,
4, 3, 7,   8, 6, 7, 5, 6, 7,   8, 5, 6, 5, 6, 7, 6, 7,   4, 5, 6, 7, 6, 5, 6, 5, 4,   5, 4,

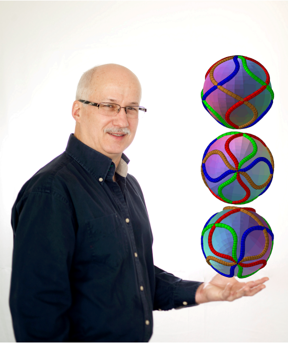
Sequence for 13, size 304: $\alpha_T =$

3, 2, 4,   3, 5, 4, 3, 2, 4,   3, 5, 4, 6, 5, 4, 3, 5,   4, 6, 5, 7, 6, 5, 4, 3, 2,   3, 4,
3, 4, 5,   4, 5, 6, 5, 4, 3,   6, 5, 6, 5, 4, 5, 4, 7,   6, 5, 4, 6, 5, 7, 6, 8, 7,   6, 5,
4, 3, 7,   8, 6, 7, 5, 6, 7,   8, 5, 6, 5, 6, 7, 6, 7,   4, 5, 6, 7, 6, 5, 6, 5, 4,   5, 4,
9, 8, 7,   6, 5, 4, 3, 2, 3,   4, 3, 4, 5, 4, 5, 6, 5,   4, 3, 5, 4, 6, 5, 4, 5, 6,   7, 6,
5, 4, 5,   6, 5, 6, 7, 6, 5,   6, 7, 6, 7, 8, 7, 6, 5,   4, 3, 5, 4, 6, 5, 7, 6, 5,   4, 6,
5, 7, 6,   8, 7, 8, 7, 6, 5,   4, 5, 6, 7, 6, 5, 4, 7,   6, 8, 7, 6, 5, 7, 6, 5, 8,   7, 6,
9, 8, 7,   6, 5, 4, 8, 7, 8,   7, 6, 7, 6, 5, 9, 8, 7,   6, 8, 7, 6, 5, 9, 8, 7, 6, 10, 9,
8, 7, 6,   5, 4, 3, 7, 8, 9, 10, 6, 7, 8, 9, 7, 8, 9, 10, 6, 7, 8, 7, 8, 9, 8, 9,   5, 6,
7, 8, 9, 10, 7, 8, 9, 6, 7,   8, 6, 7, 8, 9, 7, 8, 5,   6, 7, 8, 7, 6, 5, 6, 7, 8,   9, 8,
9, 7, 8,   6, 7, 5, 6, 7, 8,   6, 7, 5, 6, 4, 5, 6, 7,   8, 9, 8, 7, 8, 7, 6, 7, 8,   7, 6,
7, 6, 5,   6, 7, 8, 7, 6, 5,   6, 7, 5, 6, 4, 5, 6, 7,   6, 5, 6, 5, 4, 5, 4.

A simple symmetric 13-Venn diagram!



The End



Thanks for coming.
Any questions?