MATH 676

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Finite element methods in scientific computing

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Lecture 9:

A second example:

The *step-2* tutorial program
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Degrees of freedom (DoFs)

step-2

Step-2 shows:

- How degrees of freedom are defined with finite elements
- The *DoFHandler* class
- How DoFs are connected by bilinear forms
- Sparsity patterns of matrices
- How to visualize a sparsity pattern

step-2

Sparsity of system matrices:

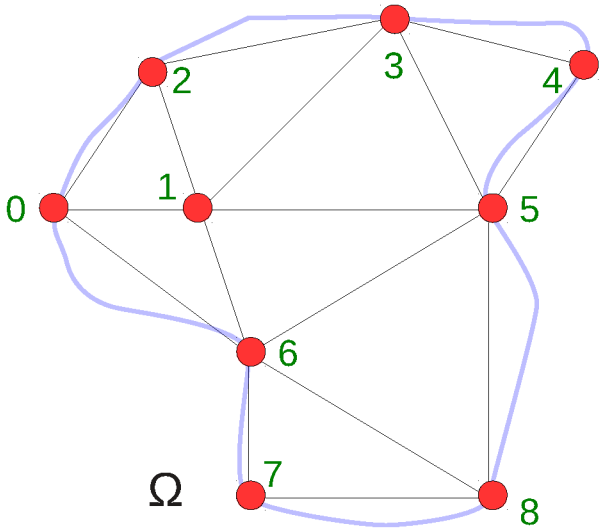
- For PDEs, finite element matrices are *always sparse*
- Result of
  – *local* definition of shape functions
  – *locality* of the differential operator

Sparsity is not a coincidence. It is a design choice of the finite element method.

Sparsity can not be overestimated as a factor in the success of the FEM!

step-2

Example: Consider this mesh and bilinear form:



$$A_{ij} = (\nabla\phi_i, \nabla\phi_j) = \int_\Omega \nabla\phi_i \cdot \nabla\phi_j \,\mathrm{d}x$$

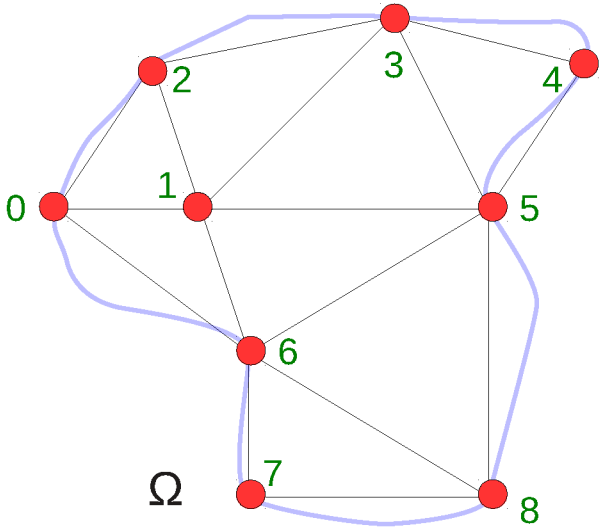
Note: In general we have that

- $A_{00} \neq 0, A_{01} \neq 0, A_{02} \neq 0, A_{06} \neq 0$
- $A_{03} = A_{04} = A_{05} = A_{07} = A_{08} = 0$

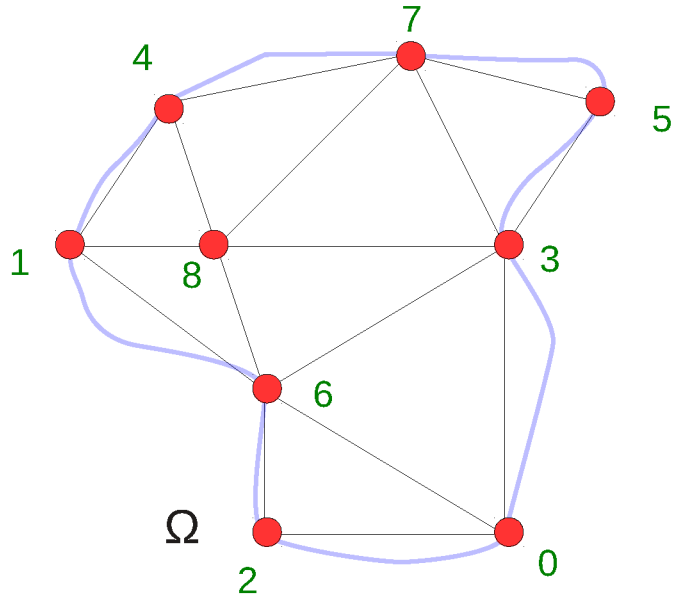
The bigger the mesh, the more zeros there are per row!

step-2

Renumbering: The order of enumerating degrees of freedom is arbitrary



vs.



Notes:

- Resulting matrices are just permutations of each other
- Both sparse, but some algorithms care

step-2

Read through the commented program at
http://www.dealii.org/7.1.0/doxygen/deal.II/step_2.html

Then play with the program:

cd examples/step-2
cmake -DDEAL_II_DIR=/a/b/c . ; make run

This will run the program and generate output files:

ls -l

Then run *gnuplot* as described in the documentation

gnuplot

Next step: Play by following the suggestions in the results section. This is the best way to learn!

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